3 Medium Questions

1. (**15 points**) An astronomer used his f/5 telescope with a diameter of 130 mm to observe a binary system. He is using an eyepiece with a field of view of 45° and a focal length of 25 mm. In this system, star A has a mass of 18.9 solar masses, and an apparent magnitude in the V filter of 9.14. Star B has a mass of 16.2 solar masses, and an apparent magnitude in the V filter of 9.60. The period of the system is 108 days, and the distance between the binary stars and the Solar System is 2.29 kpc. The binary system has an edge-on orbit relative to the Solar System.

   (a) What is the field of view of the telescope?

   (b) What is the limiting magnitude of the telescope?

(c) What is the angular resolution of the telescope?

(d) What is the angular separation between the stars?

(e) Is the astronomer able to observe both stars as distinct points in the telescope? Answer as YES or NO.

The limiting magnitude for the human eye is 6.0, and the diameter of the pupil is equal to 7.0 mm. Also consider that visible light has a wavelength of 550 nm.

2. (**20 points**) The rotation curve of a particular spiral galaxy is modeled by an exponential function of the form $V(r) = V_0(1 - e^{-r/R})$, where $V_0 = 250$ km/s, $R = 7.5$ kpc, and r is measured radially from the center of the galaxy. Throughout parts (a)-(d), you may assume the galaxy is disk-shaped. Further, we'll assume that the distribution of mass in the galaxy depends only on the radial coordinate r (and is thus radially symmetric).

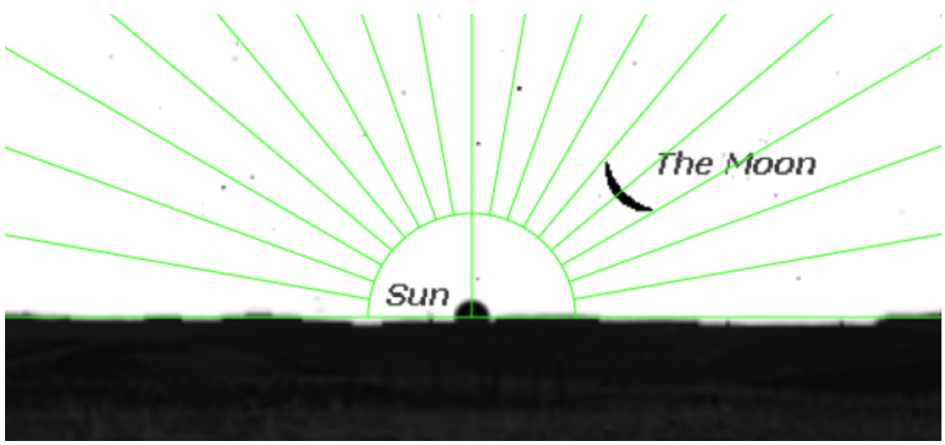
   (a) Find the period of rotation (in years) of a particle 10 kpc from the center of the galaxy. Also, find the mass enclosed within the (circular) orbit in solar masses, i.e. the mass within $r = 10$ kpc from the center of the galaxy.

   (b) Find the angular velocity of the galaxy very close to the center ($r \ll R$). Hint: $e^x \approx 1 + x$ for $|x| \ll 1$.

   (c) Determine how the (gravitational) mass per unit area must vary with distance from the center of the galaxy in order to yield the given rotation curve. Find the expressions only for regions very far from the galactic center.

   (d) An astronomer measures the absolute bolometric magnitude of the galaxy to be -21.2. For comparison, the bolometric magnitude of the sun is 4.75. Assume that the luminous mass per unit area follows a profile given by $\sigma_L = \frac{k}{r}$ for $k = 2.55 \times 10^8$ M_{Sun}/kpc and that all of the luminous mass is in the form of Sun-like stars. Approximate the percentage of the galaxy's mass that is dark matter, out to the maximum distance (radius) that is still visibly defined.

3. (**15 points**) An astro-photographer has taken the photo of the moon close to a new moon day shown below right before the sunset on December 21 (Winter Solstice) in a wide open area.



   (a) In which hemisphere (Northern or Southern) is the photographer located?

   (b) Find the latitude of the photographer. Ignore the orbital inclination of the Moon and the ellipticity of the Earth's orbit. Hint: The green equiangular lines are added to the image to help you out in measuring any relevant angle.

   (c) Calculate the sidereal time when the photo was taken.

4. (**20 points**) In general relativity, the orbit of satellites around a massive object (like a black hole) are known as geodesics and do not obey all of Kepler's laws for orbits. However, for objects that are moving at non-relativistic speeds, we can analyze the orbit using classical mechanics, with a corrective term added to Newton's Law of gravity. In this case, the potential energy of an object in orbit around a black hole is:

$$V_s(r) = -\frac{GMm}{r} - \frac{GML^2}{c^2mr^3}$$

where M and m are the masses of the black hole and the object respectively, c is the speed of light, L is the angular momentum of the object in orbit and r is the distance of that object from the black hole. Likewise, the gravitational force from a black hole has magnitude:

$$F_s(r) = \frac{GMm}{r^2} + \frac{3GML^2}{c^2mr^4}$$

You may assume that both conservation of energy and conservation of angular momentum hold in this regime.

(a) Argue which of Kepler's laws are still true.

(b) Calculate the radius of a stable circular orbit with an angular momentum L (you will get two solutions, the stable orbit generates the classical result under the proper limits)

(c) What is the radius, R_{ISCO} of the innermost stable circular orbit (the smallest stable circular obit) for a black hole of mass M? What is the numerical value of R_{ISCO} for Sagittarius A*, which has mass 3.6×10^6 solar masses?

(d) Suppose we discovered a new star orbiting Sagittarius A*, S99, that has a periapsis of $10R_{ISCO}$ and an apoapsis of 16 AU. Find the magnitude of velocity of S99 at both periapsis and apoapsis.

5. (**15 points**) The following table gives the numerical values for some physical properties of four stars. The quantities that are affected by, i.e. include the effects of, interstellar extinction are marked with a star (*). You may consider that all stars are black bodies. The temperature of a star can be calculated directly from its B-V index, by using Ballesteros' formula:

$$T_{eff} = f(B-V) = 4600\left(\frac{1}{0.92(B-V)+1.7} + \frac{1}{0.92(B-V)+0.62}\right)K.$$

Determine the numerical values of all the other physical characteristics presented in the given table. For full credit, show your full work by writing all the mathematical expressions used in the calculation.

Hint: You might use the following empirical relation:

$$\frac{A_V}{E_{B-V}} = 3.2$$

Star	κ Velorum	β Tauri	Sirius A	Sun
Annual Parallax $p^*(10^{-3} arcsec)$	6.05	24.89	379.2	-
Distance to Sun $\Delta^*(pc)$				-
Interstellar Extinction in V Band A_V (mag)	0.20	0.08	Negligible	-
$10^{0.2\times A_V}$				-
Distance to Sun $\Delta(pc)$				4.85×10^{-6}
Annual Parallax $p(10^{-3}$ arcsec)				
Distance Modulus $\mu = m - M$				
Visual apparent magnitude $m^*(mag)$	2.86	1.68	-1.47	-26.73
Visual apparent magnitude m (mag)				
Visual absolute magnitude M_V (mag)				
Color Index $(B-V)^*$ (mag)	-0.14	-0.06	+0.01	+0.65
Extinction E_{B-V} (mag)				
Color Index $(B-V)$ (mag)				
Effective Temperature $T_{eff} = f(B-V)$ (K)				
λ_m (nm)				
Radius (Solar Radius, R_S)	9.10	4.60	1.71	1.00
Total Luminosity (Solar Luminosity,L_S				1.00
Absolute Bolometric Magnitude M_{bol} (mag)				4.64
Bolometric Correction BC for V band (mag)				-0.20