National Astronomy Olympiad 2014-2015

This is the Second Round of the 2014-2015 USA Astronomy and Astrophysics Olympiad (USAAAO) competitions. You have exactly 2 hours and 30 minutes to complete the exam. There is a maximum of 67 points, and the value of each problem is indicated following each problem statement.

This test consists of 13 short answer problems and one long problem with multiple parts. Solutions should be written on blank paper, with the problem number, student's name, and school written clearly on top of each page. Solutions to multiple short answer problems may be written on the same page, but the long problem should begin on its own page. Partial credit will be given for correct work, so make sure to clearly show all steps.

You may only use a scientific calculator as aid for this exam. A table of physical constants and other information will be provided for you. This exam document, your solutions, and all used scratch paper must be turned in at the end of the exam.

Do not discuss this examination with anyone after its completion. Your results will be emailed to you shortly. The top 5 students will be invited to represent the United States at the 9th International Olympiad on Astronomy and Astrophysics competition in Central Java, Indonesia, from July 26th to August 4th, 2015.

In the event of a tie for the top 5 places, the student with the higher score on the long problem will qualify. If the tied students have the same score on the long problem, then the short answer problems with more points will be given more weight in the grading.

I hereby affirm that all work on this exam is mine, and that I took this test under a proctor's supervision, with no outside aids beyond the materials provided and allowed. Furthermore, I affirm to not discuss the test with others or provide any sort of aid to other examiners throughout the course of the examination period. I understand that failure to do so may result in disqualification from the exam.

Signature: ______________________________ Date: ______________

Section A: Short Answer [42 Points]

1. A blue star with effective temperature $T_{\text{eff}} = 10{,}000$ K and apparent magnitude $m = 5$ is located 150 pc from Earth. Find the radius of the star. [2]

2. The radial velocity curve of a nearby Solar-mass star shows that it has a planet orbiting it with a period of 3 days. This planet causes its host star to have a radial velocity semi-amplitude of 50 m/s. Assuming that this planet's orbit is perfectly inclined to Earth, and has 0 eccentricity, what is the mass of this planet, in Jupiter masses? [2]

3. The Sun has a rotation rate of 24.5 days. Jupiter has a mass of 9.54×10^{-4} solar masses, and a semimajor axis of 5.2 AU. Which object has more total angular momentum? [2]

4. What is the distance to a star cluster whose stars at the main sequence turn-off point have an apparent magnitude of $m = 10$ and an effective temperature of $T_{\text{eff}} \approx 6000$ K? How old is this cluster? [3]

5. Assuming that the universe currently is well described by a density parameter $\Omega_0 = 1$, there is no dark energy and the current temperature of the universe is 2.73 K, compute how long from the present it will take for the universe to cool down by 0.2 K. Remember that the temperature of the universe is inversely proportional to its radius (the scale factor). [3]

6. A star has apparent magnitude $m = 8$, parallax $p = 0.003''$ and effective temperature $T_{\text{eff}} \approx 6000$ K. What is the luminosity of the star? What is the likely spectral type of this star? Justify your answer. [3]

7. Assuming that the cosmic microwave background radiation has the spectrum of a blackbody throughout the evolution of the universe, determine how its temperature changes with redshift. In particular, find the temperature of the CMB at the epoch $z \approx 10$, knowing that the current temperature of the CMB is 2.73 K. [3]

8. Derive an expression for the blackbody temperature of a planet with radius r and albedo a, orbiting its sun at a certain distance D. The star has a temperature T_* and radius R. You must show your derivation for credit, not just the final expression. [3]

9. A 2048 x 3072 pixel CCD camera with 7.2 micron pixels is attached to an f/10 telescope with a 0.256 m primary mirror. What is the angular resolution of the CCD, in arcseconds/pixel? [3]

10. An interplanetary spacecraft bound for Saturn (a = 9.6 AU) is launched into a 300 km, non-inclined parking orbit around the Earth. If the spacecraft takes a Hohmann transfer to Saturn, what is the delta-V required for trans-Saturn injection, and what is the required delta-V for insertion into a 100000 km circular orbit around Saturn? On what side of each planet should the burns occur? (Assume Saturn is in the same orbital plane as the Earth, and neglect the gravitational deflection of the ship's path after injection and before orbital insertion.) [4]

11. Planet A orbits around Star A of mass $M = 0.54\ M_{\odot}$ with a period of $P = 6$ Earth years. Astronomers on this planet want to measure the distance of a distant Star B which happens to lie along the semimajor axis of Planet A's orbit, on the side of perihelion. They choose to do so using parallax, by noting the position of Star B with respect to the background stars at two different points in the orbit. These two points, X and Y, are located such that XY is perpendicular to the semimajor axis, and intersects it at the focus, i.e. where Star A is located. Astronomers measure the angle that Star B appears to change from X to Y as $\theta = 0.05''$.' If Planet A is 0.537 AU from Star A at perihelion, what is the distance to Star B in parsecs? [4]

12. On the vernal equinox, the Sun has a right ascension of 0 hours and a declination of 0°. Starting from the equinox, calculate the number of days, to the nearest tenth, that it takes for the Sun to have a right ascension of 4 hours. Assume a perfectly circular orbit and that the Earth is inclined 23.5° to the ecliptic plane. [4]

13. Rigel has equatorial coordinates RA = 05h 14m 32.3s, Dec = -08° 12' 06", and Betelgeuse has coordinates RA = 05h 55m 10.3s, Dec = +07° 24' 25". What is the angular separation between the two stars, and the position angle of Rigel relative to Betelgeuse? If a photographer wants to take a photo containing both stars, what is the maximum focal length they can use? [6]