National Astronomy Olympiad
2013-2014
Marking Criteria

Long Answers:

Please note that there are other possible methods of solving these questions. Points were awarded for any method with correct reasoning. Points were also given for partially correct solutions and errors carried forward, however one point was deducted for an incorrect final answer.

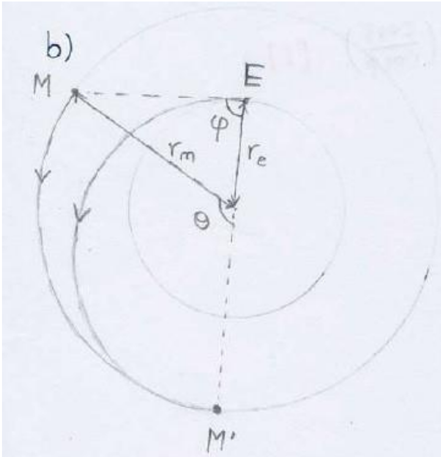
1. a)

   ⇨ Semimajor axis of orbit (a) = 0.5(1 AU + 1.52 AU) = 1.26 AU [1]

   ⇨ T^2 (in years) = a^3 (in AU)

   $T = (1.26)^{3/2} = 1.41$ years [2]

   ⇨ Time required = 0.5 × T = 0.707 years [1] b)



⇨ $T_{Mars} = (1.52)^{3/2}$ years = 1.87 years [2]

⇨ $\theta = \frac{0.707}{1.87} \times 360° = 136°$ [2]

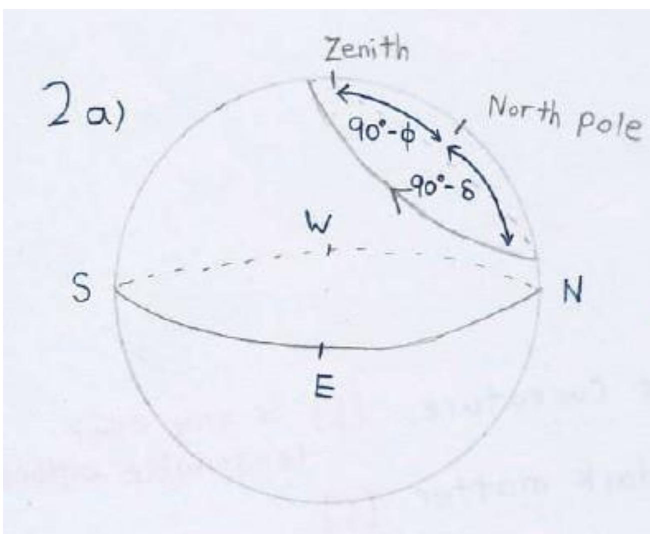
⇨ $ME^2 = r_m^2 + r_e^2 - 2r_m r_e \cos(180 - \theta)$

$ME = 1.06$ AU [2]

⇨ $r_m^2 = r_e^2 + ME^2 - 2r_e ME\cos(\varphi)$

⇨ $\varphi = 95°$ [2]

2. a)



Correct shape [1]

Correct direction [1]

Correct angles [1]

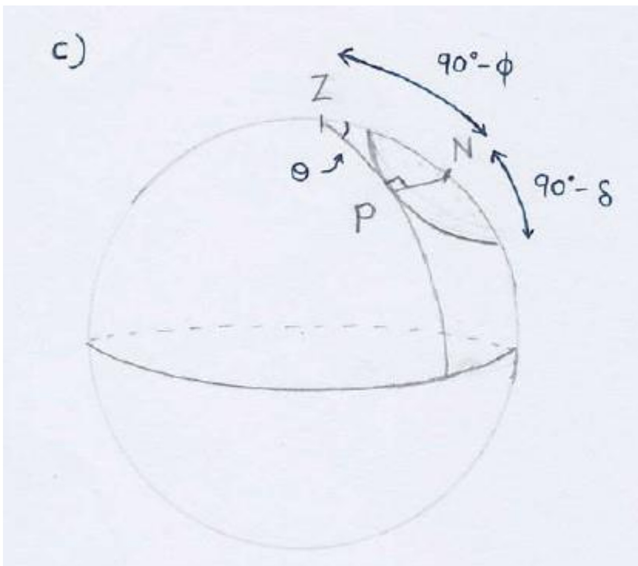
b)

⇨ $(90° - \phi) + (90° - \delta) < 90°$ [2]

⇨ $\phi + \delta > 90°$ [1]

Full points were awarded to correct answers without any working shown.

c)



For $(90° - \delta) > (90° - \phi)$, or $\phi > \delta$:

The maximum azimuth is 180°.

(According to the question, this is not the case)

For $(90° - \delta) < (90° - \phi)$, or $\phi < \delta$:

Let P be the point of greatest azimuth.

Angle ZPN = 90° [2]

Applying the spherical sine rule on ZPN:

$$\frac{\sin\theta}{\sin(90° - \delta)} = \frac{\sin 90°}{\sin(90° - \phi)}$$ [3]

$$\sin\frac{\theta = \cos\delta}{\cos\phi}$$

The maximum azimuth = $\sin^{-1}\left(\frac{\cos\delta}{\cos\phi}\right)$. [1]

3. a)

Consider a particle of mass *m* at a distance *r* from the center.

⇨ Kinetic Energy = $\frac{1}{2}mv^2 = \frac{1}{2}mH_0{}^2r^2$ [2]

⇨ Gravitational Potential Energy = $-\frac{GMm}{r} = -\frac{Gm}{r}\left(\frac{4}{3}\pi r^3\rho_C\right)$ [2]

⇨ For the particle to be critically bound:

Kinetic Energy + Gravitational Potential Energy = 0 [1]

⇨ $\rho_C = \frac{3H_0{}^2}{8\pi G}$ [1]

b)

⇨ $\rho_C = \frac{3H_0{}^2}{8\pi G}$ $= 8.66 \times 10^{-27}$ kg m^{-3} [1]

⇨ The density of the universe must be close to ρ_C [1]

⇨ As predicted by inflationary models/measurements of universe curvature/ other reasonable explanation [1]

⇨ Since $\ll \rho_C$, the remaining mass must be made up for by dark matter. [1]