3 Medium Questions

1. **(30 points)** *The speed of light*

   The year is 1671 and you are astronomer Ole Rømer, measuring the period of Io's orbit around Jupiter by timing the passages of Io into or out of Jupiter's shadow.

   In December of 1671, Jupiter is at its first quadrature and you observe eclipses at the following times:

   - December 18 at 06:17:48
   - December 20 at 00:46:09
   - December 21 at 19:14:30

   In June of 1672, Jupiter is at its second quadrature and you observe eclipses at the following times:

   - June 19 at 08:42:50
   - June 21 at 03:11:30
   - June 22 at 21:40:10

   (a) **(2 points)** What is the interval between eclipses of Io as measured in December 1671, and what is it in June 1672?

   (b) **(7 points)** These orbital periods are slightly different. Rømer hypothesized that this is evidence that light has a finite speed. Explain why he thought this.

   (c) **(15 points)** Calculate the speed of light from these observations, and what you know about the orbits of Earth and Jupiter. Explain any simplifying assumptions that you make. How close is this speed of light to the correct value? (Hint: at second quadrature Earth is moving directly away from Jupiter, and at first quadrature Earth is moving directly towards Jupiter.)

   (d) **(6 points)** In 1672, Rømer did not have an accurate measurement of the distance from the Earth to the Sun. Write the speed of light *as he would have had to write it*, in terms of the unknown Earth-Sun distance a.

   (Be careful: Rømer also did not know the gravitational constant or the mass of the sun!)

2. **(30 points)** A meteorite that is radially approaching the Earth collides with a space station that revolves around the Earth in a circular orbit with radius R. For all parts of the question, express your results in terms of the mass M of the Earth, the gravitational constant G, the mass m_1 of the meteorite, and the mass m_2 of the space station.

   (a) Assume that, after the impact, the meteorite and the space station form a conglomerate that moves in a closed orbit which approaches the center of the Earth at a minimum distance $R/2$. State what the shape of the orbit of the conglomerate is and determine:

   (i) the speed of the meteorite just before the collision,

   (ii) the minimum and maximum speeds of the conglomerate,

   (iii) the maximum distance of the conglomerate from the center of the Earth.

   Determine the condition that m_1 and m_2 must obey so that the aforementioned scenario is possible.

   (b) Determine the minimum speed that the meteorite should have just before the collision so that the conglomerate moves in an open orbit after the impact. For this minimum value of the speed of the meteorite, state what the shape of the orbit of the conglomerate would be and determine

   (i) the maximum speed of the conglomerate,

   (ii) its minimum distance from the center of the Earth,

   (iii) the angle traversed by the orbital radius from the moment of the collision until the moment when the conglomerate approaches the center of the Earth to minimum distance.