USAAAO 2015 Second Round Solutions

Problem 1

T = 10000 K, m = 5, d = 150 pc

$m - M = 5\log(d/10)$

$M = m - 5\log(d/10) = 5 - 5\log(15) = -0.88$

We compare this with the absolute magnitude of the sun, 4.83.

$\frac{L}{L_{solar}} = 100^{(4.83--0.88)/5} = 192L_{solar}$

From the Stefan-Boltzmann Law, $L = 4\pi R^2\sigma T^4$

Finding the ratio with the sun, we get:

$\frac{L}{L_{solar}} = \left(\frac{R}{R_{solar}}\right)^2\left(\frac{T}{T_{solar}}\right)^4$

$\frac{R}{R_{solar}} = \left(\frac{L}{L_{solar}}\right)^{1/2}\left(\frac{T_{solar}}{T}\right)^2 = 4.6R_{solar}$

Problem 2

T = 3 days, K = 50 m/s, M = $1M_{solar}$

$T^2 = \frac{a^3}{M}$

$\left(\frac{3}{365}\right)^{2/3} = 0.041AU$ for the planet's orbit.

For the star's orbit:

$T = \frac{2\pi r}{v}$

$r = \frac{Tv}{2\pi} = 1.38 * 10^{5}AU$

Relating the two, we have:

$m_* r_* = m_p r_p$, so we have $m_p = \frac{r_*}{r_p}m_*$

Plugging in, we find $m_p = 3.36 * 10^{-4}M_{solar}$

Problem 3

Solar rotation rate 24.5 days, $M_J = 9.54 * 10^{-4}M_{solar}$, a = 5.2 AU, solar radius 695,000 km.

$L = I\omega$

$L_{solar} = \frac{2}{5}MR^2 * \frac{2\pi}{24.5} = 5 * 10^{10}$

For Jupiter, $L = mrv$

$r = 5.2 * 149.6 * 10^6 = 7.77 * 10^8 km$

$v = \frac{2\pi r}{5.2^{2/3} * 365} = 1.13 * 10^6$ km/day.

$L = 9.54 * 10^{-4} * 7.77 * 10^8 * 1.13 * 0^6 = 8.38 * 10^{11}$

So Jupiter has the greater angular momentum.

Problem 4

m = 10, T = 6000 K at the main sequence turnoff.

Oldest main sequence stars are 6000 K, which is approximately sun-like. We therefore assume the absolute magnitude of the stars at the turnoff point is 4.83.

$m - M = 5\log(d/10)$

$d = 10 * 10^{(m-M)/5} = 108pc$

The stars at the turnoff point are sunlike, so we expect them to have a lifetime of 10 Gyr. Since these are the oldest main sequence stars in the cluster, the cluster has an age of approximately 10 Gyr.

Problem 5

Question not graded

Problem 6

m = 8, p = 0.003", and T = 6000 K.

$d = 1/p = 333pc$

$m - M = 5\log(d/10)$

$M = m - 5\log(d/10) = 0.385$

$\frac{L}{L_{solar}} = 100^{(M_{solar} - M)/5} = 59.9L_{solar}$

The star's temperature is approximately sunlike, suggesting class G, but it is significantly more luminous, suggesting a giant. G0III would be a reasonable possible spectral type.

Problem 7

From Wein's Law, $\lambda = \frac{b}{T}$. From the definition of Redshift, $z = \frac{\lambda - \lambda_0}{\lambda_0}$, where λ_0 is the emitted wavelength. Solving for λ_0, we get $\lambda_0 = \lambda/(z+1)$

Combining this with Wein's Law, we get:

$T = \frac{b * (z+1)}{lambda}$

Using Wein's Law and the given temperature of 2.73 K, we find that the recieved wavelength is 1.06 mm. Plugging this into the above expression, we obtain at temperature of 30.03 K for z = 10.

Problem 8

At blackbody equilibrium, power in is equal to power out, so we have:

$P_{out} = 4\pi R_p^2 \sigma T_p^4$

$P_{in} = A * (1-\alpha) * \frac{L}{4\pi D^2} = \pi R_p^2 (1-\alpha) \frac{4\pi R_*^2 \sigma T_*^4}{4\pi D^2}$

$P_{in} = P_{out}$

$4\pi R_p^2 \sigma T_p^4 = \pi R_p^2 (1-\alpha) \frac{4\pi R_*^2 \sigma T_*^4}{4\pi D^2}$

$T_p^4 = (1-\alpha) T_*^4 \frac{R_*^2}{4D^2}$

$T_p = (1-\alpha)^{1/4} T_* \sqrt{\frac{R_*}{2D}}$

Problem 9

7.2 micron pixels, f/10, D = 0.256 m.

f/10, so focal length is 2.56 m.

Angular resolution is given by $\frac{pixelsize}{focallength} * 206265$

Plugging in, we get a resolution of 0.58 arcseconds/pixel

Problem 10

A Hohmann transfer orbit is being used to go from the 1 AU orbit of the Earth to Saturn's orbit at 9.6 AU. The semimajor axis of the transfer orbit is thus 5.3 AU.

Using the vis-viva equation, $\frac{v^2}{2} - \frac{\mu}{r} = -\frac{\mu}{2a}$ this corresponds to a velocity of 40,080 m/s at the Earth's orbital distance.

When it starts, however, the spacecraft is already in Earth parking orbit, so benefits from both its orbital velocity around the Earth and the Earth's orbital velocity around the Sun.

When in the parking orbit, the probe has a maximum velocity relative to the Sun of $\sqrt{\frac{GM_e}{r_{orb}}} + \sqrt{\frac{GM_s}{r_{earth}}} = 37540 m/s$

The difference between the spacecraft's current maximum velocity relative to the Sun and the orbital velocity of the transfer ellipse is equal to the required Δv, since we're neglecting further attraction from the Earth. The necessary Δv is thus 2550 m/s, and the burn must occur on the night side of the Earth in order to increase the orbital radius and match Saturn's orbit (since the parking orbit is prograde).

We use the vis-viva equation again to calculate the probe's velocity at Saturn's orbital distance, and compute Saturn's orbital velocity using the same technique we used for Earth. We then subtract Saturn's velocity from the probe's velocity to find the probe's velocity relative to Saturn.

We then calculate the velocity relative to Saturn for a 100,000 km circular orbit. The difference between this value and the value above is the Δv required to make Saturn orbit, coming out to 24900 m/s. The burn must occur on the night side of Saturn in order to enter a prograde orbit.

Problem 11

M = $0.54 M_s$, P = 6 years, perihelion distance 0.537 AU, parallax 0.05".

From the parallax equation, $d = r/\vartheta$, where d is the distance, r is the baseline, and ϑ is the parallax angle. We therefore need to find the baseline, XY, which is the latus rectum of the ellipse.

Using Kepler's Third Law, we find that the semimajor axis is 2.69 AU. This corresponds to an aphelion of 2.135 AU, and therefore an eccentricity of e = 0.3.

The latus rectum of an ellipse is given by $l = a(1 - e^2)$. Plugging in, we get l = 2.45 AU, yielding a distance d = 49.0 pc

Problem 12

Simply dividing the length of the year by 6 is incorrect. Because of the inclination of the Earth, the Sun also varies in declination (from -23.5 to +23.5 degrees), but maintains (for this problem) a constant angular velocity. Spherical trigonometry is therefore required to determine the actual angle that the Sun traverses going from 0 to 4 hours (62.1 degrees). Since the Sun has constant angular velocity, traversing 360 degrees per year, the number of days can be expressed as (62.1/360)*365 = 63.0 days.

Problem 13

This question also requires spherical trigonometry. Picking RA = 0, Dec = 90 is likely the easiest third point. Now that we have a spherical triangle, we can use the spherical law of sines or cosines to find the separation angle (18.59 degrees).

The position angle is measured east of north. For our chosen triangle, this corresponds to the angle with Betelgeuse as its vertex. Again applying the spherical trigonometric relationships, we find the position angle to be 33.12 degrees.

To cover both stars, the picture must cover 18.6 degrees of the sky, which corresponds to 66960 arcseconds. Plate scale, in arcseconds/mm, is given by 206265/focal length. Solving for the focal length, we get a value of 3.08*film size, which is a focal length of 108 mm on 35 mm film or 216 mm on 70 mm.