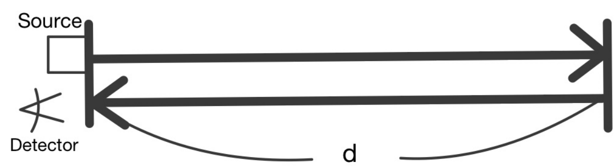
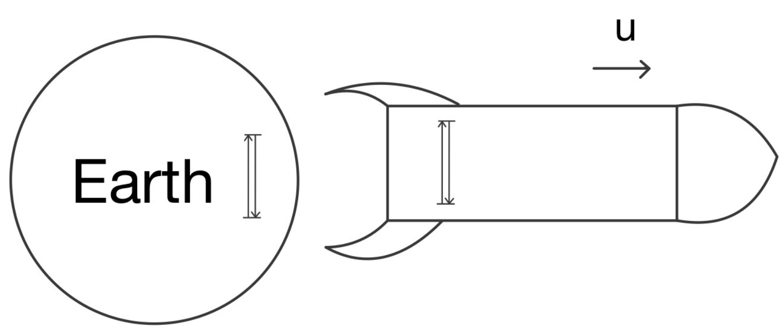
3 Medium Questions

1. (**20 points**) To measure the time accurately outside the Earth, the engineers build a special clock, with the design as follows: there is a source of light that sends light particles (photons) straight to the reflector that is located at the distance d away from the source. The reflector sends the photons back to their starting point, where there is a detector. This can measure the time accurately, because the speed of light c is constant everywhere. Then a group of engineers built a spaceship with this special clock inside. This spaceship with a clock started to move really fast at the speed u. While the observer in the spaceship reported no issues with the clock inside the spaceship, the observer on the Earth has noticed that the clock is functioning differently in a fast moving spaceship than it is on Earth.





   (a) Given that the clock is at rest, what is the total traveling time (Δt_E) of a photon from its source back to the detector?

   (b) What is the total distance traveled by a photon d_γ from the source back to the detector on the spaceship moving at the speed u away from the Earth? (Here, we denote that the total traveling time of the photon as Δt_S)

   (c) What is the total time Δt_S of a photon as it travels from the source to the detector on the moving spaceship? Answer in terms of d, c, and $\beta = \frac{v}{c}$.

   (d) If we relate Δt_E (non-moving frame) to Δt_S (moving frame), as follows: $\Delta t_S = \gamma \Delta t_E$, what does γ equal to? What is significant about the range of γ?

   (e) So far, we have only analyzed the motion on the perspective of an observer on the Earth. From the perspective of an observer on the moving spaceship, how do the time on the spaceship $\Delta t_S{}'$ and the time on the Earth $\Delta t_E{}'$ relate to each other?

   (f) What can we conclude about the relative passing on time on two different frames that are relatively in motion to one another?

2. (**30 points**) You want to send a rocket with an instrument to analyze the atmosphere of Jupiter. In order to get there, you decide to use a Hohmann transfer orbit. $r_E = 1$ AU and $r_J = 5$ AU represent the radii of Earth's and Jupiter's circular orbits around the Sun, respectively. m, M_E, M_J, and M_S represent the masses of your rocket, Earth, Jupiter, and Sun, respectively. Ignore planetary gravitational influences. You may use any other variables you would like if you clearly define them first. Refer to the figures at the end of the question. Show your work for all derivations.

(a) Explain which two (relevant) physical quantities are conserved during this transfer orbit. Write down their statements mathematically.

(b) How long will it take to reach Jupiter?

(c) Halfway through its path to Jupiter, an unrealistic comet passes right next to your rocket and its icy tail freezes your rocket fuel. What is the **maximum** amount of time that you can afford to pass until you need the fuel to be once again unfrozen?

(d) Knowing that this comet will come in the way, your colleague suggests a bi-elliptic transfer orbit instead, with a peak distance of $12r_E$. Write equations describing how long it will now take to reach Jupiter. Will this solution always avoid the comet?

Now that you've compared the orbital times, you want to try and calculate the difference in efficiency.

(e) Derive the δv for each orbital transition in the Hohmann transfer, and sum them to find the total δv.

(f) Derive the δv for each orbital transition in the Bi-elliptic transfer, and sum them to find the total δv.

(g) Factoring in all your previous results, which transfer would you like to use? Why?

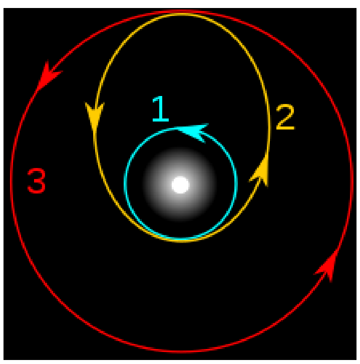


Figure 1: Hohmann Transfer

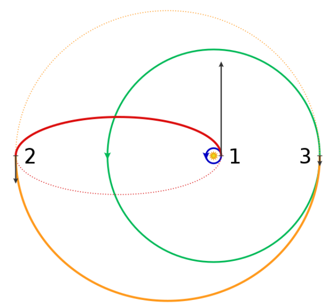


Figure 2: Bi-elliptic Transfer

3. (**30 points**) A space station of mass m is orbiting a planet of mass M_0 on a circular orbit of radius r. At a certain moment, a satellite of mass m is launched from the space station with a relative velocity $\overrightarrow{w}$ oriented towards the center of the planet. Assume that $w < \sqrt{\frac{GM_0}{r}}$.

(a) Justify the shape of the satellite's orbit after launching and, for the satellite-planet system, determine the following quantities:

(1) Satellite's velocity relative to the planet, immediately after launch, v

(2) Total angular momentum of the satellite-planet system, $L_{P,Sat}$

(3) Satellite's orbit semi-major and semi-minor axes, a_{Sat} and b_{Sat}

(4) Satellite's orbit eccentricity, ϵ_{Sat}

(5) Apogee and perigee distances, $r_{max,Sat}$ and $r_{min,Sat}$

(6) Satellite's minimum velocity, $v_{min,Sat}$ and maximum velocity $v_{max,Sat}$ on it's orbit

(7) Total energy of the satellite-planet system, $E_{Sat,P}$.

(b) Determine the shape of the space station's orbit relative to the planet, after the satellite was launched.