2 Short Questions

1. (**5 points**) The sidereal period of Mars is 687 days, while the sidereal period of Earth is 365.26 days. The most recent opposition of Mars occurred on July 27, 2018. Predict all dates in the year 2020 when Mars will be in quadrature. You may use the fact that the orbital radius of Mars is 1.52 AU and that Earth and Mars have circular orbits. Why might your answer be inaccurate?

> **Solution:** The synodic period is computed as $\frac{1}{P_{syn}} = \frac{1}{P_{Earth}} - \frac{1}{P_{Mars}}$. Using the values given in the problem, we obtain $P_{syn} = 779.9$ days. Thus, oppositions happen every 779.9 days, so the next one will be around 9/15/2020. Quadrature occurs when Mars is approximately $\arccos 1/1.52 = 48.9$ degrees behind or in front of its position at opposition, or $779.9(\frac{48.9}{360}) = 106$ days before or after opposition. The only such date in 2020 is around 6/2/2020 (accept answers within two days of this date).
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> Inaccuracies could be due to the nonzero eccentricities of Earth and Mars, as well as the nonzero relative inclination between the two planets' orbits. The actual date is nearer 6/7/2020.

2. (**10 points**) The star Betelgeuse has recently made news for its abnormal dimming. Although the dimming has now been attributed to dust, we consider in this problem that it was due to radial pulsations. Suppose that Betelgeuse's mass is 11 solar masses and its radius is 887 solar radii. Furthermore, Betelgeuse is currently rotating such that the tangential velocity of a point on its equator is 5000 m/s (assume Betelgeuse is perfectly spherical). The dimming has increased Betelgeuse's apparent magnitude by 1.05. You may neglect the contribution of pulsation to the surface velocity.

   (a) Assuming contraction and expansion are isothermal, find the (new) radius of the star (in solar radii) needed to account for the dimming.

   (b) Assuming no mass loss, find the new angular rotation velocity of the star.

> **Solution:** An increase in apparent magnitude of 1.05 corresponds to a decrease in brightness by a factor of $100^{1.05/5} = 0.38$. This is the ratio of the final to the initial luminosity. Applying the Stefan-Boltzmann law, $\frac{L_f}{L_i} = \frac{R_f^2}{R_i^2}$, so the final radius is 547 solar radii.
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> Apply conservation of angular momentum to find the final angular velocity. The initial angular velocity is $\omega_i = \frac{v_T}{R_i} = 8.10 \times 10^{-9}$ rad/s. The initial moment of inertia is $I_i = \frac{2}{5}MR_i^2 = 3.34 \times 10^{54}$ kg-m^2 and the final moment of inertia is $I_i = \frac{2}{5}MR_f^2 = 1.27 \times 10^{54}$ kg-m^2. $I_i\omega_i = I_f\omega_f$, so $\omega_f = 2.13 \times 10^{-8}$ rad/s.

3. (**10 points**) The Lyman-break galaxy selection technique makes use of the fact that any light from galaxies with wavelength shorter than the Lyman limit (the shortest wavelength in the Lyman series) is essentially totally absorbed by neutral gas surrounding the galaxies. The ionization energy of hydrogen is 13.6 eV. Suppose that we are observing galaxies in the V band, whose effective midpoint is 551 nm and bandwidth is 99 nm.

   (a) At what range of redshifts would we begin to see galaxies "disappear" (break) from images in the V band?

   (b) What range of recessional velocities (km/s) and distances (Mpc) does this correspond to? Assume only Hubble expansion contributes to the radial velocity and redshift.

Solution: The Lyman limit is obtained via $E = \frac{hc}{\lambda_L} = \frac{1}{1^2} - \frac{1}{\infty^2}$. We have $E = 13.6$ eV, so $\lambda_L = 91.2$ nm. For the V band, $\lambda_{max} = 600.5$ nm and $\lambda_{min} = 501.5$ nm. Applying $z = \frac{\delta\lambda}{\lambda_L}$, the desired range of redshifts is $z = 4.50 - 5.58$. Using relativistic Doppler shift, $v_{rel} = \frac{(1+z)^2-1}{(1+z)^2+1}c$, the range of recessional velocities is $v_{rel} = 281,000 - 286,000$ km/s. Applying Hubble's law, the range of distances is 4010-4090 Mpc.

4. (**5 points**) TRAPPIST-1d is a temperate exoplanet that orbits the ultra-cool M dwarf star TRAPPIST-1 with a semi-major axis of 0.022 AU. TRAPPIST-1 has a mass of 0.089 Solar masses and an effective temperature of 2511 K. Through transit timing variations induced by other planets in the TRAPPIST-1 system, TRAPPIST-1d is estimated to have a mass of 0.420 Earth masses. Assuming that TRAPPIST-1d has a circular orbit (which is a good approximation because the measured eccentricity is only 0.008), what is the radial velocity semi-amplitude of TRAPPIST-1 due to the orbital motion of TRAPPIST-1d, in m/s?

Solution: $M_p/M_\star = V_\star/V_p \rightarrow V_\star = M_pV_p/M_\star$. $V_p = 2\pi a/T$.
$T = ([a(AU)]^3/[M_\star(M_{\text{Sun}})])^{1/2} = (0.022AU^3/0.089M_{\text{Sun}})^{1/2} \times 365.25\, days/year = 4.00$ days.

$$V_p = \frac{2\pi(0.022AU \times 1.496 \times 10^{11}m}{4.00days \times 86400sec/day} = 59.8\text{km/s}$$

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$$V_\star = \frac{0.420 \times 5.97 \times 10^{24}kg \times 59.8 \times 10^3 m/s}{0.089 \times 1.99 \times 10^{30}kg} = 0.847 \text{ m/s}$$

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The radial velocity semi-amplitude of TRAPPIST-1 due to TRAPPIST-1d is 0.847 m/s.

5. (**5 points**) HD 209458b is a hot Jupiter exoplanet with a mass of 0.69 Jupiter masses. However, HD 209458b has an anomalous radius of 1.38 Jupiter radii that is inflated relative to Jupiter. Jupiter has an interior that is comprised of metallic hydrogen at pressures greater than 1 Mbar. Estimate the pressure, in Mbar, at the center of HD 209458b, and determine whether or not the interior of HD 209458b will also be comprised of metallic hydrogen.

Solution: Estimate: $P_c \sim \rho g R \sim 3GM^2/(4\pi R^4)$. The actual solution (if a student knows the equation or can derive it) is $P_c = 3GM^2/(8\pi R^4)$.
Plugging in, $P_c = 3 \times 6.67 \times 10^{-11} m^3/kg/s^2 \times (0.69 * 1.898 \times 10^{27} kg)^2/(8\pi(1.38 \times 69.91 \times 10^6 \text{ m})^4 =$ 1.57 Mbar. Estimate would give twice the value (because of ignoring the integral, which is acceptable because no calculus is required), which is 3.15 Mbar.
Accept answers between 1 and 5 Mbar. Yes, the interior of HD 209458b will be composed of metallic hydrogen.