2 Short Questions

1. (**5 points**) Jupiter emits more energy to space than it receives from the Sun. The internal heat flux of Jupiter can be quantified by the "intrinsic" temperature of the planet $T_{\rm int}$. The effective temperature $T_{\rm eff}$ of a planet is related to its intrinsic temperature and equilibrium temperature $T_{\rm eq}$ by $T_{\rm eff}^4 = T_{\rm eq}^4 + T_{\rm int}^4$. Given that Jupiter's albedo is 0.5, its emissivity is 1, its average separation from the Sun is 5.2 AU, and its effective temperature is 134 K, estimate its intrinsic temperature in Kelvin. You may use the Sun's surface temperature equal to 5777 K.

2. (**5 points**) The convection zone of the sun is the major region of the solar interior that is closest to the surface. It is characterized by convection currents that quickly carry heat to the surface. As a pocket of gas rises, it expands and becomes less and less dense. For it to continue to rise, the temperature gradient in the sun must be steeper than the adiabatic gradient, which is the temperature that the gas would have if it were allowed to expand without any heat input.

   In the sun, the adiabatic gradient satisfies $T \propto p^{0.4}$, where T is the temperature and p is the pressure at any given point.

   The bottom of the convection zone is about 200,000 kilometers beneath the surface of the sun, and has a temperature of about 2×10^6 K and a density of about $200\,\mathrm{kg/m^3}$. Estimate an upper bound for the temperature of the convection zone where the density is $1.2\,\mathrm{kg/m^3}$ (the density of air). You may assume the ideal gas law holds in the convective zone.

3. (**5 points**) Galaxies are very hard to spot, even those that are nearest to us. For instance, Andromeda, despite having an apparent magnitude of 3.44, appears very "dim" in the sky. This is because its light is very spread out, since its solid angle in the sky is so large (around 3 times that of the Sun!).

   Hence, it is often useful to use the surface magnitude of a galaxy, defined as the magnitude that a certain solid angle of that galaxy has. It is usually measured in $mag/arcmin^2$.

   Show that, in a non expanding universe, the surface magnitude is independent of the distance to the galaxy.

4. (**5 points**) An Earth satellite has the following position ($\vec{r}$) and velocity ($\vec{v}$) vectors at a given instant:

$$\vec{r} = 7000\hat{i} + 9000\hat{j}(\mathrm{km})$$

$$\vec{v} = -2\hat{i} + 5\hat{j}\ (\mathrm{km/s})$$

   Calculate the eccentricity of the satellite orbit. Hint: The eccentricity of the orbit is related to total energy E and angular momentum L as $e = \sqrt{1 + \frac{2EL^2}{G^2M^2m^3}}$; where M is Earth's mass and m is the mass of the satellite.

5. (**5 points**) An astronomer who lives in Chicago ($\phi = 41.88^o N$; $\lambda = 87.63^o W$) was very bored during the day of the winter solstice in the Northern hemisphere, so he started thinking about the sunset. The astronomer could not wait to see the sunset on that day. Considering that the true solar time at his location was 2:30 pm, how long did he have to wait to see the sunset? The declination of the sun on winter solstice is $\delta = -23.44^o$.