USAAAO 2022 - First Round

February 5^{th}, 2022

1. Classify the following galaxies according the Hubble galaxies classification:

Figure 1: Galaxy 1

Figure 2: Galaxy 2

Figure 3: Galaxy 3

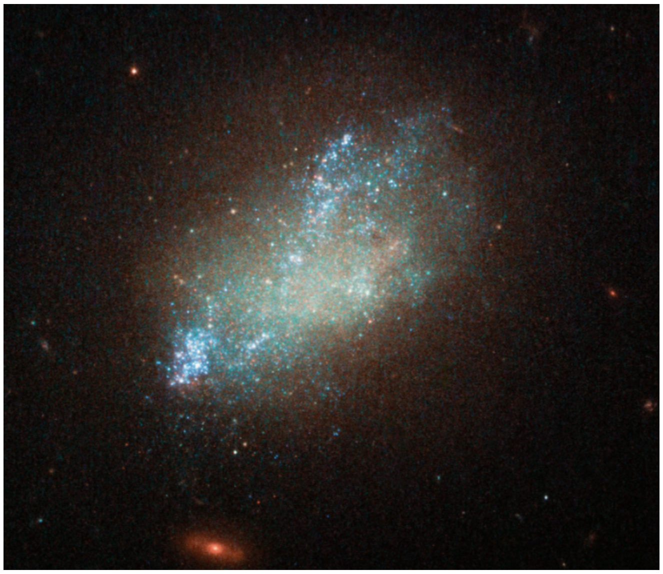


Figure 4: Galaxy 4

Figure 5: Galaxy 5

(a) Sb, Sc, Peculiar, E2, Irregular
(b) Sbc, E4, Irregular, Sb, Peculiar
(c) E3, Sbc, Sa, Peculiar, Irregular
(d) Sc,Sba, Sbc, E2, Peculiar
(e) Sa, Sbb, E3, Irregular, Peculiar

Solution:

Looking at the classification chart, we can see that the only alternative with possible correct answers would be letter e.

Answer: E

2. A comet's orbit has the following characteristics: eccentricity e = 0.995; aphelion distance $r_a = 5 \cdot 10^4 AU$. Assume we know the mass of the Sun $M_S = 1.98 \cdot 10^{30} kg$, and gravitational constant $G = 6.67 \cdot 10^{-11} Nm^2/kg^2$. Determine the velocity of the comet at its aphelion.

(a) 34.76 m/s
(b) 20.57 m/s
(c) 187.91 m/s
(d) 63.38 m/s
(e) 9.19 m/s

Solution:

$$r_a = a(1+e)$$

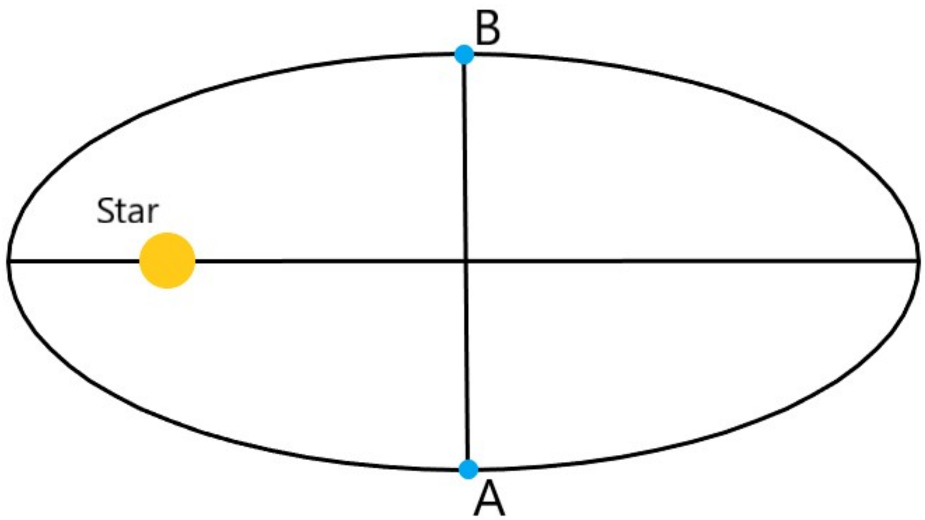
$$a = \frac{r_a}{1+e} = 2.506 \cdot 10^4 AU$$

Aphelion velocity:

$$v_a = \sqrt{GM_S\left(\frac{2}{r_a} - \frac{1}{a}\right)} = 9.19\frac{m}{s}$$

Answer: E

3. Consider the following elliptical orbit of a comet around a star:



Which of the following expressions corresponds to the time that the comet takes to go from point A to point B as a function of the period of the comet (T) and the eccentricity of the orbit (e)?

Assume that the direction of the orbit is **counterclockwise**.

(a) $\frac{T}{2}$

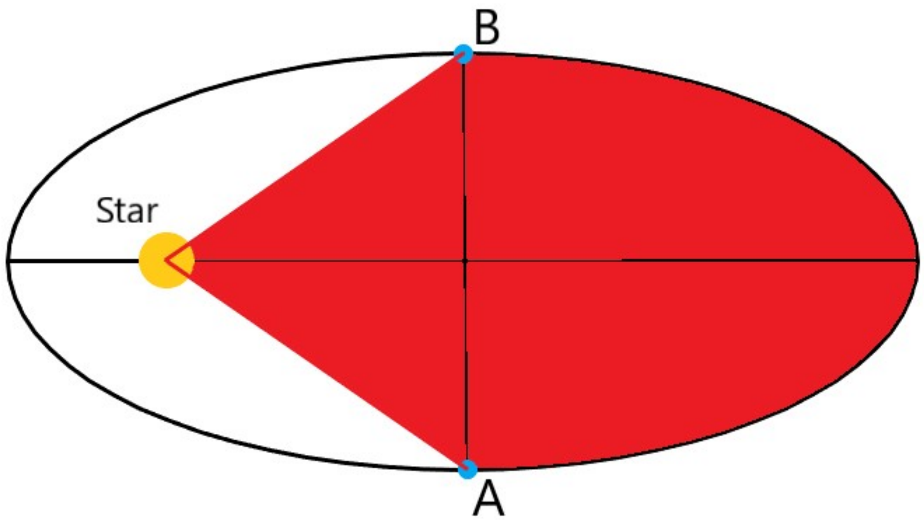
(b) $\left(\frac{e}{\pi} + \frac{1}{2}\right) * T$

(c) $\left(\frac{1}{2} - \frac{e}{\pi}\right) * T$

(d) $(1+e) * \frac{T}{2}$

(e) $\frac{T * e}{2}$

Solution:
When the comet goes from point A to point B, it sweeps out the area marked in red in the the following figure:



Using Kepler's Second Law:

$$\frac{\Delta t}{T} = \frac{A_{swept}}{A_{total}}$$

$$\frac{\Delta t}{T} = \frac{2 * \frac{aeb}{2} + \frac{\pi ab}{2}}{\pi ab}$$

$$\frac{\Delta t}{T} = \frac{aeb + \frac{\pi ab}{2}}{\pi ab}$$

$$\frac{\Delta t}{T} = \frac{e}{\pi} + \frac{1}{2}$$

$$\Delta t = \left(\frac{e}{\pi} + \frac{1}{2}\right) * T$$

Note: In the calculations, the semi-major axis was denoted by a and the semi-minor axis was denoted by b.

Answer: B

4. What is the shortest distance (along the surface of the Earth) between two points on the Equator separated by 30° of longitude? What is the shortest distance (along the surface of the Earth) between them if both the two points lie on the 60° latitude while still separated by 30° of longitude? (For simplicity, assume that the Earth is a sphere)

   (a) 3336 km, 1668 km
   (b) 3336 km, 1654 km
   (c) 6672 km, 3336 km
   (d) 3298 km, 1649 km
   (e) 3298 km, 1668 km

Solution: We know the radius of the Earth from the formula sheet, $R_\oplus = 6.371 \times 10^6$ m. For the two points on the Equator, we can easily calculate the shortest distance between them as

$$d_{\text{equator}} = R\theta \tag{1}$$

$$d_{\text{equator}} = R_\oplus \frac{\pi}{6} \tag{2}$$

$$d_{\text{equator}} \approx 3.3358 \times 10^6 \text{m} \approx 3336\text{km} \tag{3}$$

For the points on latitude 60°, we can calculate the distance between them, along the latitude, as $d_{60^\circ} = d_{\text{equator}} \times \cos 60^\circ = \frac{d_{\text{equator}}}{2} \approx 1668$km.

However, this naive approach is incorrect. The shortest distance between any two points is along the great circle connecting them, not along the latitude.
The Equator is the great circle connecting the two points on the Equator. However, the 60° latitude is not a great circle!

We use spherical trigonometry to determine the arc length of the great circle connecting the points. Using the cosine formula, we know

$$\cos c = \cos a \cos b + \sin a \sin b \cos C. \tag{4}$$

Here, we are considering a spherical triangle with two vertices as the discussed points and third one being the pole. So, c is the arc length of the great circle which we need to find. a and b are the arc length of points from pole which is $90^\circ - 60^\circ = 30^\circ$. C is the angle between the two longitudes, which is 30°. Substituting all the quantities, we get

$$\cos c = \frac{\sqrt{3}}{2}\frac{\sqrt{3}}{2} + \frac{1}{2}\frac{1}{2}\frac{\sqrt{3}}{2} \tag{5}$$

$$c = \arccos\left(\frac{6+\sqrt{3}}{8}\right) \approx 0.2595 \tag{6}$$

Arc length of great circle is

$$R_\oplus \times c = 6371\text{km} \times \arccos\left(\frac{6+\sqrt{3}}{8}\right) \tag{7}$$

$$= 1653.57\text{km} \tag{8}$$

Answer: B

5. Two (spherical) asteroids, Ek and Do, are orbiting in free space around their stationary center of mass. Ek has mass $7M_{☾}$ and Do has mass $1.4M_{☾}$, where $M_{☾}$ is the mass of moon. What is the ratio of the angular momentum of the whole system to the angular momentum of Do about the center of mass of the system?

   (a) 26
   (b) 6
   (c) 1.2
   (d) 1.04
   (e) 0.1667

Solution:

If the distance between the asteroids is $6R$, then Ek would be R distance from center of mass and Do would be at $5R$ from center of mass. If Ek is orbiting with velocity v. The orbital velocity of Do (as center of mass is stationary) would be $5v$. So, total angular momentum of system is L_{total}, which is given by

$$\begin{aligned} L_{\text{total}} &= m_1 v_1 r_1 + m_2 v_2 r_2 && (9) \\ &= 7Mvd + 1.4M \times 5v \times 5d && (10) \\ &= 42Mvd && (11) \end{aligned}$$

While the angular momentum of Do is

$$\begin{aligned} L_2 &= m_2 v_2 r_2 && (12) \\ &= 35Mvd && (13) \end{aligned}$$

So the required ratio is $\frac{L_{\text{total}}}{L_2} = \frac{42}{35} = 1.2$.

Answer: C

6. Consider a f/9 telescope with focal length $f = 1.0$ m that operates at visible wavelength $\lambda = 5000$ Å. What is the farthest distance at which an open cluster of radius $R_C = 4.1$ pc can be resolved by this telescope?

   (a) 1.2×10^6 pc
   (b) 1.5×10^6 pc
   (c) 3.0×10^6 pc
   (d) 4.2×10^6 pc
   (e) 5.8×10^6 pc

Solution:

First, note that the angular diameter of the cluster is

$$\theta = \frac{2R_C}{d}$$

where d is the distance to it.

Equating this angular diameter to the resolution limit of the telescope, we find that:

$$1.22\frac{\lambda}{D} = \frac{2R_C}{d}$$

$$d = \frac{2R_C D}{1.22\lambda}$$

$$= \frac{2R_C f}{9 * 1.22\lambda}$$

$$\approx 1.5 \times 10^6 \text{ pc}$$

Answer: B

7. Imagine the you observe transits of earth across the sun from a far away exoplanet. Assuming earth's orbit has 0 eccentricity and it transits directly across the sun's diameter (the impact parameter is 0), what is the duration of earth's transit?

   (a) 3.24 hrs
   (b) 25.93 hrs
   (c) 6.48 hrs
   (d) 1.62 hrs
   (e) 12.97 hrs

Solution:

Assuming a circular orbit, the earth has velocity $2\pi 1\text{AU}/1\text{yr}$. If the observer is far enough away, then the earth must travel a distance of $2R_\odot$. This will take a total time of $t = d/v =$ 12.97 hrs

Answer: E

8. An exoplanet was observed during its transit across the surface of a bright star. Estimate the variation of the apparent magnitude (Δm) of the star caused by exoplanet's transit. During the transit, assume an Earth-based astronomer observes that the area covered by the exoplanet on the projected surface of the star represents $\eta = 2\%$ of the star's projected surface.

   (a) -4.247
   (b) 0.003
   (c) 0.022
   (d) 0.679
   (e) -0.003

Solution:

Using Pogson's formula for apparent magnitudes:

$$\log \frac{F}{F_0} = -0.4(m - m_0)$$

$$F = (1 - \eta)F_0$$

$$\log(1 - \eta) = -0.4\Delta m$$

$$\Delta m = -2.5 \log(0.98)$$

$$\Delta m = 0.022$$

Answer: C

9. Estimate the mass of a globular cluster with a radial velocity dispersion $\sigma_r = 16.2$ km/s. The cluster has an angular diameter of $\theta = 3.56'$ and is a distance $d = 9630$ pc away from us.

(a) 6.05×10^{35} kg
(b) 9.71×10^{35} kg
(c) 1.01×10^{36} kg
(d) 3.03×10^{36} kg
(e) 5.96×10^{36} kg

Solution:

In order to estimate the mass of the globular cluster, we use the Virial theorem, which relates the time-averaged values of the kinetic and potential energy of a gravitationally bound system:

$$\langle T \rangle = -\frac{1}{2}\langle U \rangle$$

Note that $\sigma^2 = \sigma_x^2 + \sigma_y^2 + \sigma_z^2$ and $\sigma_r^2 = \sigma_x^2 = \sigma_y^2 = \sigma_z^2$ (assuming velocity dispersions are randomly distributed in a globular cluster). Thus, $\sigma^2 = 3\sigma_r^2$.

Additionally, we use the expression

$$\langle U \rangle = -\frac{3}{5}\frac{GM^2}{R}$$

for the gravitational potential energy of a uniform sphere.

Then,

$$\frac{1}{2}M\sigma^2 = -\frac{1}{2}\left(-\frac{3}{5}\frac{GM^2}{R}\right)$$

$$\frac{1}{2}M\left(3\sigma_r^2\right) = \frac{3}{10}\frac{GM^2}{R}$$

We also note that:

$$R = \frac{D}{2} = \frac{d\theta}{2}$$

,

where *theta* needs to be converted from arcminutes to radians before being substituted.

Solving for M, replacing R with the expression from above, and substituting numerical values, we find that:

$$M = \frac{5\sigma_r^2 R}{G}$$

$$= \frac{5\sigma_r^2 d\theta}{2G}$$

$$\approx 3.03 \times 10^{36} \text{ kg}$$

Answer: D

10. Jupiter's deep atmosphere is very warm due to convection leading to an adiabatic temperature profile that increases with increasing pressure. Assuming (for simplicity) that this outer layer of Jupiter has a temperature of 500 K, perform a back-of-the-envelope estimate of the characteristic thickness (or e-folding scale) of the envelope of Jupiter (you may find that this is independent of pressure level). You may further use that the specific gas constant in Jupiter's atmosphere is 3600 J kg^{-1} K^{-1}.

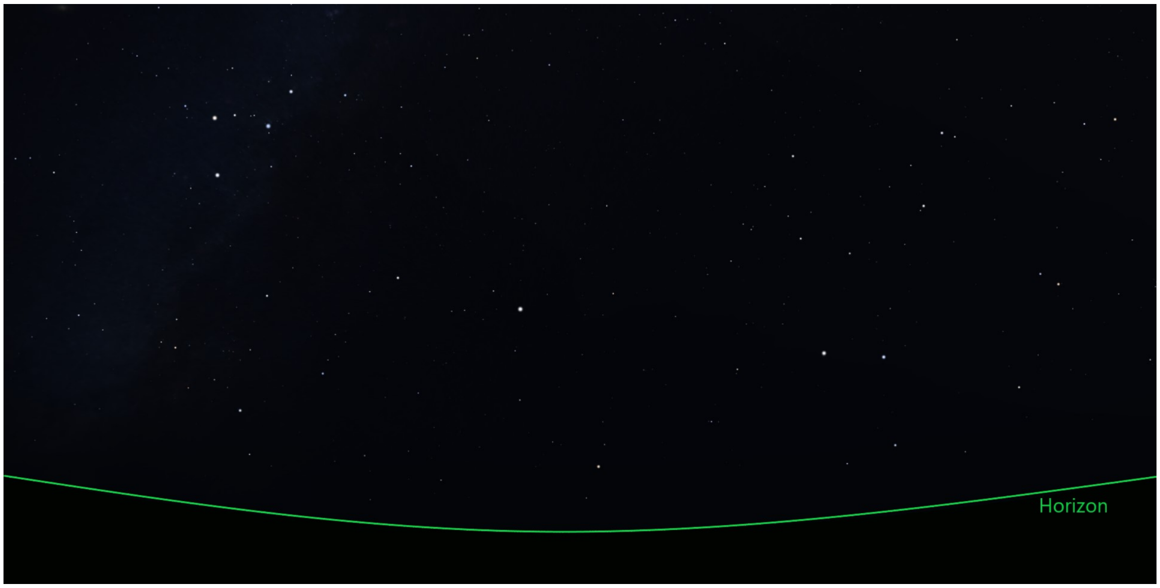
    (a) 20 km
    (b) 73 km
    (c) 568 km
    (d) 3,120 km
    (e) 10,233 km

Solution:

From hydrostatic equilibrium, $p = \rho g H$.
From the ideal gas law, $\rho = p/RT$
Solving for H, we find:
$H = \frac{p}{\rho g} = \frac{pRT}{pg} = \frac{RT}{g}$
A more accurate answer would integrate hydrostatic equilibrium to determine the e-folding length scale (H), which is equivalent to the derived H in the isothermal limit.
Plug in for R, T, and $g = GM/r^2$ to find:
$H = 73km$

Answer: B

11. An astronomer took the following picture while observing the night sky:



What is the latitude of the place where the astronomer took the picture?

(a) 70° S

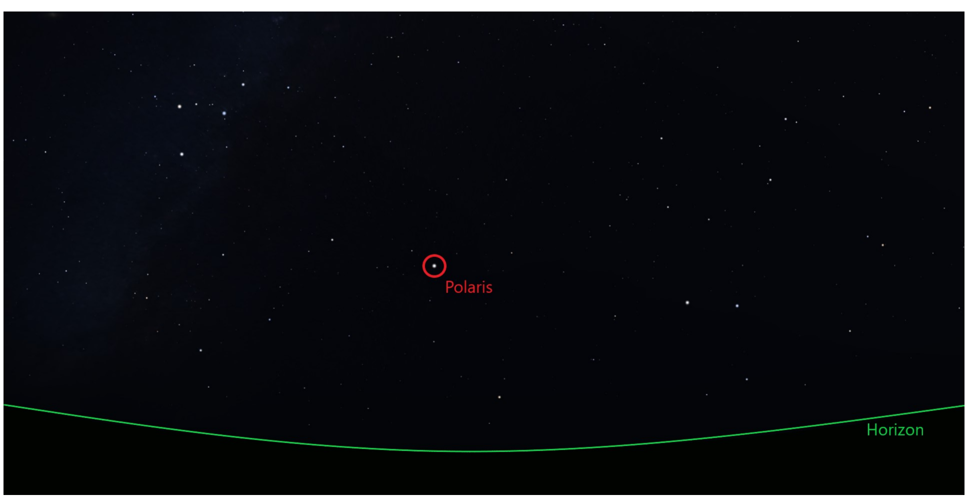
(b) 20° S

(c) 2° N

(d) 20° N

(e) 70° N

Solution:



The latitude of the observer corresponds to the altitude of the celestial pole above the horizon. The North Celestial Pole is very close to Polaris (α UMi).

Since the North Celestial Pole is above the horizon, the observer is in the Northern hemisphere.

Polaris is clearly much higher than 2° above the horizon, but lower than 70°, so the only possible answer is 20° N.

Answer: D

12. Order the following phases of the Sun's evolution from first to last chronologically.

    1. Helium flash
    2. White dwarf
    3. Red giant branch
    4. Asymptotic giant branch
    5. End of hydrogen fusion in the core

    (a) 5, 4, 1, 3, 2
    (b) 5, 3, 1, 4, 2
    (c) 1, 5, 3, 4, 2
    (d) 5, 2, 4, 1, 3
    (e) 3, 5, 1, 4, 2

Solution:

The Sun's post-main-sequence evolution will begin when nuclear fusion exhausts the hydrogen in the core and a helium core forms. The Sun will then expand into a red giant. Eventually, the temperature of the degenerate helium core will become high enough to initiate triple-alpha helium fusion, leading to a runaway chain reaction in which much of the helium core is rapidly converted into carbon. This is known as the helium flash. After the helium flash, the Sun will enter the horizontal branch and then the asymptotic giant branch once the helium in the core has been exhausted. Finally, the Sun will shed off its outer layers in a planetary nebula and the remaining exposed core will become a white dwarf.

Answer: B

13. The orbit of some planet to its star has an eccentricity of 0.086. What is the ratio of the planet's closest distance to its star to the farthest on its orbit?

    (a) 0.842
    (b) 0.188
    (c) 1.188
    (d) 0.158
    (e) None of the above

Solution:

The ratio of the planet's closest distance to its star to the farthest on its orbit is

$$\frac{1-e}{1+e}.$$

$$\frac{1-0.086}{1+0.086} = 0.842.$$

Answer: A

14. Figure 6 shows a 6-hr root-mean-square (rms) Combined Differential Photometric Precision (CDPP) curve for 150,000 stars observed by the *Kepler* space telescope. CDPP is a measure of the white noise contained in a light curve, so for a target with 6-hour CDPP of 100 parts per million (ppm), a 6-hour transit with depth 100 ppm would be considered a 1-σ detection.

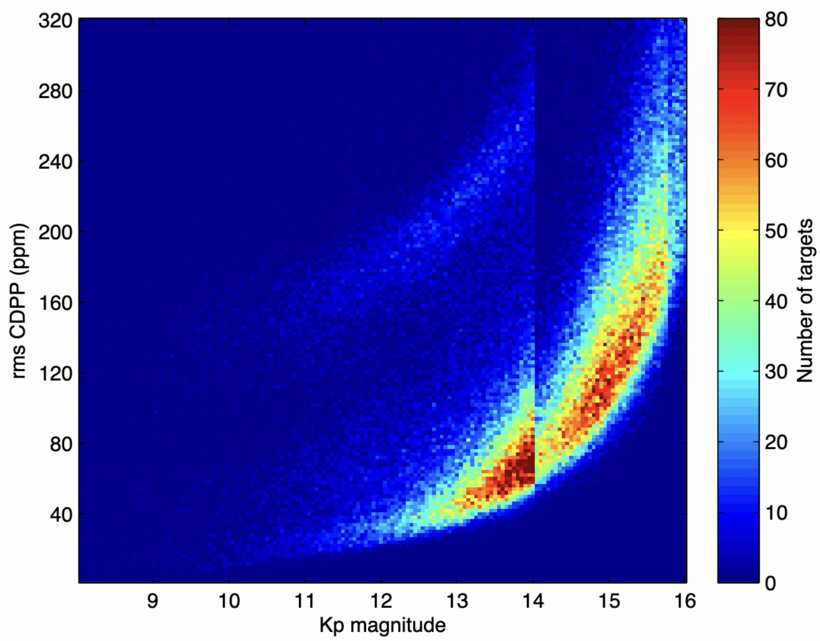


Figure 6: From Christiansen et al. (https://arxiv.org/abs/1208.0595). Original caption: The distribution of the 6-hour rms CDPP values with Kp magnitude for all Quarter 3 planetary targets.

Consider a $1R_\odot$ target with Kepler magnitude $K_p = 13.5$ that's among the best targets for its magnitude with respect to noise in Figure 6. Also consider three independent exoplanet scenarios for exoplanets with radii:

- I. 0.5 $R_\oplus$
- II. 1 $R_\oplus$
- III. 10 $R_\oplus$

Using a $1-\sigma$ detection threshold (and assuming 6-hour transit durations), which planet(s) transits would we likely **fail** to observe due to noise?

(a) I

(b) III

(c) I and II

(d) II and III

(e) I, II, and III

Solution:

If the target has a $K_p = 13.5$, it's 6 hour CDPP is $40ppm$ according to Figure 6. This means we'll only be able to detect transits deeper than $40ppm$ (assuming we're using a 1 sigma threshold, which is a fairly low threshold).

The planets have expected depths of:

- $(0.5R_{\oplus}/1R_{\odot})^2 = 21$ppm
- $(1R_{\oplus}/1R_{\odot})^2 \approx 80$ppm
- $(10R_{\oplus}/1R_{\odot})^2 \approx 8000$ppm

21 is less than 40, so we likely wouldn't be able to detect planet I due to the noise in the light curve.

Note that there are many reasons we could fail to detect an exoplanet: e.g we didn't catch enough transits, the exoplanet isn't perfectly edge on so the depth and duration are particularly low, the field is particularly crowded, etc. In other words, we can't *guarantee* that we'll detect planets II and III, but we do know that planet I will fall below our noise floor.

Answer: A

15. A star with mass M goes through an energy generating nuclear reaction $4\ {}^1H \rightarrow\ {}^4He + Energy$. Here, the burning efficiency of the p-p(proton-proton) chain is 0.007, meaning that each mass m yields $0.007mc^2$ of energy. Assuming that the has a total available hydrogen mass for nuclear reaction amounts to half of its original mass, and the luminosity(L) stays constant throughout the burning phase, get an expression of the hydrogen burning lifetime of the star.

(a) $1.625 \times 10^{18} s(\frac{M}{M_{\odot}})^{-2}$

(b) $3.15 \times 10^{14} s(\frac{M}{M_{\odot}})^{-2}$

(c) $1.625 \times 10^{18} s(\frac{M}{M_{\odot}})^{2}$

(d) $3.15 \times 10^{14} s(\frac{M}{M_{\odot}})^{2}$

(e) None of the above

Solution:

Total amount of energy available for burning is

$$0.007(0.5M)c^2.$$

Since L remains constant, the burning lifetime is

$$\frac{0.0035Mc^2}{L}$$

Now using the mass-luminosity relationship

$$\frac{L}{L_\odot} = (\frac{M}{M_\odot})^3$$

we obtain

$$\frac{0.0035Mc^2}{L} = 0.0035c^2 \frac{(M_\odot)^3}{L_\odot M^2}$$

$$= 0.0035c^2(\frac{M_\odot}{L_\odot})(\frac{M}{M_\odot})^{-2}$$

$$= 1.625 \times 10^{18} s(\frac{M}{M_\odot})^{-2}$$

Answer: A

16. In 2025, the Parker Solar Probe will pass just 6.9×10^6 km from the Sun, becoming the closest man-made object to the Sun in history. It will make five orbits, passing close to the Sun once every 89 days, before the planned end of the mission in 2026. How fast will the Parker Solar Probe be traveling at its closest approach to the Sun?

(a) 38 km/s

(b) 48 km/s

(c) 139 km/s

(d) 190 km/s

(e) 196 km/s

Solution:

Knowing that the orbital period of the probe is 89 days, we can use Kepler's Third Law to solve for the semi-major axis of the orbit:

$$T^2 = \frac{4\pi^2}{GM_{\text{Sun}}} a^3$$

$$a = \left(\frac{GM_{\text{Sun}}T^2}{4\pi^2}\right)^{1/3} = 5.8 \times 10^7 \text{ km}$$

To find the velocity of the probe at its closest passage to the Sun, we can apply the vis-viva equation:

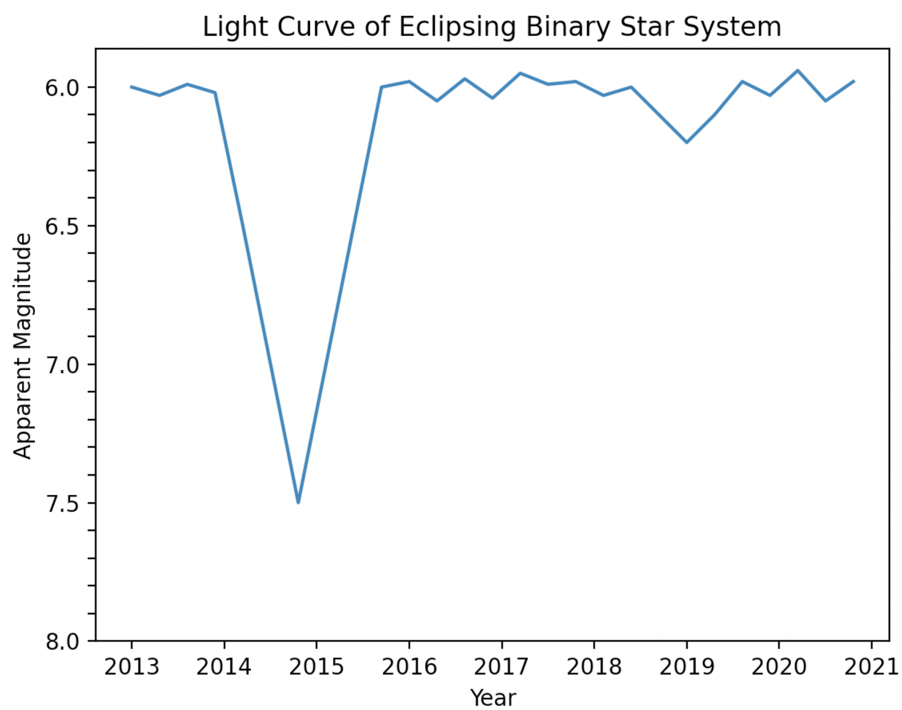
$$v^2 = GM_{\text{sun}} \left(\frac{2}{r} - \frac{1}{a} \right)$$

$$v = 190 \text{ km/s}$$

Here a is the semi-major axis, and r is the distance from the Sun given in the problem.

Answer: D

17. An astronomer observes an eclipsing binary star system from Earth, and he plots the following light curve.



Suppose that both stars have circular orbits and the distance between the stars is 14.8 AU. What is the total mass of the binary star system in terms of solar masses?

(a) $2.3M_{\odot}$
(b) $5.7M_{\odot}$
(c) $6.8M_{\odot}$
(d) $23M_{\odot}$
(e) $46M_{\odot}$

Solution:

The time difference between the primary and secondary eclipses is 4.2 years. These eclipses when one star covers the other. Thus, we observe that the period of orbit is twice this, or 8.4 years. The semimajor axis is 14.8 AU, so by Kepler's Third Law, we have

$$T^2 = \frac{a^3}{M} \implies M = \frac{a^3}{T^2} = \frac{14.8^3}{8.4^2} = 46\, M_\odot.$$

Answer: E

18. Assume that the smaller star in the above binary star system is brighter than the larger star. What is the ratio of the radius of the smaller star to the radius of the larger star?

(a) 0.21

(b) 0.76

(c) 0.82

(d) 0.95

(e) 0.98

Solution:

If the smaller star is brighter, then the primary eclipse occurs when the smaller star is eclipsed, and the secondary eclipse occurs when the larger star is eclipsed. Let A and B denote the fractions of the brightness of the total system from the smaller and larger stars, respectively. Note that as defined $A + B = 1$. The increase in apparent magnitude during the primary eclipse is 1.5, so we have

$$B = 10^{-0.4\Delta m_1} = 0.25.$$

Thus, $A = 0.75$. The increase in apparent magnitude during the secondary eclipse is 0.2. Let k be the requested ratio of radii. Note that $(1 - k^2)$ of the larger star is visible during the secondary eclipse, so

$$B(1 - k^2) + A = 10^{-0.4\Delta m_2} = 0.83.$$

This implies $1 - k^2 = 0.32$, and $k = 0.82$.

Answer: C

19. The resolution of a space telescope is theoretically limited by diffraction from its primary mirror. In this problem, we will compare the diffraction limit of the Hubble Space Telescope (HST) (primary mirror diameter $d = 2.4\,\text{m}$) and the James Webb Space Telescope (JWST) ($d = 6.5\,\text{m}$). The operating wavelengths for the two telescopes are 500 nm and 10 µm respectively. Calculate the ratio of the diffraction limited angular resolution $\frac{\theta(\text{HST})}{\theta(\text{JWST})}$. Which telescope can resolve smaller angular features if limited only by diffraction?

(a) 0.014, JWST

(b) 0.14, HST
(c) 1.4, JWST
(d) 14, HST
(e) 140, JWST

Solution:
The diffraction limit is given as $\theta = \frac{1.22\lambda}{d}$. Using the given values, we get the answer as $\frac{\theta(\text{HST})}{\theta(\text{JWST})} \approx 0.14$. Hence, HST can resolve smaller angular features.

Answer: B

20. The eccentricity of Pluto's orbit is 0.25. Estimate the maximum change in magnitude of Pluto as seen from Earth in one orbit of Pluto. You may assume that the semi-major axis of Pluto's orbit is much greater than 1 A.U.

(a) 0.2
(b) 1.2
(c) 2.2
(d) 3.2
(e) 4.2

Solution: The ratio of aphelion to perihelion distance for an elliptical orbit is $\frac{1+e}{1-e}$ where e is the eccentricity of the orbit. Since, Pluto reflects light from the Sun, the reflected flux received at Earth scales as $\frac{1}{d_{SP}^2}\frac{1}{d_{EP}^2}$ where d_{SP} is the Sun Pluto distance and d_{EP} is the Earth - Pluto distance. Since the semi-major axis of Pluto is much larger than 1 A.U., we can approximate $d_{SP} \approx d_{EP}$.

The required change in magnitude is $-2.5 \log \left(\frac{1-e}{1+e}\right)^4 = 2.2$.

Answer: C

21. A satellite is in a circular, equatorial orbit, and can fire its engines to accelerate in any of the following directions:

1. In the direction of motion
2. Against the direction of motion
3. Towards Earth, perpendicular to direction of motion (against radial vector)
4. Away from Earth, perpendicular to direction of motion (along radial vector)
5. Towards the North Celestial Pole (perpendicular to both direction of motion and radial vector)

Consider a small change of velocity in each of these directions. For how many of these maneuvers will the perigee of the orbit decrease?

(a) 1
(b) 2
(c) 3
(d) 4
(e) 5

Solution:

Applying the vis-viva equation, a burn in the direction of motion increases instantaneous speed, increasing a. As the spacecraft still travels in the same direction, its vertical speed is 0 at this instant; therefore the current point is the perigee of the new orbit, and the perigee stays the same.

Similarly, a burn opposite the direction of motion decreases instantaneous speed, decreasing a, meaning that the perigee decreases.

The other four burns are perpendicular to the direction of motion, so increase the satellite's speed, though only slightly. For the burns towards/away from Earth, vertical speed is nonzero at this point. Therefore, after a short time (or a short time ago), the spacecraft is (or was) at a lower position, which means perigee is lower.

For the last burn, they are perpendicular to the radial vector, so vertical speed remains 0 at this point. As the satellite's speed has slightly increased, the same logic as a small prograde burn applies, meaning perigee stays the same.

Answer: C

22. For the five maneuvers described above, rank the resulting apogees from lowest to highest. Assume the change in velocity is small relative to orbital velocity, but not negligible.

(a) $2 < 3 = 4 = 5 < 1$
(b) $2 = 3 < 5 < 4 = 1$
(c) $2 < 3 = 4 < 5 < 1$
(d) $2 < 5 < 3 = 4 < 1$
(e) $2 < 3 < 5 < 4 < 1$

Solution:

The burn against the direction of motion decreases the semi-major axis, while keeping the apogee constant. The radial burns and burn towards the NCP increase semi-major axis, though only slightly. The burn towards the direction of motion increases the semi-major axis by the greatest amount, and keeps the perigee the same, increasing the apogee by the greatest amount.

Now to distinguish the radial and NCP burns, note that the radial burns also slightly decrease the perigee, while the burn towards the NCP keeps the perigee constant. As the semi-major axes of the resulting orbits are constant, the radial burns therefore lead to a slightly higher apogee.

Answer: D

23. An exoplanet discovered by the radial velocity method is found to have an orbital period of 2.45 days around a Sun-like star. Assuming the planet has zero albedo (i.e., absorbs all incoming starlight) and perfectly transports heat across its surface, estimate the temperature at the photosphere of the planet.

(a) 395 K
(b) 954 K
(c) 1231 K
(d) 1476 K
(e) 2071 K

Solution:

This is the equilibrium temperature assuming zero albedo and full redistribution.
$L_\star/(4\pi d^2)\pi R_p^2 = 4\pi R_p^2 \sigma T_p^4$.
First, solve for d using Kepler's third law, assuming the planet's mass is much smaller than the Sun:
$d(au)^3 = T(year)^2$
$d = 0.036AU$.
Solve for T_p in the above expression:
$T_p = \left(\frac{L}{16\sigma\pi d^2}\right)^{1/4}$.
Plug in for d and $L = 3.827 \times 10^{26}$ Watts to find the temperature:
$T_p = 1476K$

Answer: D

24. Deneb is a very important star in the Northern hemisphere as it is one of the three stars in the Summer Triangle. Deneb (α Cyg) is also the brightest star in the Cygnus constellation. Knowing the following information calculate the distance between Deneb and Albireo (β Cyg).

	Deneb	Albireo
Parallax (π)	2.29 mas	7.51 mas
Declination (δ)	45 ° 17'	27 ° 57'
Right ascension (α)	20h 41min	19h 31min

(a) 569 pc
(b) 102 pc
(c) 432 pc
(d) 317 pc
(e) 459 pc

Solution:

First, we must calculate the angular distance between the 2 stars using the spherical law of cosines:

$$\cos\theta = \sin\delta_D \sin\delta_A + \cos\delta_D \cos\delta_A \cos\Delta\alpha$$

From what we find $\theta = 22.2°$. Then using that the distance to a star is $1/\pi$ and the law of cosines, we can find the distance between the stars:

$$d = \sqrt{\frac{1}{\pi_A^2} + \frac{1}{\pi_D^2} - 2\frac{1}{\pi_A}\frac{1}{\pi_D}\cos\theta}$$

Finally we find that the distance between them is $d = 317pc$.

Answer: D

25. Suppose you are in Houston ($29°46'N$ $95°23'W$) on the fall equinox and you just observed Deneb culminating (upper culmination). Knowing the data in the table of exercise 24, what is the hour angle of the Sun?

(a) 8h41min
(b) 20h41min
(c) 12h00min
(d) 14h19min
(e) 18h22min

Solution:

Using Deneb's right ascension and hour angle (0h, since the star is culminating), we can find the local sidereal time:

$$LST = H_D + \alpha_D = 20h41min$$

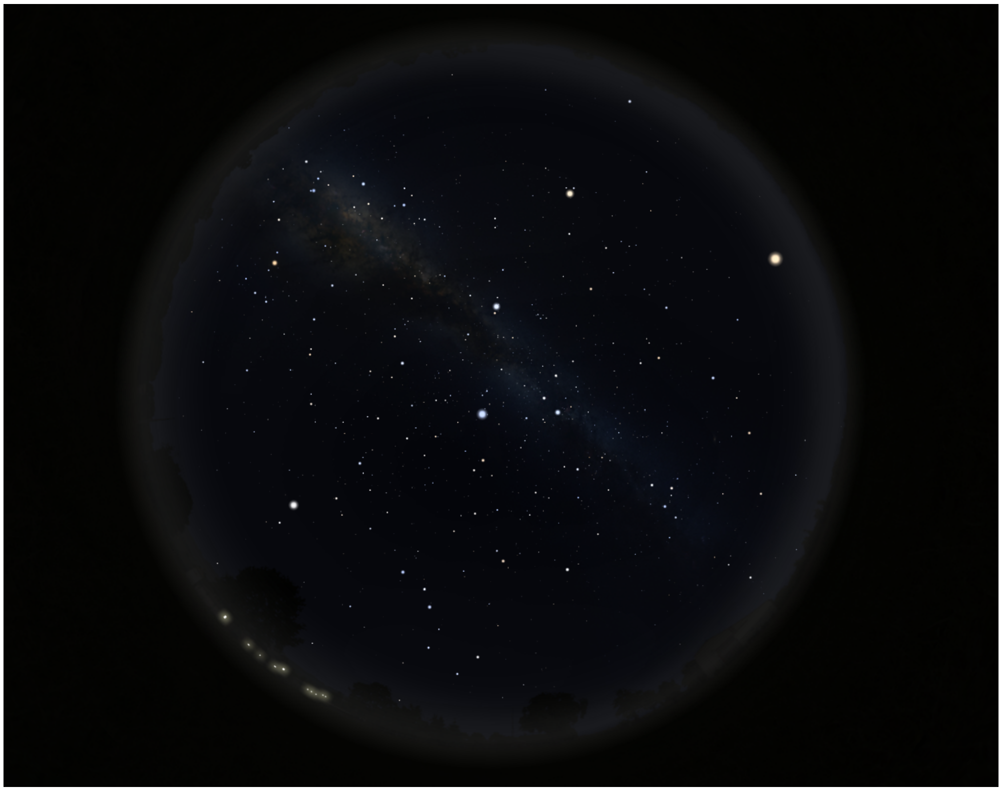
Using the same equation for the Sun, that has $\alpha = 12h$:

$$LST = H_\odot + \alpha_\odot$$

We then obtain that $H_\odot$ = 8h41min

Answer: A

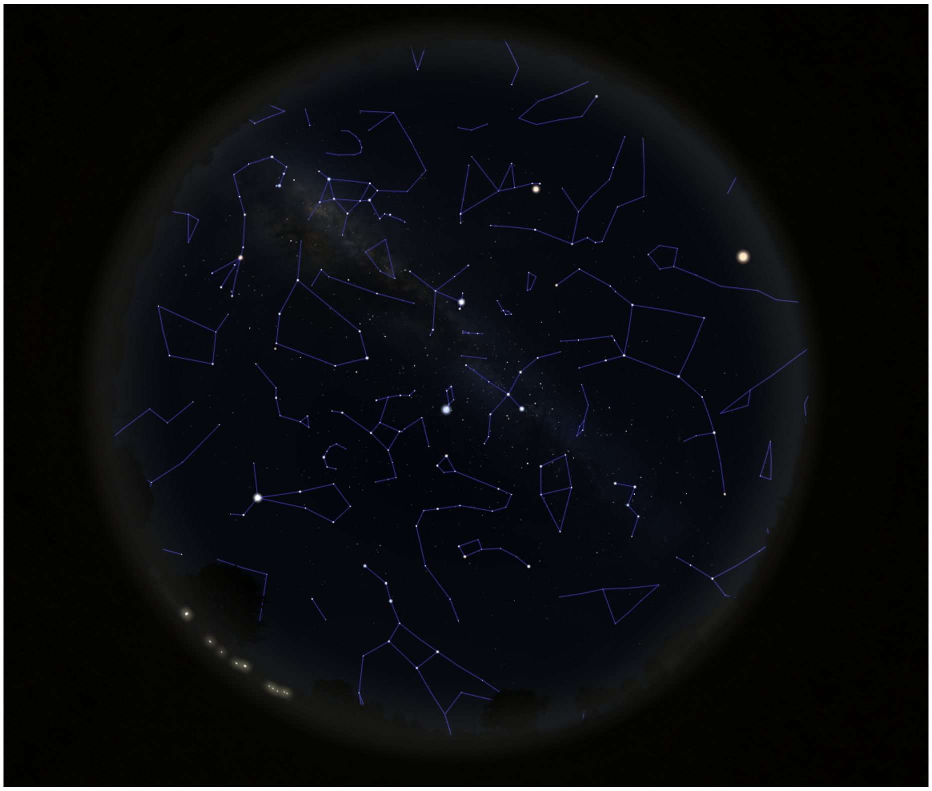
26. Knowing that the following image was taken at at 11:59pm, determine the name of which constellation was the sun passing in front of in that same day.



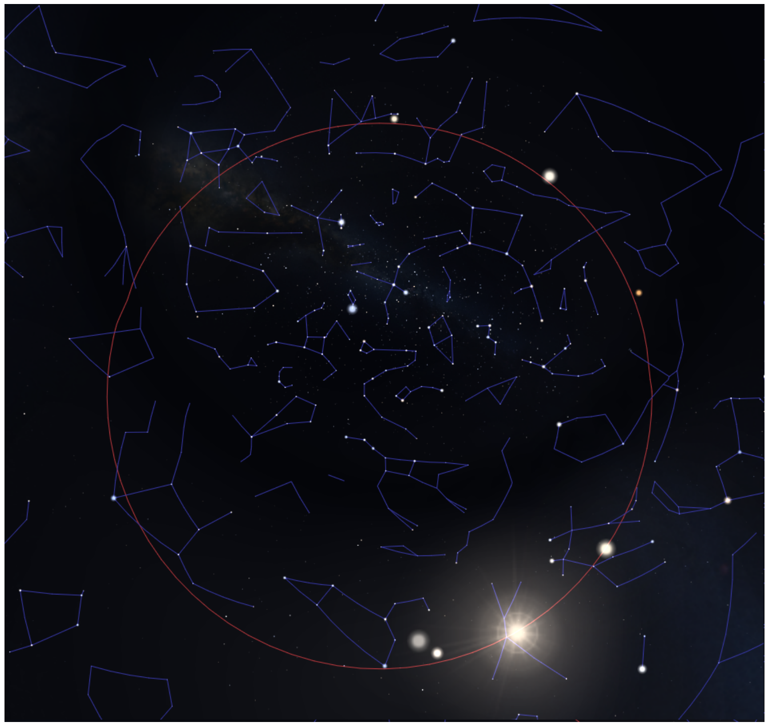
(a) Scorpius
(b) Virgo
(c) Big Dipper
(d) Cancer
(e) Taurus

Solution:

Looking at the constellations present in the image, we see that Scorpius, Sagittarius, Libra, Capricornus, Aquarius, and Pisces are above the horizon. Another important detail is that the constellations on the ecliptic that are closer to the center of the image are both Capricornus and Sagittarius.



Now looking at the ecliptic constellations, we see that that only opposite constellations to Capricornus and Sagittarius is either Cancer or Gemini. As Gemini is not in the options, Cancer would be the right answer.



Answer: D

27. An astronomer observes a galaxy in very foggy weather. So far, she has an image of signal-to-noise ratio of approximately 1, imaging for about 5 seconds. if she wants to reach a signal-to-noise ratio of 10. How long, in total, must she observe the galaxy for?

(a) 15 seconds

(b) 20 seconds

(c) 25 seconds

(d) 50 seconds

(e) 500 seconds

Solution:

Signal-to-noise ratio scales with the square root of time, so she would need 100 times more time to reach a 10 times greater signal-to-noise ratio. Hence, the answer is 500 seconds.

Answer: E

28. Dubhe (declination $\delta = 61.75^\circ$) is a star in the constellation of Ursa Major. Is it circumpolar from the city of San Francisco (latitude $\lambda = 37.7^\circ$ N)? How about from the city of Miami (latitude $\lambda = 25.8^\circ$ N)?

(a) Yes, Yes
(b) Yes, No
(c) No, Yes
(d) No, No
(e) Need more information

Solution:

A star is circumpolar in the northern hemisphere if $\delta > 90 - \lambda$.

Answer: B

29. An astronomer takes a spectrum of a galaxy and observes that the hydrogen-alpha emission line is at a wavelength of 721.9 nanometers. In a laboratory on Earth, this same emission line is observed at a wavelength of 656.3 nanometers. Approximately what is the (proper) distance to this galaxy?

(a) 66 Mpc
(b) 430 Mpc
(c) 480 Mpc
(d) 3900 Mpc
(e) 4700 Mpc

Solution:

The redshift of this galaxy is

$$z = \frac{\lambda_{obs} - \lambda_{rest}}{\lambda_{rest}} = 0.10$$

According to Hubble's law, for small redshifts the proper distance to a galaxy is related to its redshift by

$$cz \approx H_0 D$$

where H_0 is the Hubble constant, 70 (km/s)/Mpc. Therefore, $D \approx 430$ Mpc.

Answer: B

30. What is the time difference between the longest day of the year and the shortest day of the year in San Francisco (37.7° N, 122.4° W)?

Neglect atmospheric refraction.

(a) 2h30min
(b) 3h32min

(c) 4h08min

(d) 5h12min

(e) 6h25min

Solution:

The hour angle of the Sun during sunrise and sunset is given by the following formula:

$$H = arccos(-tan(\delta)tan(\phi))$$

The negative solution corresponds to the hour angle at sunrise and the positive solution corresponds to the hour angle at sunset.

In the Northern Hemisphere, the longest day of the year occurs when the Sun is at its maximum declination. In that case:

$$H = arccos(-tan(23.45^\circ)tan(37.7^\circ))$$

$$H = \pm 7h18min$$

Since the hour angle of the Sun has a variation of 24 hours in a day, the amount of daylight hours will be the difference between the hour angles:

$$\Delta H_1 = 7h18min - (-7h18min)$$

$$\Delta H_1 = 14h36min$$

In the shortest day of the year corresponds to the minimum declination of the Sun:

$$H = arccos(-tan(-23.45^\circ)tan(37.7^\circ))$$

$$H = \pm 4h42min$$

The amount of daylight hours in that case will be the following:

$$\Delta H_2 = 4h42min - (-4h42min)$$

$$\Delta H_2 = 9h24min$$

The difference in the amount of daylight hours:

$$\Delta t = \Delta H_1 - \Delta H_2$$

$$\Delta t = 14h36min - 9h24min$$

$$\Delta t = 5h12min$$

Answer: D