USAAAO 2021 - First Round

January 30^{th}, 2021

PHYSICAL AND ASTRONOMICAL CONSTANTS

c	Speed of light in vacuum	2.998×10^8m s^{-1}
e	Elementary charge	1.602×10^{-19} C
m_n	Neutron rest mass	1.675×10^{-27} kg
m_p	Proton rest mass	1.6725×10^{-27} kg
m_e	Electron rest mass	9.110×10^{-31} kg
m_{He}	Helium-4 rest mass	6.644×10^{-27} kg
h	Planck's constant	6.626×10^{-34} J s
H_0	Hubble's constant	70 (km/s)/Mpc
k_B	Boltzmann's constant	1.381×10^{-23} J K^{-1}
b	Wien's constant	2.898×10^{-3} m K
G	Gravitational constant	6.673×10^{-11} N m^2 kg^{-2}
σ	Stefan-Boltzmann constant	5.670×10^{-8} J m^{-2} K^{-4} s^{-1}
c_1	First Radiation Constant $(= 2\pi hc^2)$	3.742×10^{-16} J m^2 s^{-1}
c_2	Second Radiation Constant $(= hc/k)$	1.439×10^{-2} m K
N_A	Avogadro constant	6.022×10^{23} mol^{-1}
R	Gas constant	8.314 J K^{-1} mol^{-1}
a_0	Bohr radius	5.292×10^{-11} m
μ_B	Bohr magneton	9.274×10^{-24} J T^{-1}
$M_\odot$	Solar mass	1.989×10^{30} kg
$R_\odot$	Solar radius	6.96×10^8 m
$L_\odot$	Solar luminosity	3.827×10^{26} J s^{-1}
$T_\odot$	Solar temperature	5770 K
$M_\oplus$	Earth mass	5.976×10^{24} kg
$R_\oplus$	Mean Earth radius	6.371×10^6 m
$I_\oplus$	Earth moment of Inertia	8.04×10^{37} kg m^2
$R_{☾}$	Mean Moon radius	1.737×10^6 m
$M_{♃}$	Mean Jupiter mass	1.9×10^{27} kg
$R_{♃}$	Mean Jupiter radius	7.1492×10^7 m
$a_{♃}$	Mean orbital radius of Jupiter	5.2 AU
$a_{☾}$	Mean semimajor axis Moon orbit	3.84399×10^8 m
1 light year		9.461×10^{15} m
1 AU	Astronomical Unit	1.496×10^{11} m
1 pc	Parsec	3.086×10^{16} m
1 year		3.156×10^7 s
1 sidereal day		86164 s
1 erg		1×10^{-7} J
1 bar		10^5 N m^{-2}

ENERGY CONVERSION : 1 joule (J) = 6.2415×10^{18} electronvolts (eV)

1. On December 21, 2020, Jupiter was at $(\alpha, \delta) = (20^h\,10^m, -20^\circ\,34')$. Which constellation was Saturn in?
   (a) Capricornus
   (b) Aquarius
   (c) Pisces
   (d) Aquila
2. What is the spectral type of a star with a luminosity of $5.86 * 10^{26}$ W and radius of $8.51 * 10^8$ m?
   (a) A
   (b) F
   (c) G
   (d) K
   (e) M
3. The exoplanet HD 209458b has a mass of 0.71 Jupiter masses and orbits HD 209458 with an orbital period of 3.53 days. HD 209458 has a mass of 1.15 Solar masses. Assuming that the orbit of HD 209458b is circular (which is a good approximation here) and that its orbit lies perfectly in our line of sight, what is the radial velocity semi-amplitude of HD 209458 due to the orbital motion of HD 209458b, in m/s?
   (a) 69.6 m/s
   (b) 85.9 m/s
   (c) 94.2 m/s
   (d) 120.8 m/s
4. The photon number density of a blackbody depends on temperature as $n_d = a\left(\frac{k_B T}{\hbar c}\right)^n$ where k_B is the Boltzman constant, $\hbar$ is the reduced Planck's constant, c is the speed of light and T is the blackbody temperature. Here, a is a dimensionless numerical constant. What is the value of n?
   (a) 1
   (b) 2
   (c) 3
   (d) 4
5. HD 209458b has a radius of 1.35 Jupiter radii, while the radius of HD 209458 is 1.20 Solar radii. What is the transit depth of HD 209458b, in percent?
   (a) 0.013%
   (b) 0.13%
   (c) 1.3%
   (d) 13%
6. Which of the following is a problem of the conventional Big Bang theory that is resolved by the theory of inflation?
   (a) Under the conventional Big Bang theory, it is extremely unlikely for our universe to be flat or nearly flat today, contrary to observation.

(b) Under the conventional Big Bang theory, it is impossible for the Cosmic Microwave Background to have come into thermal equilibrium by the time of recombination, despite its observed uniform temperature.

(c) The conventional Big Bang theory predicts a huge abundance of magnetic monopoles, while no magnetic monopoles have ever been discovered.

(d) All of the above

7. Comet C/2020 F3 (NEOWISE) last reached perihelion on July 3, 2020. Comet NEOWISE has an orbital period of ≈ 4400 years and its eccentricity is 0.99921. What is the perihelion distance of Comet NEOWISE, in AU?

(a) 0.0123 AU

(b) 0.212 AU

(c) 2.69 AU

(d) 26.8 AU

8. An astronomer detected a galaxy and decided to analyze its different parts and physical aspects. The frequency generated by a "spin-flip" transition of atomic hydrogen is $\nu_0 = 1420.406MHz$, however it was detected on the galaxy as $\nu = 1422.73$. He finds that:

 1. Population I stars are (1) and are metal-(2).
 2. The galaxy is (3) from us with a speed of (4) $km * s^{-1}$.

Choose the alternative that correctly completes sentences above.

(a) (1) young; (2) poor; (3) distancing; (4) 245

(b) (1) old; (2) rich; (3) approaching; (4) 490

(c) (1)old; (2) poor; (3) distancing; (4) 490

(d) (1) young; (2) rich; (3) approaching; (4) 490

(e) (1) young; (2) rich; (3) approaching; (4) 245

9. A stable open cluster of about $N = 1000$ sun-like stars has an angular diameter of $\theta = 30$ arc minutes and distance of $d = 500$ pc. Assuming the cluster can be approximated by a sphere of uniform density, estimate the average velocities of stars in the cluster.

The gravitational potential energy of a sphere of uniform density and radius r is

$$U_{sphere} = -\frac{3}{5}\frac{GM_{sphere}^2}{r}$$

(a) 507 m/s

(b) 643 m/s

(c) 894 m/s

(d) 1021 m/s

(e) 771 m/s

10. What would happen to the analemma of the Sun if the obliquity of the Earth's orbit suddenly went to zero degrees and its eccentricity remained unchanged?

(a) The anallema would be perfectly symmetric in both axes and would have the shape of an "8".

(b) The analemma would look like a dot.

(c) The analemma would be the arc of a great circle.

(d) The analemma would look like a circle.

(e) The analemma would be a spherical triangle.

11. Let $T_{\odot,C}$ and $T_{\odot,S}$ be the temperatures at the core and the surface of the sun, respectively. Similarly, let $T_{A,C}$ and $T_{A,S}$ be the temperatures at the core and surface of the red giant Arcturus, and let $T_{S,C}$ and $T_{S,S}$ be the temperatures at the core and surface of the white dwarf Sirius B. Which of the following inequalities is true?

(a) $\frac{T_{\odot,C}}{T_{\odot,S}} < \frac{T_{A,C}}{T_{A,S}} < \frac{T_{S,C}}{T_{S,S}}$

(b) $\frac{T_{\odot,C}}{T_{\odot,S}} < \frac{T_{S,C}}{T_{S,S}} < \frac{T_{A,C}}{T_{A,S}}$

(c) $\frac{T_{A,C}}{T_{A,S}} < \frac{T_{\odot,C}}{T_{\odot,S}} < \frac{T_{S,C}}{T_{S,S}}$

(d) $\frac{T_{S,C}}{T_{S,S}} < \frac{T_{\odot,C}}{T_{\odot,S}} < \frac{T_{A,C}}{T_{A,S}}$

(e) $\frac{T_{S,C}}{T_{S,S}} < \frac{T_{A,C}}{T_{A,S}} < \frac{T_{\odot,C}}{T_{\odot,S}}$

12. The spectral line H_α in the spectrum of a star is recorded as having displacement of $\Delta\lambda = 0.043 \times 10^{-10}$ m. At rest, the spectral line has a wavelength of $\lambda_0 = 6.563 \times 10^{-7}$ m. Calculate the period of rotation for this star, if it is observed from its equatorial plane. We also know: $R_{star} = 8 \times 10^5$ km.

(a) 29.59 days

(b) 14.63 days

(c) 21.15 days

(d) 34.39 days

13. The reflector telescope built by Sir Issac Newton was a $f/5$ telescope and had a primary mirror of diameter 30mm. He used an eyepiece with a focal length of 5mm. What is the focal length and magnification obtained by this telescope?

(a) 150mm, 30×

(b) 300mm, 15×

(c) 300mm, 30×

(d) 150mm, 15×

14. Take a look at the following image:

Three Messier objects are circled in the image. Select the alternative that correctly matches each object with its type.

(a) 1 - Open cluster; 2 - Open cluster; 3 - Nebula.

(b) 1 - Open Cluster; 2 - Nebula; 3 - Galaxy.

(c) 1 - Galaxy; 2 - Nebula; 3 - Globular cluster.

(d) 1 - Open cluster; 2 - Galaxy; 3 - Globular cluster.

(e) 1 - Open cluster; 2 - Nebula; 3 - Open cluster.

15. An interesting phenomena that happens in the Solar System is the capture of comets in the interstellar medium. Assume that a comet with a mass of $7.15 * 10^{16}$ kg is captured by the solar system. The perihelion of this comet's orbit after it is captured is equal to 4.64 AU, and its velocity with respect to the Sun before being captured by the Solar System was very small. Calculate the velocity of the comet at the perihelion.

(a) 87.1 km/s

(b) 45.9 km/s

(c) 5.67 km/s

(d) 105.4 km/s

(e) 19.6 km/s

16. In a certain day, when it is 0h UT, the sidereal time of Prime Meridian is 5h 56min 9.4s. For this day, with start and end based on UT, find the civil time of Chicago, whose longitude and time zone are respectively, 87.65004722^o W and UT-6, when the sidereal time there is 20h. The difference between solar time and sidereal time **SHOULD** be accounted for.

(a) 14h 1min 32s
(b) 13h 26min 17s
(c) 14h 36min 47s
(d) 14h 0min 43s
(e) 13h 51min 11s

17. Consider the following horrifying scenario. The Sun has become a Red Giant, and its radius is doubling every 100 years. Rank the following of humanity's concerns in order of immediate importance.

I: Orbital decay due to direct contact between the Earth and Sun

II: As the distance between the Earth and Sun shrinks, the Earth will enter the Sun's Roche limit and start to be ripped apart

III: Orbital decay due to tidal effects on the Sun's outer atmosphere, the same way the Moon loses energy when forming the Earth's tides

IV: Runaway greenhouse effect due to extreme temperatures, leading to the Earth becoming a hot, Venus-like planet with no habitability

(a) III, IV, II, I
(b) IV, III, II, I
(c) IV, II, III, I
(d) IV, III, I, II

18. TESS Object of Interest (TOI) 402.01 has an orbital period of 4.756 ± 0.000023 (days) and was last observed to transit on 2139.1 ± 0.0027008 (in TESS Julian days, i.e. BJD - 2457000). For follow-up observation, we would like to predict the next transit – this would be the 23rd transit since the last observation. In TESS Julian days, when will the next transit occur?

(a) 2243.732 ± 0.0027238 TESS JD
(b) 2248.488 ± 0.000529 TESS JD
(c) 2248.488 ± 0.0027238 TESS JD
(d) 2248.488 ± 0.0032298 TESS JD

19. Jupiter has a mass of 1.90×10^{27} kg and a radius of 7.15×10^{7} m. To the closest order of magnitude, estimate the pressure at the center of Jupiter, in Megabars.

(a) 0.1
(b) 1
(c) 10
(d) 100

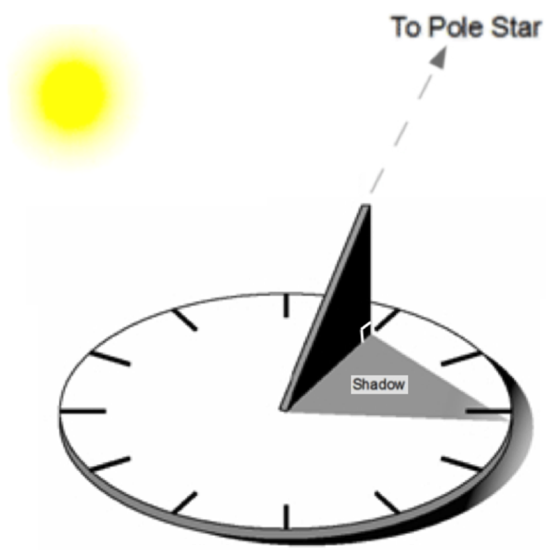
20. Which of the following statements is wrong?

(a) It is believed that elements with atomic number greater than that of iron are formed mostly by the explosion of supernovas.
(b) What holds a star together is the hydrostatic equilibrium between pressure and gravity.

(c) The granulations of the Sun happen on its corona.
(d) Protostars are actually not stars because their main source of heat is not fusion.
(e) The earlier type the main-sequence star, the more massive it is.

21. Consider a horizontal sundial where the triangular gnomon rises at an angle equal to the sundial site's latitude, $\phi = 38°$. If the area of the triangular gnomon is 2 m^2, what would be the area of the shadow in m^2 three hours after the noon in the first day of spring (vernal equinox)?

(a) 3.0
(b) 3.5
(c) 2.5
(d) 1.5
(e) 4.0



22. Consider an eclipsing binary star system observed (in some fixed band) to have a **combined** apparent magnitude of 5.67. During the secondary transit, the second star is totally eclipsed by the first star, and the apparent magnitude dims to 6.28. What percent of the combined flux is produced by the **second** star?

(a) 10.8%
(b) 43.0%
(c) 57.0%
(d) 89.2%

23. An astronomer observes that a Solar type star has an apparent V magnitude of 6.73 when seen from the Earth. Assuming that the average interstellar extinction in V is 1.00 mag/kpc, determine the distance between this star and the Solar system.

(a) 11.5 pc
(b) 49.5 pc

(c) 34.2 pc
(d) 23.7 pc
(e) 18.9 pc

24. 1. The temperature of the Sun is 5000K while that of Sirius is 10000K. Which star has a higher integrated radiance i.e. net energy emitted per unit time per unit area?

(a) Sun
(b) Sirius
(c) Depends on value of the respective radii
(d) The integrated radiance is equal.

25. Suppose a spaceship is attempting a slingshot maneuver on a gas giant with mass 100 times that of the spaceship. Because the spaceship somewhat entered the planet's atmosphere, kinetic energy was not conserved—only momentum. What is the ratio of the spaceship's change in velocity to the planet's change in velocity, $\frac{\Delta v_s}{\Delta v_p}$?

(a) 10
(b) 100
(c) -10
(d) -100

26. Let's imagine that our Universe would be filled with basketballs, each having a mass of $m_b = 0.62$ kg. What would be the necessary numerical density (n_b) of basketballs in the Universe such that the mass density of the basketballs would equal the current critical density of our Universe?

(a) 1.5×10^{-26} balls/m^3
(b) 1.7×10^{26} balls/m^3
(c) 1.5×10^{-27} balls/m^3
(d) 1.7×10^{27} balls/m^3

27. When binary systems are really close together, they can execute an accretion process, in which one star (called the primary star) "eats" the mass of the other (called the secondary star), whose mass spirals down into the primary star, creating an accretion disk!

For an accretion disk with the outer edge $3R$ from the center of the primary star (radius R and mass M), calculate the energy lost by a test mass (mass m) where it touches the primary star from where it first enters the accretion disk.

Consider the orbits to be Keplerian.

(a) $\frac{GMm}{R}$
(b) $\frac{1}{2}\frac{GMm}{R}$
(c) $\frac{5}{2}\frac{GMm}{R}$
(d) $\frac{2}{3}\frac{GMm}{R}$
(e) $\frac{3}{4}\frac{GMm}{R}$

28. An often-repeated fun fact is that humans produce more power per unit volume than stars. If the sun were the same size, but it produced the same amount of power per unit volume as a human, what would its surface temperature be? Assume the "average human" produces 100 watts of power and has a volume of 66400 cubic centimeters.

    (a) 3500 K
    (b) 10000 K
    (c) 25000 K
    (d) 40000 K
    (e) 50000 K

29. Given that the redshift of cosmic microwave background (CMB) is 1100, what was the temperature of the Universe when photon decoupled from matter and neutral hydrogen started to get formed? The present temperature of the CMB is 2.73 K.

    (a) 10000 K
    (b) 30000 K
    (c) 3000 K
    (d) 1000 K
    (e) 300 K

30. Where and when should we place a radio telescope such that, when combined with a radio telescope on Earth, the system could "see" the supermassive black hole in Sculptor's Galaxy (NGC 253)?

    Sculptor's Galaxy's supermassive black hole's mass is estimated to be around $5 \cdot 10^6 M_\odot$, and its distance is estimated to be around 3.5 Mpc.

    Out of the options below, pick the one closest to the estimate you obtain, rounding up. Consider the energy of a radio wave to be around 10^{-5} eV.

    Use the following formula to estimate the angular resolution: $\theta = \frac{\lambda}{D}$

    (a) On the Moon when it is at its apogee.
    (b) On Mars when it is in conjunction.
    (c) On Venus when it is in its greatest elongation.
    (d) On one of Jupiter's moons when it is in opposition.
    (e) Somewhere in the farthest points of the Oort cloud when Earth is at its perihelion.