Chapter 25

Astronomy and cosmology

The Andromeda galaxy contains a vast collection of stars and is similar to our own. Astronomers have deduced that it lies at a distance of 2.40×10^{19} km from our own galaxy. How can they know that? And, how can we calculate distances in the Universe and get an idea of its size?

Prior understanding

From Chapters 13 and 18 you may remember how you used the inverse square law for gravitational and electric forces to calculate force as a function of distance from a point source. You may recall from Chapter 7 how the Doppler effect affects the observed frequency of a source of sound waves moving relative to an observer, and what you learnt in Chapter 22 about absorption line spectra. It may be useful to look back at Chapter 12 to remind yourself how radians are used as a unit of angle measurement.

Learning aims

In this chapter you will learn how to calculate the distances of very distant stars and galaxies using their brightness. You will show how a distance scale using the idea of standard candles can be constructed which enables us to deduce how far away galaxies are. You will see how we can estimate how big stars are and their surface temperatures.

By examining the light from distant galaxies, you will see how this leads scientists to conclude that the Universe is expanding and once had a beginning far back in time in an event called the Big Bang.

25.1 Standard candles (Syllabus 25.1.1–25.1.4)

25.2 Stellar radii (Syllabus 25.2.1–25.2.3)

25.3 Hubble's Law and the Big Bang theory (Syllabus 25.3.1–25.3.4)

25.1 Standard candles

MEASURING ASTRONOMICAL DISTANCES

How far away are stars? Astronomers use a number of non-SI units to measure distances in space. A natural starting point as a unit for astronomical distances is the mean distance from the Earth to the Sun. This distance is called the **astronomical unit** (AU) and is equal to 1.50×10^{11} m. This unit is only appropriate on interplanetary scales as the distances to other stars (and galaxies) are too great to make the AU useful.

One way to measure their distances is by using parallax. Imagine looking at a candle held at arm's length. If you alternately open one eye and close the other, several times, then the candle will appear to jump back and forth relative to a fixed point in the background. The angle that the candle makes with your eye as it shifts its apparent position is called the **parallax angle** of the candle.

We can see parallax happening on an astronomical scale, as the Earth orbits the Sun. This time the candle is a nearby star, and the fixed point corresponds to the background of distant stars. These stars are so far away that they do not appear to change their positions as the Earth orbits the Sun (Figure 25.1). Suppose we record the angle p to a nearby star at two points on the Earth's orbit separated by a time interval of six months. This will give us a base line equal to the diameter of the Earth's orbit, or 2 AU and the maximum shift in position of the star from which to record the value of the parallax angle p.

Link

You saw in Topic 2.3, how parallax can produce a shift in position when viewing a marker from different lines of sight.

1 AU

1 parsec

1 ly

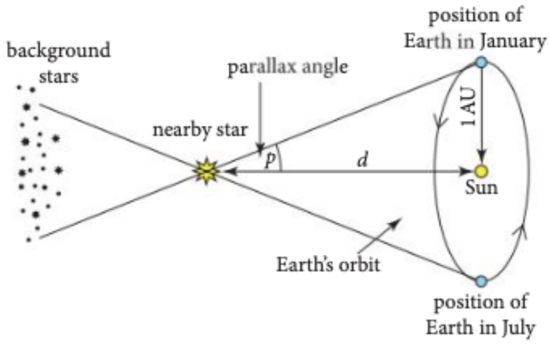


Figure 25.1 As the Earth orbits the Sun, a nearby star appears to shift its position with respect to the background stars.

By using simple trigonometry and a small angle approximation we can show that p (measured in radians) is related to the distance d of the star by

$$d = \frac{1\,\text{AU}}{\tan p} \approx \frac{1\,\text{AU}}{p}$$

Parallax angles even for nearby stars are very small so astronomers measure them in units of **arcminutes** and **arcseconds**. Since one radian = 57° 17′ 45″:

$$1\text{ rad} = (57 \times 3600)'' + (17 \times 60)'' + 45'' = 206\,265''.$$

$$\text{So one arcsecond} = \frac{1\,\text{rad}}{206265}$$

Since p (in arcseconds) is found from p (rad) ÷ 206 265, substituting for p in the equation above we get

$$\text{distance (m)} = \frac{1\,\text{AU} \times 206265}{p(\text{arcsec})}$$

An object whose parallax is 1 arcsec is defined to be at a distance of 1 **parsec** (pc). The word is an abbreviation of *par*allax and *sec*ond.

Tip

When measuring angles in radians we can use the fact that for angles close to 0 then $\tan\theta \approx \theta$.

Tip

An arc is a part of a circle that contains 360°. Just as a circle is divided into 360° then each degree is divided into 60 arcminutes (′) and each arcminute into 60 arcseconds (″). So 1° = 60′ = 3600″.

From the definition of the parsec, 1 pc = 1 AU × 206 265 = 1.50×10^{11} m × 206 265 = 3.09×10^{16} m.

Then we can write distance (pc) = $\dfrac{1}{p(\text{arcsec})}$ and $d = \dfrac{1}{p}$

where the unit of d is parsec and the unit of p is arcsecond.

Common multiples of the parsec are the kiloparsec (1 kpc); and the megaparsec (1 Mpc).

Worked example

A star observed in two images taken six months apart is observed to have a parallax angle of 0.078 arcsec. How far away from the Earth is it in parsecs (pc)?

Answer

To find the distance in parsecs we use

$$d = \frac{1}{p} = \frac{1}{0.078} = 13 \text{ pc (2 sf)}$$

Another distance unit commonly used by astronomers is the **light year** (ly) which is the distance that a photon of light would travel through space in one year:

1 ly = (3.00×10^{8} m s^{-1}) × (365 × 24 × 60 × 60 s) = 9.46×10^{15} m.

Since the speed of light is constant then if we are observing a galaxy which is 2.5 million ly away, then we are now observing light that was emitted from that galaxy 2.5 million years ago.

1. A measurement of the parallax of the star 61 Cygni in the constellation of Cygnus (the Swan) is found to be 0.316″. What is its distance from the Earth in each unit?

(a) parsec **(b)** metre **(c)** light year

(1 pc = 3.09×10^{16} m; 1 ly = 9.46×10^{15} m)

LUMINOSITY

As the distance to an astronomical object increases, the parallax angle becomes smaller to the point where it can be no longer measured. To measure stellar distances beyond this limit, astronomers use astronomical objects whose **luminosity** is known and from which we can infer their distances.

The luminosity L of an object is the total amount of energy in joules it radiates per second (that is, its power) and is measured in watts, W. Astronomers often express luminosities as multiples of the luminosity of the Sun ($L_{Sun} = 3.8 \times 10^{26}$ W).

If we imagine a star as a point source at the centre of a sphere of radius d, the energy passing through each square metre every second is the luminosity L divided by the surface area of the sphere. This quantity is the **radiant flux intensity**, F, of a star and is measured in watts per square metre (W m^{-2}).

Brightness

$F = \frac{L}{A}$

$A = 4\pi r^2$

$$F = \frac{L}{4\pi d^2}$$

The energy radiated from a star illuminates an ever-increasing area of a sphere as the distance from the star increases, so the flux intensity decreases as the square of the distance in an inverse square law (Figure 25.2).

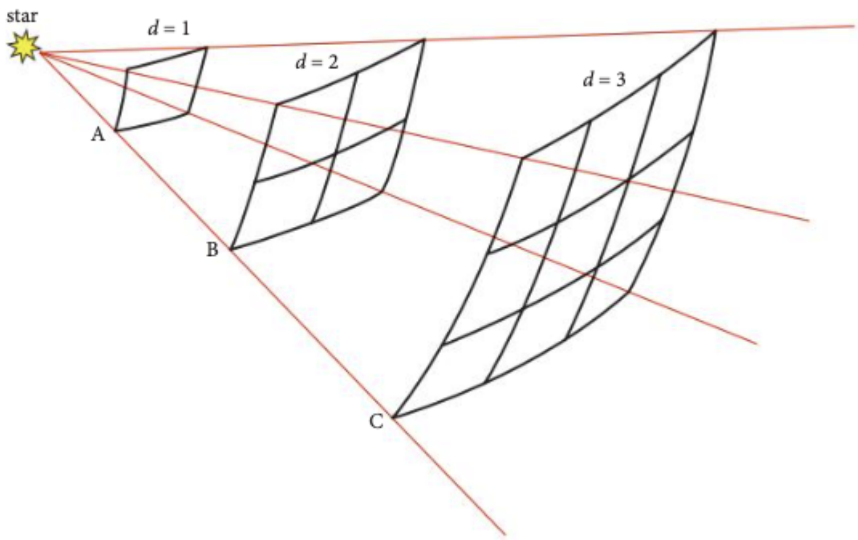


Figure 25.2 The radiant flux intensity of the star is nine times less at C than it is at A

Key ideas

- The astronomical unit (AU) and light year (ly) are astronomical distance units.
- One light year (1 ly) is the distance travelled by light in one year.
- The parsec (pc) is a distance unit defined using the parallax of a star, which is the apparent shift in position of the star with respect to the background of distant stars as the Earth orbits the Sun.
- The luminosity L of a star is the amount of energy in joules that it radiates per second. It is measured in watts, W.
- The radiant flux intensity F of a star varies in an inverse square law with its distance d, $F = \frac{L}{4\pi d^2}$. It is measured in W m^{-2}.

Worked example

The Sun has a luminosity of 3.8×10^{26} W. What is the radiant flux intensity of the Sun when viewed at a distance of 4 ly? Use 1 ly = 9.46×10^{15} m.

Answer

Use the equation for radiant flux intensity:

$$F = \frac{L}{4\pi d^2} = \frac{3.8 \times 10^{26}\,\text{W}}{4\pi \times (4 \times 9.46 \times 10^{15}\,\text{m})^2} = 8.5 \times 10^{-8}\ \text{W m}^{-2}$$

Now suppose we look at a series of candles of equal luminosity but at further and further distances. What we would observe is that the more distant candle would appear dimmer than the nearer ones (Figure 25.3). Similarly if all stars all had the same luminosity then they would appear dimmer at greater distances.

Astronomers can measure the radiant flux intensity of a star and if the luminosity of the star is known or can be estimated (see Cepheid variables and Topic 25.2), then its distance can be directly calculated from the equation for radiant flux intensity.

Figure 25.3 Candles of equal luminosity appear dimmer at further distances

Worked example

The flux intensity F from a distant star of known luminosity 5.7×10^{26} W is measured as 2.7×10^{-8} W m^{-2}. Calculate how far it is away from us.

Answer

$$F = \frac{L}{4\pi d^2}$$

Rearranging this equation we obtain:

$$d = \sqrt{\frac{L}{4\pi F}} = \sqrt{\frac{5.7 \times 10^{26}\ \text{W}}{4\pi \times (2.7 \times 10^{-8}\ \text{W m}^{-2})}} = 4.1 \times 10^{16}\ \text{m}.$$

2. The star Proxima Centauri has a known luminosity 2.8×10^{23} W. Astronomers measure the radiant flux intensity on Earth as 1.09×10^{-11} W m^{-2}. Calculate how far the star is away from us.

It is important to understand that this method of distance determination only works if we can be sure of what the actual luminosity of the star is. At any point in its life cycle the luminosity of a star is an intrinsic property of the star. Different stars will of course appear brighter or dimmer if they have different luminosities but what we are looking for are objects whose luminosities are known.

STANDARD CANDLES

Cepheid variable stars

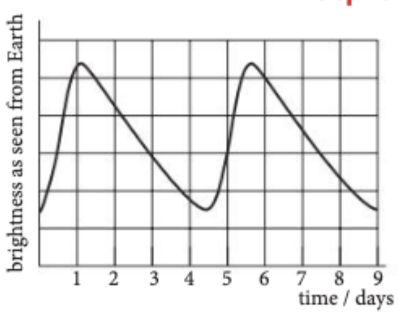


Figure 25.4 Light curve for a Cepheid variable

Astronomers have identified objects in space whose luminosity is known. These objects are called **standard candles** and are used to calculate very large distances to star clusters or galaxies. One type of standard candle is a type of star called a **Cepheid variable**. Cepheid variable stars vary in brightness but each Cepheid has a constant period. Observing their brightness at different times results in a **light curve** such as that shown in Figure 25.4.

3. Estimate the period of the Cepheid variable shown in Figure 25.4.

Around 1908, the astronomer Henrietta Leavitt discovered from observing many such stars in a nearby galaxy that the period of the variability was closely linked to the star's average brightness as seen from Earth: brighter stars had longer periods. From this, astronomers determined a relationship between period and luminosity for Cepheids (Figure 25.5). All Cepheids of a given period have the same luminosity. Astronomers can therefore measure the period of a Cepheid from the observed variation in its radiant flux intensity, and use the relationship shown in Figure 25.5 to deduce its actual luminosity.

Note that in Figure 25.5 the period and luminosity are both plotted on a logarithmic scale.

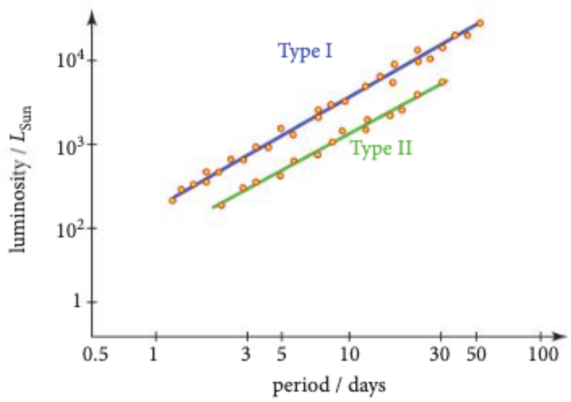


Figure 25.5 Period-luminosity relationship for the two types of Cepheid variable stars. (Cepheids classed as types I and II have different chemical compositions).

Tip

A logarithmic scale is a measurement scale where the difference between equally spaced marks on the scale is not the same, but increases by a constant factor depending on the base of the logarithm (usually base 10).

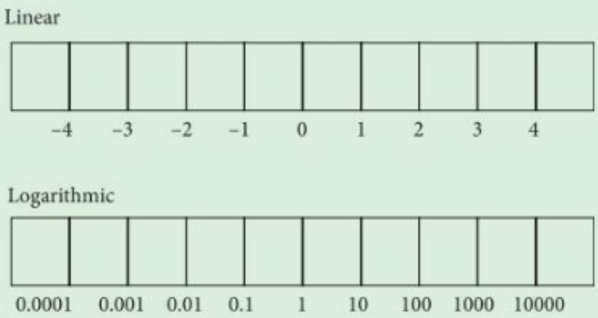


Figure 25.6 Linear and logarithmic scales. The top (linear) scale increases in equal intervals. In the bottom (logarithmic) scale the intervals increase by a factor of 10.

We use a logarithmic scale on a graph axis when we have a large range of values from small to very large and want to display them all on the same axis. A logarithmic scale compresses a wide range of values and makes the graph more manageable.

You may ask how we can know the actual luminosity of Cepheids if we do not know their distance? The answer is that distances to the nearer Cepheid variable stars can be measured by the method of parallax to establish their absolute luminosities. From these we can calibrate the relationship between period and luminosity, and so find the luminosities of Cepheids at even greater distances which we cannot measure using parallax.

Worked example

A Type 1 Cepheid variable star observed in the Andromeda galaxy is observed to have a period of 30 days. The observed radiant flux intensity of the star is 5.4×10^{-16} W m^{-2}, and the radiant flux intensity of the Sun at the Earth is 1.37×10^{3} W m^{-2}. The average distance between the Sun and the Earth is 1.50×10^{11} m.

Use this information and the relationship between period and luminosity shown in Figure 25.5 to estimate the distance to the Andromeda galaxy.

Answer

From Figure 25.5 for a Type 1 Cepheid with a period of 30 days $L \div L_{\text{Sun}} = 10^4$.

Using the relationship $F = \dfrac{L}{4\pi d^2}$ and the inverse square law

$$\frac{L_{\text{star}}}{L_{\text{Sun}}} = \left(\frac{d_{\text{star}}}{d_{\text{Sun}}}\right)^2 \times \frac{F_{\text{star}}}{F_{\text{Sun}}}$$

Rearranging, $d_{\text{star}} = \sqrt{\dfrac{L_{\text{star}}}{L_{\text{Sun}}} \times \left(\dfrac{F_{\text{Sun}}}{F_{\text{star}}}\right)} \times d_{\text{Sun}} = \sqrt{10^4 \times \left(\dfrac{1.37 \times 10^{3}\,\text{W m}^{-2}}{5.4 \times 10^{-16}\,\text{W m}^{-2}}\right)} \times 1.50 \times 10^{11}\,\text{m}$

$= 2 \times 10^{22}$ m

Note, only 1 significant figure is justified because the luminosity can only be estimated from the graph to the nearest order of magnitude.

Assignment 25.1: Calculating the distance to a galaxy using Cepheid variables as a standard candle

Figure 25.7 is a long exposure image of the Large Magellanic Cloud (LMC), a neighbouring galaxy to the Milky Way as seen from Earth.

Using telescopes, astronomers have detected Cepheid variable stars in the LMC and measured their brightness. The variation in brightness of a particular Type 1 Cepheid variable in the LMC, as seen from Earth, is plotted as a light curve (Figure 25.8).

Figure 25.7 The Large Magellanic Cloud

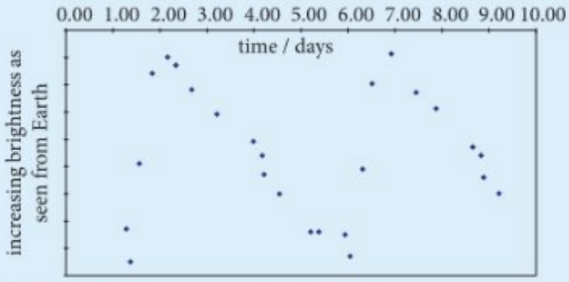


Figure 25.8 Light curve of a LMC Cepheid

Questions

A1. Estimate the period of this Cepheid using Figure 25.8.

A2. Estimate the luminosity of this Cepheid using the period-luminosity relationship shown in Figure 25.5. Luminosity of Sun = 3.8×10^{26} W.

A3. The average radiant flux intensity of the Cepheid is measured as 1.2×10^{-14} W m^{-2}.

- **(a)** Estimate the distance to the LMC in kilometres, using the fact that the Sun's radiant flux intensity at the Earth is 1.37×10^3 W m^{-2}.
- **(b)** Give your answer to part (a) in light years. (1 ly = 9.46×10^{15} m)

A4. This measurement is only an estimate.

- **(a)** Suggest what the sources of uncertainty in this measurement are.
- **(b)** How could astronomers improve the accuracy of the distance estimate?

A5. By considering your answer to question A3(b), how long ago are we viewing the LMC when we observe it today?

Figure 25.9 Supernova in galaxy M82

Type 1a supernovae

At very large distances, we cannot see individual stars in galaxies, so Cepheid variables cannot be used as standard candles for the further galaxies. How do we measure distance in the Universe to galaxies whose distances are so great that we cannot use Cepheids? The answer is that astronomers use another kind of standard candle called a **Type 1a supernova**.

A **supernova** is a star that suddenly and very rapidly increases luminosity because of an explosion that ejects most of its mass. Type 1a supernovae are so bright that when they explode they can be seen in distant galaxies (Figure 25.9). They then dim over a period of days and months (Figure 25.10).

All Type 1a supernovae explosions occur from the same mass of star and so they all reach nearly the same luminosity at the peak of their outburst. This allows Type 1a supernovae to be used as standard candles.

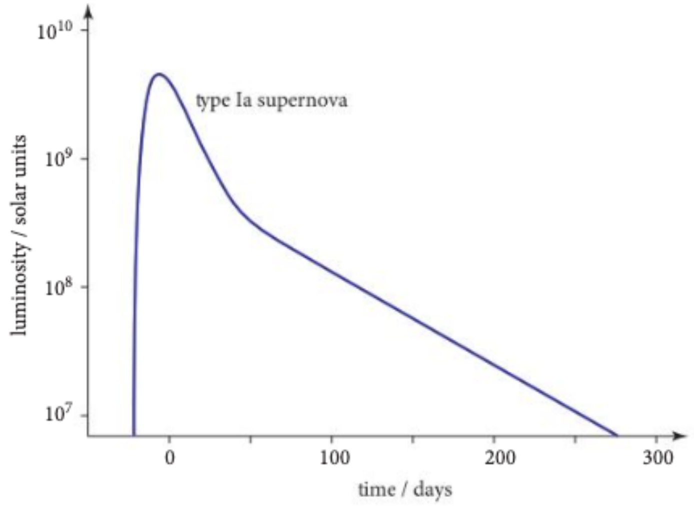


Figure 25.10 Light curve of a typical Type 1a supernova.

When astronomers observe a Type Ia supernova, they can measure its peak radiant flux intensity and use the inverse square law for radiant flux intensity F in terms of luminosity L to calculate the distance d of the supernova from Earth. Type Ia supernovae can be used to measure distances to remote galaxies from about 1 million light years to over 5 billion light years.

Worked example

A Type Ia supernova is observed in another galaxy with a peak radiant flux intensity of 9×10^{-18} W m^{-2}.

(a) If we assume that the peak luminosity of all Type Ia supernovae is about 10^{36} W, estimate the distance of the galaxy from Earth.

(b) How long ago did it explode? Use 1 ly = 9.46×10^{15} m.

Answer

(a) $F = \dfrac{L}{4\pi d^2}$ so $d = \sqrt{\dfrac{L}{4\pi F}} = \sqrt{\dfrac{10^{36}\ \text{W}}{4\pi \times 9 \times 10^{-18}\ \text{W m}^{-2}}} = 9 \times 10^{25}$ m (1 sf)

(b) 9×10^{25} m $= 9 \times 10^{25} \div 9.46 \times 10^{15}$ ly $\approx 10^{10}$ ly

The supernova exploded about 10^{10} or 10 billion years ago.

Assignment 25.2: Supernova light curves

Figure 25.11 shows the light curve for a Type 1a supernova observed in another galaxy.

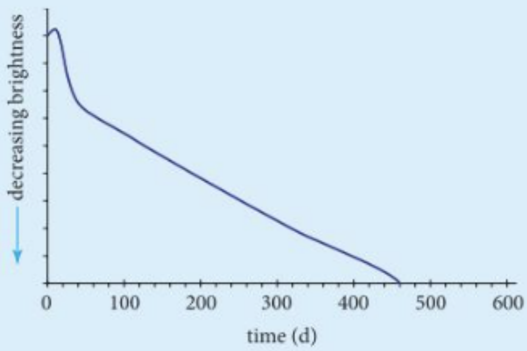


Figure 25.11 Type 1a supernova light curve.

Questions

A1. What property of Type 1a supernovae make them useful as standard candles?

A2. Describe the variation in brightness with time of the supernova, as seen from Earth.

A3. About how many days after the explosion did the supernova reach peak luminosity?

A4. At its peak luminosity, the radiant flux intensity recorded at the surface of the Earth was 7.5×10^{-15} W m^{-2}. Assuming that the peak luminosity of all Type Ia supernovae is about 10^{36} W, estimate the distance to the galaxy in light years. Use 1 ly = 9.46×10^{15} m.

A5. Approximately how long ago did the supernova explode?

A6. Suggest why your answer to A4 may only be an estimate.

A7. Suggest a reason why it can be difficult to measure distances using Type 1a supernovae compared with measuring distances using Cepheid variables.

Key ideas

- A standard candle is an object in space whose luminosity is known or can be reliably estimated.
- The distance to a standard candle can be calculated from its luminosity and how bright it appears from Earth, by using the equation $F = \frac{L}{4\pi d^2}$.
- Astronomers use Cepheid variable stars and Type 1a supernovae as standard candles to estimate the distances to stars and galaxies.
- Cepheid variables are stars whose luminosity varies with a regular period.
- Distances to the nearer Cepheid variable stars can be measured by the method of parallax to establish their absolute luminosities.
- All Cepheids of a given period have the same luminosity, so if the period of the variability of any Cepheid is measured, its luminosity can be found, and hence its distance estimated.
- At large distances where we cannot observe the light from individual stars in galaxies, Type 1a supernovae can be used as a standard candles to measure the distance to distant galaxies.
- Type 1a supernova can be used as standard candles as they all reach the same peak luminosity.

25.2 Stellar radii

How big are stars? Some stars are close enough for their diameters to be measured. The Sun has an angular size of about 0.5° or 30 arcminutes (30′).

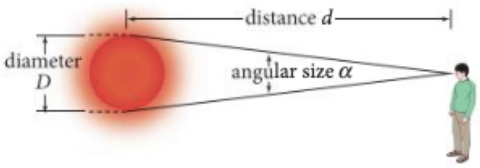


Figure 25.12 Angular size of a star

The angular size of a star refers to the star's apparent size expressed as an angle as seen by an observer on Earth. The angular size of the star is determined by its diameter and its distance from the observer. For a star of fixed diameter, the larger the distance, the smaller its angular size. For stars at a fixed distance, the larger the star's diameter, the larger its angular size (Figure 25.14).

In Figure 25.12 the angular size α is related to the distance d of the star by $\tan\frac{\alpha}{2} = \left(\frac{\frac{D}{2}}{d}\right)$

so $\frac{\alpha}{2} = \tan^{-1}\frac{D}{2d}$

For small angles α where $d >> D$ then $\tan \alpha \approx \alpha$ (measured in radians), so α (radians) $\approx \frac{D}{d}$.

The distances to stars are very large but many of the nearer ones are large enough to have measurable angular sizes, most commonly measured in arcseconds.

Worked example

The star Betelgeuse is at a distance of 6.1×10^{15} km from the Earth and has a measured angular size of 0.044″. What is the radius of Betelgeuse?

Answer

1 rad = 206 265 arcsec

diameter of Betelgeuse, $D = \frac{\alpha \times d}{206265} = \frac{0.044 \text{rad} \times 6.1 \times 10^{15} \text{km}}{206265}$

$= 1.3 \times 10^9$ km, so radius $= 6.5 \times 10^8$ km

Stars come in a huge range of sizes. The largest stars are supergiants which have radii several hundred times or even 1000 or more times the radius of the Sun. Giant stars have radii of up to a few hundred times that of the Sun. A white dwarf has a radius similar to that of the Earth whereas neutron stars are relatively tiny and have radii of the order of 10 km.

The size of a star depends on the way it was formed and its mass when it first begins to shine. Stars have a lifetime in which their radius remains largely constant but changes together with luminosity and temperature as they near the end of their lives.

Most stars appears as simple points of light even when viewed through telescopes. However, by measuring their luminosities astronomers can calculate their size. To do this they must first calculate their temperature. Fortunately the colour of a star is a good guide to its approximate surface temperature.

CALCULATING THE SURFACE TEMPERATURE OF A STAR

Suppose we take a metal bar and heat it with a blowtorch. At first the bar will glow a dull red. As it grows hotter, the bar will change colour from red through to orange and then yellow; and if it gets extremely hot (and could be prevented from melting), to a brilliant bluish white (Figure 25.13). This shows that, as an object is heated, more radiation is emitted, particularly at shorter wavelengths.

Figure 25.13 The hotter part of the metal is radiating strongly across the visible part of the spectrum, and appears yellow. Cooler parts emit most of their light in the red part of the spectrum.

Experiments show that all objects emit electromagnetic radiation over a continuous range of wavelengths, but for a particular temperature there will be one wavelength called the peak wavelength, for which the emission of radiation has its maximum intensity (rate of emission).

In Topic 22.4 you saw that the idea of a **black body** was developed to explain the shape of curves for the continuous spectra from hot objects at different temperatures. An ideal black body is an object which absorbs all the electromagnetic radiation that falls upon it.

In 1894, the German physicist Wilhelm Wien discovered a simple relationship between the absolute temperature T of an ideal black body and the wavelength at which the energy radiated from the object reaches its maximum intensity. The wavelength of the peak emission intensity, λ_{max}, is inversely proportional to the absolute temperature of the object:

$$\lambda_{max} \propto \frac{1}{T}$$

where T is the temperature on the absolute (kelvin) scale. This relationship is known as **Wien's displacement law** or sometimes simply Wien's law. It shows that the dominant wavelength of a black-body radiator decreases as it gets hotter, just as we observe when we heat the metal bar. An object at room temperature (300 K), for example, emits mainly infrared radiation. A very cold object of temperature a few kelvin above absolute zero emits primarily microwaves, whereas an object of a few million kelvin would emit mainly at X-ray wavelengths.

A graph of the intensity of radiation emitted from a black body against wavelength is a continuous curve and always has a characteristic shape. Figure 25.14 shows the intensity distribution of black-body radiators at different temperatures. Notice that the higher the temperature, the shorter the wavelength of maximum intensity, just as we would expect from Wien's law.

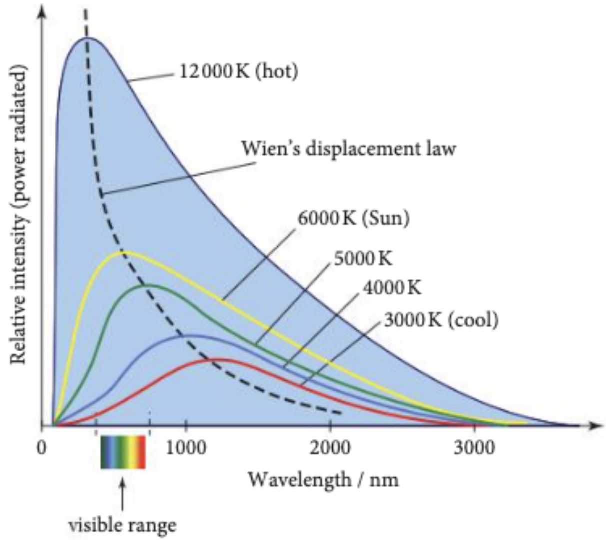


Figure 25.14 The intensity-wavelength curves of black-body radiators at different temperatures

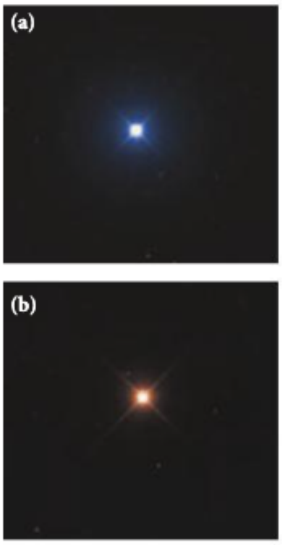


Figure 25.15 The colour of stars depends on their temperature (a) Vega is a hot bluish star with a surface temperature of 9600 K. (b) Aldebaran is a cooler red giant star with a surface temperature of 3900 K.

These curves are called **black-body curves**. The Sun and other stars emit radiation very much like an ideal black body. The Sun has a surface temperature of around 6000 K. The Sun emits radiation across the entire electromagnetic spectrum, but most of the radiation emitted is in the visible part of the spectrum, with a peak in the yellow region. The variations in spectra among stars are mostly due to their temperature and not their chemical composition. Hotter stars emit most of their radiation at shorter wavelengths and will appear to be bluer, whereas cooler stars emit at longer wavelengths and will appear to be redder.

Worked example

The Sun has a surface temperature of 5780 K, and the wavelength of light for which the maximum rate of emission occurs is 480 nm. The radiation from another star in our galaxy is found to have its maximum intensity at a wavelength of 250 nm. Estimate its surface temperature.

Answer

Using Wien's displacement law, $\lambda_{max} \propto \frac{1}{T}$

so $T = \left(\frac{5780\,\text{K} \times 480\,\text{nm}}{250\,\text{nm}}\right) = 11\,000\ \text{K}$

4. The wavelength of light for which the maximum rate of emission of light occurs from Vega is 300 nm. Using the values given for the surface temperatures of Vega and Aldebaran given in Figure 25.15, calculate the value of the wavelength in nm that corresponds to the peak rate of emission for Aldebaran.
5. Figure 25.16 shows the intensity-wavelength curves for 3 stars A, B and C.
   (a) Which star is the coolest?
   (b) Which star is the hottest?

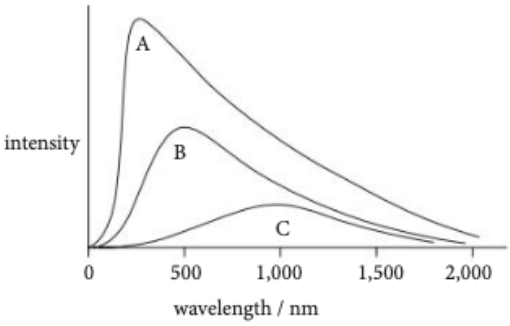


Figure 25.16

LUMINOSITY, SURFACE TEMPERATURE AND SURFACE AREA OF A STAR

We have seen that the luminosity of a star is the rate at which energy is radiated, in watts. The luminosity depends on the temperature of the star and its size.

In the late 19th Century, the Austrian physicist Josef Stefan carried out a series of experiments which showed the relationship between the rate of thermal energy emitted by a hot object and its temperature. This empirical relationship was also derived theoretically by another physicist, Ludwig Boltzmann. They both concluded that the total energy that the object emits per second is proportional to the fourth power of the object's absolute temperature, T^4. The total energy that a star emits per second defines the luminosity of the star L, hence $L \propto T^4$.

We know from the equation for radiant flux intensity that luminosity is directly proportional to the star's surface area ($L \propto A$). This leads to an equation for the luminosity of a star of surface area A at a given temperature T:

$L = \sigma A T^4$

where σ is a constant of proportionality called the Stefan-Boltzmann constant and has the value 5.67×10^{-8} W m^{-2}K^{-4}.

For a spherical star of radius r, the surface area is $4\pi r^2$. This gives the **Stefan–Boltzmann law**:

$$L = 4\pi\sigma r^2 T^4$$

This is known as **Stefan's law** and shows that the stellar luminosity is proportional to r^2 and T^4 where T is the star's surface temperature, in kelvin. In the case of the Sun, taking the Sun's surface temperature to be 5780 K, and its radius to be 6.96×10^8 m, the luminosity of the Sun is

$L_{sun} = 4\pi\sigma r^2 T^4 = 4\pi \times 5.67 \times 10^{-8}\ \text{W m}^{-2}\text{K}^{-4} \times (6.96 \times 10^8\ \text{m})^2 \times (5780\ \text{K})^4 = 3.85 \times 10^{26}\ \text{W}$

Tip

In an exam you will be expected to look up values of constants from the data and formulae given at the start of the question paper. A copy of this information is in the appendices.

CALCULATING THE RADIUS OF A STAR

Stefan's law forms the basis for all estimates of stellar size.

Worked example

The star Sirius A has a surface temperature of about 10 000 K and its luminosity is about 9.9×10^{27} W. Estimate the radius of Sirius A.

Answer

$L = 4\pi\sigma r^2 T^4$

so $r = \sqrt{\dfrac{L}{4\pi\sigma T^4}} = \sqrt{\dfrac{9.9 \times 10^{27}\ \text{W}}{4\pi \times 5.67 \times 10^{-8}\ \text{W m}^{-2}\text{K}^{-4} \times (10000\ \text{K})^4}} \approx 1 \times 10^9$ m

We can also combine Wien's displacement law and Stefan's law to estimate the radius of a star from its observed wavelength of peak emission and the star's luminosity. From Wien's law we first estimate its absolute temperature. Once this has been done then we can rearrange Stefan's law as

$$r = \sqrt{\frac{L}{4\pi\sigma T^4}}$$

to find the radius r, substituting in the values of T, L and σ.

Worked example

A black body emitting radiation at 2000 K has its maximum intensity at a wavelength of 1450 nm. The star Betelgeuse has been measured to have a wavelength of 850 nm at peak intensity and has a luminosity of 3.1×10^{31} W. Estimate its radius.

Answer

From Wien's law $\lambda_{max} \propto \frac{1}{T}$

T for Betelgeuse = 2000 K × 1450 nm ÷ 850 nm = 3410 K

Rearranging Stefan's law $L = 4\pi\sigma r^2 T^4$

$$r = \sqrt{\frac{L}{4\pi\sigma T^4}} = \sqrt{\frac{3.1\times10^{31}\,\text{W}}{4\pi\times5.67\times10^{-8}\,\text{W m}^{-2}\,\text{K}^{-4}\times(3140\,\text{K})^4}} = 6.7\times10^8\ \text{km}$$

6. The star Rigel appears bluish white and the star Arcturus appears orange when viewed through a telescope. Which of these stars is hotter?
7. The star Arcturus has a surface temperature of about 4300 K. The wavelength at which maximum intensity of emission occurs is about 670 nm. The star Rigel has a surface temperature of about 11 000 K. Show that for Rigel, the wavelength at which maximum intensity of emission occurs is about 260 nm.
8. Rigel has a luminosity of about 2.5×10^{31} W. The diameter of the Sun is 1.4×10^6 km. How many times larger is Rigel than the Sun?

Key ideas

- Stars are good approximations to black bodies so we can tell how hot a star is by observing its colour, which is related to the wavelength at maximum intensity of emission, λ_{max}.
- Wien's displacement law, $\lambda_{max} \propto \frac{1}{T}$, allows the peak surface temperature of a star to be estimated.
- The luminosity of a star depends on its size and its temperature.
- Stars come in different sizes and its radius r can be estimated from Stefan's law: $L = 4\pi\sigma r^2 T^4$ if its surface temperature T and luminosity L are known.

25.3 Hubble's law and the Big Bang theory

The study of the structure and development of the Universe as a whole is called **cosmology**. The task of the cosmologist is to construct theories of how different phenomena of nature, from small elementary particles and fundamental forces, right up to very large-scale structures in the Universe such as clusters of galaxies, all fit together.

Observational data and mathematical theory are both needed – often together with creative inspiration – to try to understand how the Universe formed and what might happen to it in the future.

When we use telescopes to look at distant regions of the Universe, we are looking back in time. This is because, although it is high, the speed of light (3.00×10^8 m s^{-1}) is finite. It takes light from the nearest star about four years to reach us across space. Some very distant galaxies are millions or even billions of light years away, and so we see the farthest galaxies as they were in the early Universe – the light that left them then is finally reaching us just now (Figure 25.17).

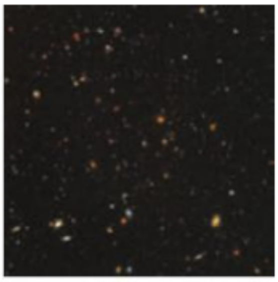


Figure 25.17 The Hubble ultra deep field. This image, taken by the Hubble space telescope in the direction of the constellation Fornax, shows an estimated 10 000 distant galaxies. The most distant objects in the image are over 1.3×10^{10} light years away, so we see them as they were 13 billion years ago.

A vast amount of cosmological information is available to us because of a seemingly everyday effect of physics. When a high-speed train is coming towards you while you are standing on a railway station platform, you may have noticed that the note of its sound is higher and then drops in frequency as it passes by and starts to recede. This is an example of the **Doppler effect**.

The Doppler effect depends on the relative motion of the source and the observer and works for any situation where a source of waves is in relative motion to an observer. So, if a light source is stationary and the observer is moving towards or away from it, the same shift to the blue (higher frequency) or red (lower frequency) occurs. Unlike with sound, we do not generally notice this effect with light, because the relative speed of source and observer needs to be very high.

Link

You learned about the Doppler effect in Topic 7.3.

THE SPECTRA OF STARLIGHT

In Topic 22.4, we saw that light with specific frequencies is emitted when a substance is vaporised and heated at low pressure. The frequencies of light correspond to particular transitions of electrons between energy levels in the atoms of the gas. Each element has an **emission line spectrum;** a unique set of photon frequencies that it can emit (Figure 25.18).

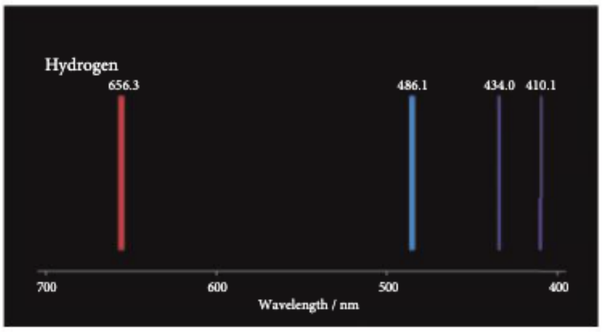


Figure 25.18 Emission line spectrum of hydrogen, showing bright lines on a dark continuous background

In a hot star, the gases present in the star's atmosphere are at high pressure. The atoms in the gas have considerable kinetic energy and undergo multiple collisions. By the time electrons in an **excited state** return to a lower energy level, further collisions between atoms in the gas have occurred. This results in a blurring of the individual bright lines in the emission line spectrum, giving rise to a continuous spectrum as shown in Figure 25.19.

Figure 25.19 A continuous spectrum of a star like the Sun

The atmosphere of the star acts as a source of visible light. This light then passes through the outer layers of the star which are much cooler and composed mainly of hydrogen gas. Photons of the characteristic energies of the transitions in the gas will be absorbed and atomic electrons raised to an excited state. As electrons fall back to the first level (the ground state), or intermediate levels, photons are emitted, but in random directions.

The resulting spectrum comprises dark lines (undetected photons) on a continuous background characteristic of an **absorption line spectrum** (Figure 25.20).

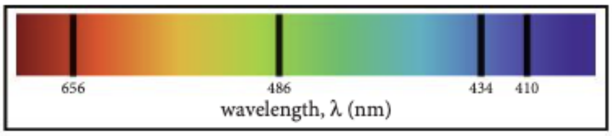


Figure 25.20 An absorption spectrum for hydrogen, showing dark lines on a continuous background. The dark lines in an absorption spectrum correspond exactly to the bright lines in an emission line spectrum produced by the same gas. Compare this spectrum with that in Figure 25.18.

Many stars like the Sun which are at high temperatures have a large amount of hydrogen in their atmospheres and display a hydrogen absorption line spectrum together with other dark lines corresponding to other elements that are present within the gas in the outer layers (Figure 25.21).

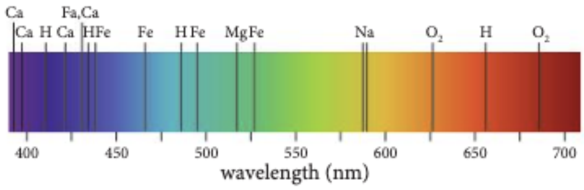


Figure 25.21 Absorption spectrum of the Sun showing the absorption of light by other elements as well as hydrogen in the cooler outer part of the Sun's atmosphere (although the O_2 absorption lines at 628 and 687 nm are actually due to the absorption of light in the Earth's atmosphere, before reaching a ground-based telescope).

As a consequence of the Doppler effect, a star that is in relative motion with respect to an observer on Earth will show an absorption spectrum that is shifted with respect to the same absorption lines as measured in a laboratory, as shown in Figure 25.22.

If the star and the Earth are moving towards each other, then the wavelengths of the absorption lines are shortened, that is, shifted towards the blue end of the spectrum (or blueshifted), and the effect is called blueshift (bottom diagram in Figure 25.22) Conversely, if the star and the Earth are moving away from each other, then the wavelengths of the absorption lines are lengthened, that is, moved towards the red end of the spectrum (or redshifted) and the effect is called **redshift** (top diagram in Figure 25.22). The name redshift arose from optical observations but now more generally means shifted to longer wavelength/lower frequency and not necessarily redder.

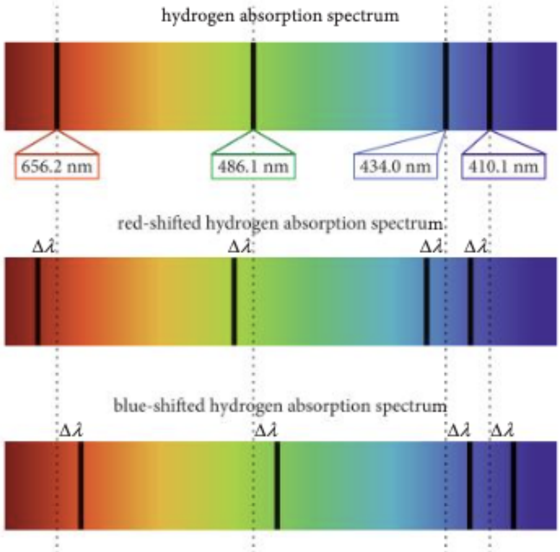


Figure 25.22 Doppler shift of absorption lines in the hydrogen spectrum of a star. The top diagram shows the hydrogen spectrum from a source at rest with respect to the observer (that is, the spectrum as observed in a laboratory). The centre diagram shows the observed hydrogen lines from the same star redshifted by an amount $\Delta\lambda$ (the star is receding from the observer). The bottom diagram shows the observed hydrogen lines from a similar stars blueshifted by an amount $\Delta\lambda$ (the star is approaching the observer).

The size of the wavelength shift is $\Delta\lambda = \lambda' - \lambda$, where λ the true wavelength of the absorption line and λ' is the apparent wavelength of the observed absorption line on Earth and depends on the relative velocity v of the star and the observer on Earth. The relationship is given by the equation:

$$\frac{\Delta\lambda}{\lambda} = \frac{\lambda' - \lambda}{\lambda} \approx \frac{v}{c}$$

where c is the velocity of light. The equation is only valid for objects where v is much less than c. Also note that v is the relative velocity along the line of sight between the source and observer.

The value $\frac{v}{c}$ is known as the **Doppler redshift** and is denoted by the symbol z. If z is positive, the star is receding from the observer. If z is negative the star is moving towards the observer. The Doppler redshift may also be expressed in terms of the change in frequency Δf as

$$\frac{\Delta f}{f} \approx \frac{v}{c}$$

where f is the frequency of the stationary source. The equation works for shifts in all parts of the electromagnetic spectrum, but is only valid where $v \ll c$.

Worked example

A particular spectral line in the spectrum of a star is found to have a wavelength of 600.80 nm compared to 600.00 nm as measured in the laboratory. What is the velocity of the star? Is it moving towards us or away from the Earth?

Answer

$\Delta\lambda = (600.80 - 600.00)\text{ nm} = 0.80\text{ nm}$

$$v = c\frac{\Delta\lambda}{\lambda} = \frac{(3\times10^{8}\text{ m s}^{-1}\times0.80\times10^{-9}\text{ nm})}{(600.0\times10^{-9}\text{ nm})} = 4\times10^{5}\text{ m s}^{-1}\text{ or }400\text{ km s}^{-1}$$

The observed wavelength is longer so it is redshifted, therefore the star it is moving away from Earth.

9. The H-alpha emission line in the hydrogen spectrum is at 656.00 nm when measured in the laboratory. Star A is observed to have that line at 656.60 nm, star B at 655.90 nm and star C at 656.40 nm.
   (a) Which star is moving the fastest relative to Earth (along the line of sight)?
   (b) What is the direction of motion of each of the stars?
10. Neutral, atomic hydrogen gas in the spiral arms of the Milky Way emits a spectral line of wavelength 21 cm, which is in the microwave part of the electromagnetic spectrum. The spectral line when detected by a radio telescope in a certain orientation is observed to be shifted by 0.1 mm less than 21 cm. How fast is this part of the galaxy moving relative to us along the line of sight? Is it moving towards us or away from the Earth?
11. The frequency of a calcium line in the absorption spectrum of the star Alpha Centauri is observed to have a frequency of 7.560×10^{14} Hz. The same line when observed in the spectrum of the Sun is measured at 7.559×10^{14} Hz. Calculate the speed at which Alpha Centauri is moving away from our solar system.

Key ideas

→ A star has an absorption spectrum (a series of dark lines) superimposed on its continuous emission spectrum characteristic of the elemental composition of its outer layers.

- The Doppler shift in a star's spectral lines compared with the same spectral lines observed in a laboratory on Earth, can be used to measure its velocity v relative to the Earth along the line of sight.
- In terms of wavelength, the Doppler redshift is $\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$.
- In terms of frequency, the Doppler redshift is $\frac{\Delta f}{f} = \frac{v}{c}$.

THE EXPANDING UNIVERSE

Observations of distant galaxies cannot be resolved into individual stars. Instead, the light from the whole galaxy is analysed spectroscopically. In the vast majority of cases, the absorption (or emission) spectra from distant galaxies are found to be redshifted. This indicates that all of these galaxies are moving away from us and so is evidence of an expanding Universe.

Worked example

The K absorption line in singly ionised calcium normally has a wavelength of 393.4 nm. In a spectrum from galaxy NGC 4889, the line occurs at 401.8 nm. Determine the redshift of this galaxy and its recession velocity.

Answer

$\Delta\lambda = (401.8 - 393.4)\text{ nm} = 8.4\text{ nm}$

$$z = \frac{v}{c} = \frac{\Delta\lambda}{\lambda} = \frac{8.4\,\text{nm}}{393.4\,\text{nm}} = 0.02$$

$v = c \times 0.02 = 3\times10^8\text{ m s}^{-1} \times 0.02 = 6.0 \times10^6\text{ m s}^{-1}$

Figure 25.23 shows that the redshift and the recession velocity (relative to our own galaxy) is greater for galaxies father away.

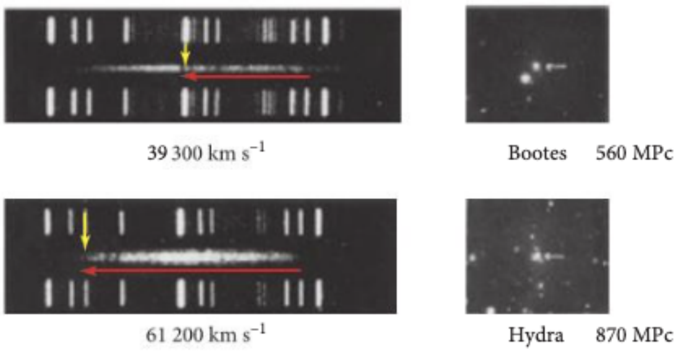


Figure 25.23 The optical spectra for two elliptical galaxies. Both have been taken with the same magnification. The yellow arrow indicates a pair of dark absorption lines that are shifted to longer wavelengths (redshifted). The figures on the right give the distance of the galaxy in Mpc and those below each spectrum give the recession velocity in km s^{-1}.

Hubble's law

The spectra of all galaxies, apart from a few very near to our own Milky Way, all show redshift. A plot of the recession velocity against distance for galaxies is close to a straight line and is called a Hubble diagram, named after Edwin Hubble, who published the relationship in 1929. Hubble had measured the distances of galaxies using Cepheid variables within them out to distances of about 20 Mpc (Figure 25.24(a)). Recent observational data has extended this to include galaxies as far distant as 5000 Mpc, around 16 billion light years (Figure 25.24(b)), where recession velocities are extremely high.

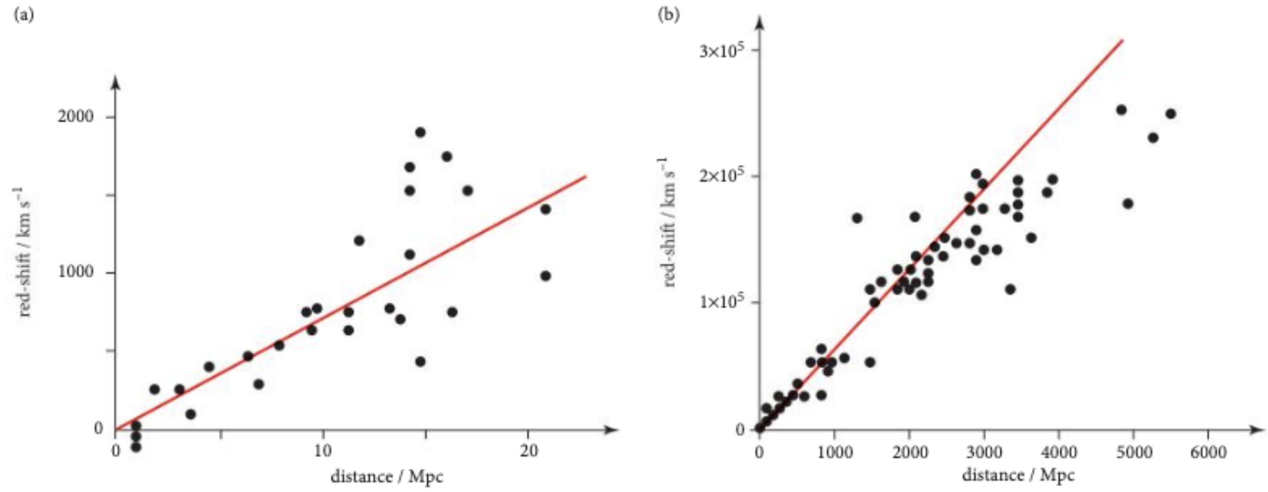


Figure 25.24 (a) Hubble's original data (replotted) showing the recession velocity of 28 nearby galaxies against their distance. Notice that some galaxies exhibited a small blue-shift. (b) Recent galactic data. Hubble's original data were confined to distances in the region between 0 and 20 Mpc.

The data show that the rate at which a galaxy recedes is directly proportional to its distance from us, that is

$$v \approx H_0 \times d$$

where v is the recession velocity and d is the distance of the galaxy. **Hubble's law** states that the speed of recession of galaxies from the Earth is proportional to the distance they are away from the Earth. The constant of proportionality in the equation, H_0, is the **Hubble constant**, which is determined from the gradient of a Hubble diagram. In Figure 25.24 the recession velocity is in km s^{-1} and the distance of the galaxy is in Mpc, so the gradient of the line of best fit indicates a value for the Hubble constant of about 68 km s^{-1} Mpc s^{-1}. Some current best estimates give $H_0 = 67.3 \pm 0.9$ km s^{-1} Mpc^{-1}. However, this value is constantly under review as more data is collected.

If we use SI units and express v in m s^{-1} and d in metres and take 1 Mpc = 3.1×10^{22} m, then $H_0 = 67.3 \times 10^3$ m $s^{-1} \div 3.1 \times 10^{22}$ m $= 2.2 \times 10^{-18}$ s^{-1}.

Once a value of a distant galaxy's recession velocity is known, Hubble's law can be used to estimate its distance.

Worked example

The recession velocity of the galaxy NGC 4889 has been determined to be $v = 6.4 \times 10^6$ m s^{-1}. Estimate its distance in m. Take $H_0 = 2.2 \times 10^{-18}$ s^{-1}.

Answer

$d \approx v \div H_0 = 6.4 \times 10^6$ m $s^{-1} \div 2.2 \times 10^{-18}$ $s^{-1} = 2.9 \times 10^{24}$ m

12. Show that the reciprocal of the Hubble constant has the unit of second.

13. **(a)** Suggest sources of error that may contribute to uncertainties in estimates of the Hubble constant.

(b) What features of Figure 25.24(b) suggest that it represents a better estimate of the Hubble constant than Figure 25.24(a)?

(c) There is some uncertainty in the value of H_0. Some estimates put its value at 2.2×10^{-18} $s^{-1} \pm 10\%$. A galaxy is moving away from us with a recession velocity of 5500 km s^{-1}. Calculate its maximum and minimum distance from us.

14. **(a)** Figure 25.25 shows the recession velocity against distance for a number of galaxies. Estimate from it the value of the Hubble constant in SI units. (1 Mpc = 3.1×10^{22} m.)

(b) Use your value of H_0 to work out the distance, in km, of a galaxy with $z = 0.002$.

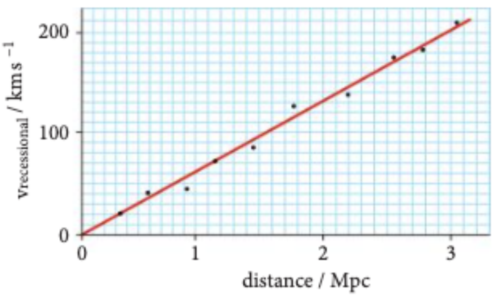


Figure 25.25

Hubble's law is a simple statement but with huge consequences. It is observational evidence that shows that the Universe is expanding. An expanding Universe means that it is cooling down – so the further back in time, the hotter and denser the Universe was. Cosmologists theorise that at a time $t = 0$ the Universe came into being from an infinitely hot, infinitely dense point called a **singularity** and has been expanding ever since. This is the **Big Bang theory**.

The Big Bang is mistakenly imagined as a literal bang, or an explosion. In the earliest history the Universe was similar to a tremendous, expanding fireball full of particles and antiparticles of different types all which were moving away from each other at great speed accompanied by high energy electromagnetic radiation all of which were cooling down as it expanded. However, the Big Bang is not an explosion as it is not expanding into anything. Instead we should think of it as space itself expanding from a point of infinite density. The Earth and everything else in the Universe are all inside this expansion having formed within over vast periods of time.

In fact, cosmologists do not think of distant galaxies as moving through space away from us, but regard space itself to be expanding, and the light waves being stretched along with it. The wavelength of light will increase as it crosses the expanding Universe, between its point of emission and where it is detected, by the same amount that space has expanded during the crossing time.

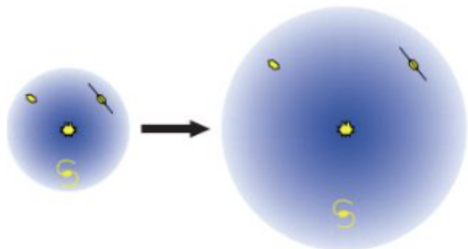


Figure 25.26 The cosmological redshift is space itself expanding from a Big Bang (the centres of the spheres). As a result, an observer anywhere in the Universe would see all distant galaxies receding from one another.

The redshift of a galaxy due to the expansion of the space between distant galaxy and our own galaxy is known as the cosmological redshift. The Doppler redshift and the cosmological redshift cannot distinguished from one another by observing the spectrum of the light source.

It should be noted that having v (speed of recession of a galaxy from the Earth) proportional to d (the distance the galaxy is from Earth) in Hubble's law does not imply that the Earth is at the centre of the expansion, rather it shows that as the Universe expands all the galaxies are receding from each other (Figure 25.26).

The age of the Universe

An accurate value of the Hubble constant, and the assumption that this has remained constant through all time, allows an estimate of the age of the Universe. If in time t a galaxy has moved outwards a distance d at velocity v, then $d = v \div t$.

But from Hubble's law we have $v = H_0 \times d$ so if we assume that H_0 has been constant then

time (age of Universe) $= \dfrac{d}{v} = \dfrac{1}{H_0}$.

Using the value of $H_0 = 2.2 \times 10^{-18}\ s^{-1}$ gives the estimated age of the Universe as $1 \div 2.2 \times 10^{-18}\ s^{-1} = 4.6 \times 10^{17}\ s = 14.6$ billion years.

The Hubble constant is one of the most fundamental quantities of nature, as it specifies the rate of expansion of the entire Universe. What we can infer from this is that the

Universe must have been much more dense in the past and began about 14.6 billion years ago.

Key ideas

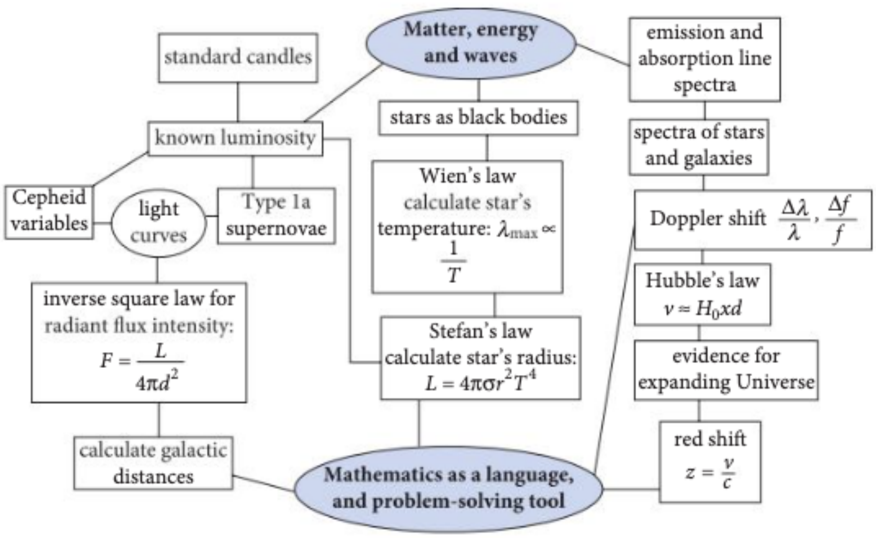
- The size of the Doppler shift for a galaxy gives the galaxy's recession velocity.
- Hubble's law results from observational data and states that the recession velocity v of a distant galaxy is proportional to its distance d: $v \approx H_0 \times d$ where the constant of proportionality is called the Hubble constant H_0.
- Hubble's law is evidence that the Universe is expanding and at one time in the distant past must have been much more dense.
- The reciprocal of the Hubble constant indicates the current age of the Universe.
- It is thought the Universe began from a point of infinite density called a singularity and rapidly expanded in an event known as the Big Bang.
- The cosmological redshift is a measure of how fast galaxies are receding from each other.

15. Explain why variations in measurements of the Hubble constant affects the estimated age of the Universe.

16. There is some uncertainty in the value of H_0. Some estimates put its value as high as 2.45×10^{-18} s^{-1} or as low as 2.10×10^{-18} s^{-1}. What is the range of estimated ages of the Universe with these values of the Hubble constant?

CHAPTER OVERVIEW

Try copying this mini mind map and expanding upon it. Use your notes from other chapters to help you explore how the essential ideas, theories and principles can be linked further together.



WHAT YOU HAVE LEARNED

- Understand that luminosity is the total output power of a star, in watts (W).
- Understand the unit of radiant flux intensity as the power radiated per square metre.
- Recall and use the inverse square law for radiant flux intensity, $F = \frac{L}{4\pi d^2}$, to calculate stellar distances.

- Understand that a standard candle is an object that has a known luminosity and can be used to determine the distances to stars or galaxies.
- Know that stars emit light of many different wavelengths, but the wavelength of light for which maximum rate of emission occurs indicates its temperature.
- Recall and use Wien's displacement law $\lambda_{max} \propto \frac{1}{T}$ to find the wavelength for which maximum rate of emission occurs from a star at a given temperature, or vice versa.
- Know that stars come in different sizes, and that the luminosity of a star depends on its radius as well as its temperature: Stefan's law, $L = 4\pi\sigma r^2 T^4$.
- Know how to use Wien's displacement law and Stefan's law to calculate the temperature of a star and estimate its radius given its luminosity.
- Understand that the lines in the emission spectra from distant objects show an increase in wavelength from their known values.
- Be able to use the redshift equations $\frac{\Delta\lambda}{\lambda} \approx \frac{\Delta f}{f} \approx \frac{v}{c}$ for the redshift of radiation from a source moving relative to observer.
- Be able to explain that the redshift of galaxies is evidence that the Universe is expanding.
- Recall and use Hubble's law $v \approx H_0 \times d$ and explain how this leads to the Big Bang theory.

CHAPTER REVIEW

1. What do astronomers call objects that have known luminosities?
2. The star Altair has a luminosity of 10.6 times the luminosity of the Sun and lies at a distance of 1.6×10^{17} m from the Earth. The luminosity of the Sun is 3.8×10^{26} W. Calculate the radiant flux intensity received on Earth from Altair.
3. A distant galaxy contains a star whose luminosity is known to be 3.4×10^{27} W. The radiant flux intensity from the star is measured by an astronomer on Earth. She finds its value to be equal to 8.2×10^{-11} W m^{-2}. How far is the galaxy from Earth?
4. The star Vega appears blue when viewed through a telescope whereas the star Arcturus appears orange. Which of these stars is hotter than the other?
5. The stars Procyon A and Procyon B both emit a continuous spectrum of radiation. Procyon A has a surface temperature of about 6400 K and Procyon B has a surface temperature of about 7700 K. The wavelength for which the peak intensity of emission from Procyon A occurs is 453 nm. Will Procyon B have a shorter, or longer wavelength at its maximum intensity of emission? Explain your reasoning.
6. Procyon A has a luminosity of 2.6×10^{27} W. Use Stefan's law, $L = 4\pi\sigma r^2 T^4$ to estimate its radius.
7. What is meant by the term redshift?
8. The wavelength of a spectral line measured in the light from a distant galaxy has a wavelength of 460 nm. On Earth this spectral line occurs at a wavelength of 430 nm. Determine the redshift of this galaxy and its recession velocity.
9. What does Hubble's law tell us about the nature of the Universe?
10. What is the Big Bang theory?

CHAPTER 25

E1. **(a)** State what is meant by a standard candle. [1]

(b) A Cepheid variable star in galaxy G1 is found to have a measured flux intensity of 1.18×10^{-4} W m^{-2}. The distance to G1 is known to be 1.5×10^{21} m. A telescope observed a Cepheid with the same period in galaxy G2 with a measured flux intensity of 6.1×10^{-15} W m^{-2}.
Calculate the distance to galaxy G2. [4]

E2. Figure 25.27 shows the light curve of a Type 1 Cepheid variable star. It is found to have a measured flux intensity on Earth of 500 nW m^{-2}.

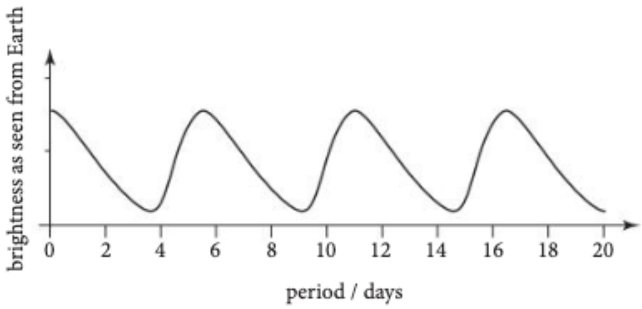


Figure 25.27

(a) Determine the period. [1]

(b) Figure 25.28 is a period-luminosity diagram for Type 1 Cepheid variable stars.

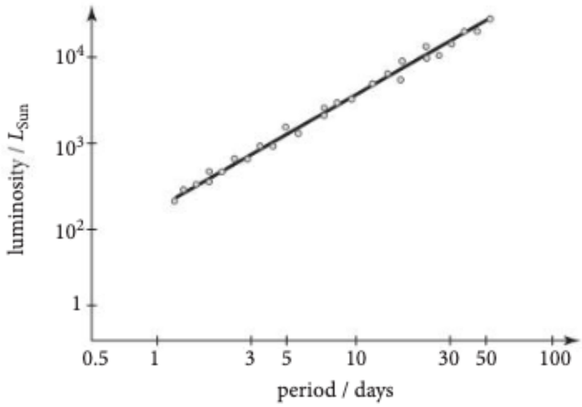


Figure 25.28

Use your answer to part (a) to estimate the luminosity of the Cepheid and hence calculate the distance of the Cepheid from the Earth. (Luminosity of Sun = 3.8×10^{26} W) [4]

E3. **(a)** The emission lines of hydrogen in the spectra of almost all galaxies show a redshift. Explain the meaning of the term redshift. [2]

(b) One of the lines in the hydrogen spectrum has a wavelength of 21.1 cm. Measurements of the redshift of this line in the spectrum of galaxy M84 show it is redshifted by 0.0633 cm.
Calculate the velocity of galaxy M84. [2]

(c) Given that this galaxy is 60 million light years distant, calculate a value for the Hubble constant in s^{-1}.
Use 1 ly = 9.46×10^{15} m. [1]

(d) Suppose at some distant time in the future, astronomers observed that most galaxies were showing a blue shift in their spectra. What could you deduce about the expansion of the Universe? [2]

E4. The star Antares has a surface temperature of 3400 K and the wavelength of peak intensity of emission is 852 nm. Another star Spica has a wavelength of peak intensity of emission of 126 nm.

(a) Which of these stars is the coolest? Explain your reasoning. [2]

(b) Estimate the surface temperature of Spica. [2]

(c) The luminosity of Spica is about 4.5×10^{30} W. Estimate the radius of Spica. [2]

Practice exam-style questions

E5. **(a)** State Hubble's law. [1]

(b) A distant galaxy is observed to have a recession velocity of 2800 km s^{-1} and lies at a distance of 1.2×10^{24} m. Estimate the value of the Hubble constant. Give the unit for your answer. [3]

(c) Show that you can take $\frac{1}{H_0}$ as an estimate for the age of the Universe. [1]

(d) From your answer to part (b) estimate the age of the Universe in years.
1 year = 3.2×10^7 s. [2]

E6. **(a)** Explain what is meant by the Big Bang and state the evidence that supports it. [2]

(b) A spectral line in a distant galaxy is observed to have a wavelength of 420 nm. The same line observed in a laboratory on Earth has a wavelength of 395 nm.

(i) In what direction is the galaxy moving? [1]

(ii) What is its velocity? [1]

CHAPTER 25 – ASTRONOMY AND COSMOLOGY

In-text questions

1. (a) $d = \frac{1}{p} = \frac{1}{0.316} = 3.16$ pc

   (b) $3.16 \times 3.09 \times 10^{16} = 9.76 \times 10^{16}$ m

   (c) $\frac{9.76 \times 10^{16}}{9.46 \times 10^{15}} = 10.3$ ly

2. $F = \frac{L}{4\pi d^2}$; $d = \sqrt{\frac{2.8 \times 10^{23}}{4\pi \times (1.09 \times 10^{-11})}} = 4.5 \times 10^{16}$ m.

3. From peak to peak about 4.5 days (4.3 to 4.6 days)

4. $\lambda_{max} \propto \frac{1}{T}$, so $\lambda_{max}T$ for Vega $= \lambda_{max} T$ for Aldebaran

   λ_{max} for Aldebaran $= 300 \times \frac{9600}{3900} = 740$ nm

5. (a) C ($\lambda_{max} \propto \frac{1}{T}$, so the wavelength that corresponds to the peak intensity of a star's emission will be longer for cooler stars)

   (b) A.

6. Rigel ($\lambda_{max} \propto \frac{1}{T}$, so the wavelength that corresponds to the peak intensity of a star's emission will be shorter for hotter stars)

7. using Wien's law
   $\lambda_{max} T$ for Arcturus $= \lambda_{max} T$ for Rigel
   λ_{max} for Rigel $= 670 \times \frac{4300}{11000} = 260$ nm

8. Using Stefan's law, radius of Rigel $= \sqrt{\frac{2.5 \times 10^{31}}{4\pi \times 5.67 \times 10^{-8} \times (11000)^4}}$
   $= 4.9 \times 10^7$ km
   or a diameter of 9.8×10^7 km. Rigel is therefore $\frac{(9.8 \times 10^7 \text{ km})}{(1.4 \times 10^6 \text{ km})}$
   $= 70$ times bigger than the Sun.

9. The shifts for the three stars are as follows.
   Star A: $\Delta\lambda = 656.60 - 656.00 = 0.60$ nm
   Star B: $\Delta\lambda = 655.90 - 656.00 = -0.10$ nm
   Star C: $\Delta\lambda = 656.40 - 656.00 = 0.40$ nm

   (a) Star A shows the greatest shift, so is moving the fastest relative to Earth

   (b) $\frac{v}{c} = \frac{\Delta\lambda}{\lambda}$, so star A is receding from Earth (redshift, positive velocity), star B is approaching Earth (blue-shift, negative velocity), and star C is receding (redshift, positive velocity).

10. $\Delta\lambda = 20.99 - 21 = -0.01$ cm $= -1.0 \times 10^{-4}$ m, $\frac{v}{c} = \frac{\Delta\lambda}{\lambda}$, so
    $v = \frac{(3 \times 10^8) \times (1.0 \times 10^{-4})}{21 \times 10^{-2}}$
    $= -1.4 \times 10^5$ ms^{-1}. This is negative so it is moving towards us.

11. $\Delta f = (7.560 - 7.559) \times 10^{14}$ Hz $= 0.001 \times 10^{14}$
    $\frac{\Delta f}{f} = \frac{v}{c}$ so $v = \frac{(3 \times 10^8) \times (0.001 \times 10^{14})}{(7.559 \times 10^{14})} = 39.6$ kms^{-1}

12. $v = H_0 \times d$ so $\frac{1}{H_0} = \frac{d}{v}$, which in SI units is $\frac{\text{km}}{(\text{km s}^{-1})}$. The km cancel out, leaving the unit of second.

13. (a) The motion of the galaxy may not be along the line of sight between the source and observer. The galaxy may also have rotational motion (some parts of the galaxy are rotating towards us, while others are rotating away from us). Motion of some galaxies may be influenced by nearby ones, due to their stronger gravitational attraction.

    (b) Less scatter around the line of best fit. More data points.

    (c) Maximum and minimum values of H_0 for ±10% of 2.2×10^{-18} s^{-1} are 2.42×10^{-18} s^{-1} and 1.98×10^{-18} s^{-1}
    Using Hubble's law, minimum distance of galaxy
    $= \frac{(5500 \times 10^3)}{(2.42 \times 10^{-18})} = 2.3 \times 10^{24}$ m

    maximum distance of galaxy $= \frac{(5500 \times 10^3 \text{ ms}^{-1})}{(1.98 \times 10^{-18} \text{ s})} = 2.8 \times 10^{24}$ m.

14. (a) The Hubble constant is found from the slope of the graph, which is about $\frac{203}{3} = 68$ km s^{-1} Mpc^{-1} or in SI units,
    $= \frac{(68 \times 10^3 \text{ ms}^{-1})}{(3.1 \times 10^{22} \text{ m})} = 2.2 \times 10^{-18}$ s^{-1}

    (b) $v = zxc = 0.002 \times 3.00 \times 10^8 = 600$ km s^{-1} So distance $= \left(\frac{v}{H}\right)$
    $= \frac{(600 \times 10^3)}{(2.2 \times 10^{-18})}$
    $= 2.7 \times 10^{23}$ m $= 2.7 \times 10^{20}$ km

15. The reciprocal of the Hubble constant indicates the age of the Universe from its creation in the Big Bang so variations in its value mean that there are variations in how old the Universe is thought to be.

16. For $H_0 = 2.45 \times 10^{-18}$ s^{-1}, age of Universe $= \frac{1}{(2.45 \times 10^{-18} \text{ s}^{-1})}$
    $= 4.1 \times 10^{17}$ s or 12.9 billion years. For $H_0 = 2.10 \times 10^{-18}$ s^{-1}, age of Universe $= \frac{1}{(2.10 \times 10^{-18} \text{ s}^{-1})} = 4.1 \times 10^{17}$ s or 15.1 billion years.

    The smaller the value of H_0 then the older the estimated age of the Universe is thought to be.

Assignment 25.1

A1. about 5 days

A2. 10^3 or 1000 times the luminosity of the Sun, i.e. 3.8×10^{29} W. (Estimate of L between 900 and 1100).

A3 (a) Using the inverse square relationship

$$d_{Cepheid} = \sqrt{\frac{L_{Cepheid}}{L_{Sun}} \times \left(\frac{F_{Sun}}{F_{Cepheid}}\right)} \times d_{Sun}$$

$$= \sqrt{10^3 \times \left(\frac{1.37 \times 10^3}{1.2 \times 10^{-14}}\right)} \times (1.50 \times 10^{11}) = 1.6 \times 10^{21} \text{ m, or}$$

2×10^{18} km (to 1 sf)

(b) $\frac{2 \times 10^{21}}{9.46 \times 10^{15} \text{ m}} = 200\,000$ ly (to 1 sf)

A4. (a) Uncertainties in the period-luminosity relationship shown in the graph / how well the graph is calibrated for distances to the nearby Cepheid variables (random and systematic errors involved in calculating the distance to these stars); difficulty in reading the scale of the graph by eye; possible sources of error in measuring the variation in radiant flux intensity of the Cepheid: The observations are taken over a number of days and the observing conditions could be different on each night (meaning more or less of the light from the star is absorbed or scattered by the atmosphere) or there may be variations in the sensitivity of the detectors attached to the telescope.

(b) By searching for other Cepheid variable stars in the LMC and finding a mean for the distance calculation

A5. A light year is the distance that light will travel in one year. Therefore we are viewing the LMC as it was when the light was emitted, about 200,000 years ago.

Assignment 25.2

A1. Type 1a supernovae all reach the same peak luminosity.

A2. The brightness rose rapidly to a peak value in a few days after it was first detected and then declined rapidly over the next 20 days and then more slowly over the next few 100 days.

A3. About 12 days (answers in the range 5 to 15 days.)

A4. $F = \frac{L}{4\pi d^2}$ so $d = \frac{L}{4\pi F}$

$= \sqrt{\frac{10^{36}}{4\pi \times (7.5\times10^{-15})}}$

$= 3.26 \times 10^{24}$ m

or $\frac{3.26\times10^{24}}{9.46\times10^{15}}$ ly

$\approx 300 \times 10^6$ ly

A5. About 300 million years ago

A6. The light from the supernova has to pass through the interstellar medium which contains gas and dust. Some of the light will be absorbed by the interstellar medium before it reaches the Earth and also by the atmosphere of the Earth. Also each type 1a supernova may not reach exactly the same luminosity as the explosion s may not be uniform and symmetric and lead to variations in luminosities

A7. The difficulty of measuring distances with Type 1A supernovae is that unlike Cepheid variable stars they have a limited life. You have to spot them before they reach maximum brightness, so that you can be sure that you are measuring their radiant flux intensity at their peak luminosity. You also cannot predict where and when a supernova will explode, so cannot arrange in advance to observe them with a telescope.

Chapter review

1. Standard candles. Examples of standard candles are Cepheid variable stars and Type 1a supernovae.

2. $F = \frac{L}{4\pi d^2} = \frac{10.6\times3.8\times10^{26}W}{4\pi(1.6\times10^{17}m)^2} = 1.2\times10^{-8}$ W m^{-2}

3. From the inverse square law for radiant flux intensity, $d = \sqrt{\frac{L}{4\pi F}} = \sqrt{\frac{3.4\times10^{27}}{4\pi\times8.2\times10^{-11}}} = 1.8\times10^{19}$ m

4. Vega. Using Wien's law, $\lambda_{max} \alpha \frac{1}{T}$, so the wavelength that corresponds to the peak intensity of a star's emission (the colour it appears) will be shorter (more blue) for hotter stars

5. Shorter. Using Wien's law, $\lambda_{max} \alpha \frac{1}{T}$, so the hotter the object the shorter the peak wavelength

6. Using Stefan's law, $L = 4\pi\sigma r^2 T^4$, radius of Procyon $= \sqrt{\frac{2.6\times10^{27}}{4\pi\times5.67\times10^{-8}\times(6400)^4}} = 1.5\times10^6$ km. This is only an estimate because the temperature is an approximate temperature.

7. The redshift is the shift of lines in an emission spectrum to longer wavelengths due to the Doppler effect / the motion of the light source away from the observer on Earth.

8. redshift $z = \frac{\Delta\lambda}{\lambda} = \frac{(30nm)}{(430nm)} = 0.07$, $Z = \frac{v}{c} = \frac{\Delta\lambda}{\lambda}$ so $v = z\times c =$ $0.07 \times 3\times10^8$ ms^{-1} $= 2.1\times10^7$ ms^{-1}

9. That distant galaxies are all receding from the Earth with a velocity proportional to its distance and that the Universe is expanding.

10. That at a time $t = 0$ the Universe came into being from an infinitely hot, infinitely dense point and has been expanding ever since.

Practice exam-style questions

E1. (a) Astronomical objects whose luminosity is known or can be reliably estimated [1]

(b) Since the Cepheids have the same period we assume they have the same luminosity [1]

Using the radiant flux equation $F = \frac{L}{4\pi d^2}$ for the same luminosity $F_1 d_1^2 = F_2 d_2^2$ [1]

so $F_{G1} \times d_{G1}^2 = F_{G2} \times d_{G2}^2$ $d_{G2} = \sqrt{\frac{F_1}{F_2}} \times d_{G1}$ [1]

$= \sqrt{\frac{1.18\times10^{-14}\,W^{-2}}{6.1\times10^{-17}\,W^{-2}}} \times (1.5\times10^{21}m)$ 2.1×10^{21} m [1]

E2. (a) from graph period = 5.5 days (answers in range 5.4–5.6 days) [1]

(b) $2\times10^3\ L_{Sun}$ (accept values between 10^3 and $3\times10^3\ L_{Sun}$) [1]

$2 \times 3.8\times10^{26}$ W $= 7.6\times10^{26}$ W [1]

Using inverse square law $d = \sqrt{\frac{L}{4\pi F}}$ [1]

$\sqrt{\frac{7.6\times10^{26}}{4\pi\times500\times10^{-9}}} = 1.1\times10^{16}$ m [1]

E3. (a) The redshift is the increase in wavelength of radiation [1] emitted by an object that is moving away from the observer [1]

(b) $\frac{\Delta\lambda}{\lambda} \approx \frac{v}{c}$

$v \approx 3\times10^8$ ms^{-1} $\times \frac{0.0633}{21.1}$ [1] $= 900\,000$ ms^{-1} [1]

(c) $v = H_0 d$, $H_0 \frac{900000}{(60\times10^6\times9.46\times10^{15})}$ [1]

$= 1.6\times10^{-18}$ (s^{-1}) [1]

(d) A blue shift would indicate that all the galaxies were moving towards each other [1]

This would suggest that the Universe had stopped expanding and was now contracting [1]

E4. (a) *Antares* is cooler [1] $\lambda_{max} \alpha \frac{1}{T}$ *Antares* has a longer wavelength so therefore a lower surface temperature [1]

(b) $\lambda_{max}T$ for Antares $= \lambda_{max}T$ for Spica [1]

T for Spica $= \frac{852\times10^{-9}\times3400}{126\times10^{-9}} = 23000$ K [1]

(c) Using Stefan-Boltzmann law, radius of Spica
$= \sqrt{\dfrac{4.5\times10^{30}}{4\pi\times5.67\times10^{-8}\times(23000)^4}}$ [1] $= 4.8 \times 10^{9}$ m [1]

E5. (a) The speed of recession of a galaxy is proportional to its distance (from Earth / observer) [1] (accept $v = H_0 d$)

(b) $H_0 = \dfrac{v}{d}$ [1] $\dfrac{2.8\times10^{6}}{1.2\times10^{24}} = 2.3 \times 10^{-18}$ [1] s^{-1} [1]

(c) time of travel $= \dfrac{d}{v} = \dfrac{d}{H_0 d}$ or $\dfrac{1}{H_0}$ [1]

(d) $\dfrac{1}{H_0} = \dfrac{1}{2.3\times10^{-18}} = 4.3 \times 10^{17}$ s [1] $\dfrac{4.3\times10^{17}}{3.2\times10^{7}} = 1.4 \times 10^{10}$ (y) [1]

E6. (a) the Universe was infinitely hot, infinitely dense at the start *or* the universe has been expanding since it was created [1] Hubble's law shows that galaxies are receding from each other [1] (ignore references to microwave background radiation)

(b) (i) The spectral line from the galaxy is redshifted / wavelength increased showing that it is moving away from us [1]

(ii) $v = c\dfrac{\Delta\lambda}{\lambda} = (3\times10^{8})\dfrac{(420-395)}{(420)}$ [1] $= 1.8 \times 10^{7}$ ms^{-1} [1]