Long Problem

a. Drawing the transit light curve, we can see that between the first and second contacts, the relative motion of the stars is equal to twice the radius of the transiting star. Similarly, the time between second and third contacts is proportional to twice the primary star's radius. Dividing, we get $\frac{r_1}{r_2} = \frac{t_2 - t_1}{t_3 - t_2} = 13.67$

We can use Wein's Law and the provided blackbody peaks to calculate the temperature of each star. We can now use the Stefan-Boltzmann Law to find the ratio of the luminosities:

$$\frac{L_1}{L_2} = \left(\frac{R_1}{R_2}\right)^2 \left(\frac{T_1}{T_2}\right)^4 = 934$$

b. We use the provided chart to determine the orbital period of the binary system (approximately 5.7 days) as well as the semi-amplitude (using $\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$). Using the period and velocity of each star, we determine the semimajor axes according to $r = \frac{vT}{2\pi}$ for semimajor axes of 0.0450 and 0.0771 AU for stars 1 and 2 respectively.

Applying Kepler's Third Law for the entire system, we get a total mass of 1.90 solar masses. Since $m_1 r_1 = m_2 r_2$, we can determine the individual mass of each star, 1.20 solar masses for star 1 and 0.70 solar masses for star 2.

c. Using the velocity of each star, we can find the relative velocity of the two stars (just sum). This can then be used to solve the equations from part a directly, giving radii of 1.35 and 0.099 solar radii for stars 1 and 2, respectively. We can now apply the Stefan-Boltzmann Law, dividing by the solar expression, to determine the luminosity of each star (2.54 solar luminosities for star 1, 0.0027 for star 2).

d. Apparent magnitude can be found by finding the total flux of the system and using the Sun's apparent magnitude and the solar flux as a standard candle, applying $\Delta M = 2.512 \log \frac{F_1}{F_2}$. Similarly, using the luminosity of the system with the Sun's absolute magnitude as a standard candle can provide an absolute magnitude for the system. Now that we have an apparent and absolute magnitude, we apply the distance modulus to get d = 5.94 pc.

e. Applying the small angle formula, we get a maximum angular separation of 0.021". The best possible resolution of the telescope is given by $\vartheta = 1.22\frac{\lambda}{D} * 206265 = 0.017$", so the stars are distinguishable. The smallest visible size can be found by applying the small angle formula with the limiting resolution, and is 0.10 AU.

f. Examining the given plot, there appears to be a longer period variation with a period of approximately 214 days. Based on the change in wavelength, the variation has an amplitude of approximately 83.2 km/s. Using this velocity and the period of oscillation, we can determine the semimajor axis of the binary stars' orbit (1.62 AU)

We now apply the fact that $m_1a_1 = m_2a_2$ and Kepler's third law for the binary-unknown system, solving for a_2 and m_2. We find that the unknown object has a mass of 16.2 solar masses and a semimajor axis of 0.19 AU. Given the high mass and lack of a visible counterpart, the object is likely a stellar mass black hole.