## **Section B: Long Questions** [34 points]

1. One way to send a spacecraft to mars would be to employ an elliptical orbit with Earth at its perihelion and Mars at its aphelion. You may assume that Mars revolves around the Sun in a circular orbit of radius 1.52 AU.
   (a) How long does it take for the spacecraft to reach Mars? [4]
   (b) Assuming that the spacecraft is launched in the prograde direction, what elongation angle of mars should it be launched at to ensure that the spaceship reaches its destination? *(A prograde direction is in the same direction as that of the orbits of the planets. The elongation angle of mars is the angle between the Sun and mars as seen from the earth.)* [8]

2. Circumpolar stars are stars that never set below the horizon. Consider such a star with declination $\delta$, as seen by a northern observer at latitude $\Phi$.
   (a) Draw a figure that illustrates the motion of such a star in the sky. Label all relevant angles. [3]
   (b) Find the condition $\delta$ and $\Phi$ must fulfill for the star to be circumpolar. [3]
   (c) Write an expression (in terms of $\delta$ and $\Phi$) for the maximum azimuth of a circumpolar star that cannot reach 180 degrees. *(The azimuth of a star is its angle from north)* [6]

3. The critical density of the Universe is the density at which the gravitational attraction of matter within the universe is balanced with its expansion in such a way that neither will ultimately prevail. If the density of the universe were lower than this critical density, expansion will continue indefinitely. On the other hand, if the density of the universe were any higher, it would re-collapse upon itself.
   (a) The critical density of the universe is given by:

$$\rho_c = \frac{3{H_0}^2}{8\pi G}$$

   It is possible to arrive at this result through classical means. By modeling the universe as of having infinite extent and uniform density, expanding linearly with velocity $v = H_0 x$ derive the above expression. [6]
   (b) The density of ordinary (baryonic) matter in the universe is determined to be about $3.8\times10^{-28}$ kg m$^{-3}$. Hence, provide a justification for the existence of dark matter. *(Latest estimates of the Hubble constant suggest a value of $2.20\times10^{-18}$ s$^{-1}$)* [4]