1. **Famous astronomical events** **(10 p)**

Arrange the following astronomy-related events in chronological order from the oldest to the most recent. **Write the correct serial number (between 1 and 11) in the appropriate box on the answer sheet.**

1. Launch of the Hubble Space Telescope
2. Viking probes arrived at planet Mars
3. Discovery of Phobos and Deimos
4. Latest perihelion of comet 1/P Halley
5. Discovery of Ceres (asteroid / dwarf planet)
6. Discovery of Uranus (planet)
7. First successful measurement of a stellar parallax
8. Discovery of the first planetary nebula
9. Discovery of stellar populations (I and II)
10. First identification of a quasar with an optical source
11. Discovery of the expansion of the Universe

2. **Deflection of radio photons in the gravitational field of solar system bodies** **(10 p)**

A. Eddington and F. Dyson from Principe, and C. Davidson and A. Crommelin from Sobral, Brazil measured the deflection of light coming from stars apparently very close to the Sun during the total solar eclipse in 1919. The deflection was found to agree with the theoretically predicted value of 1.75".

A light ray (or photon) which passes the Sun at a distance d is deflected by an angle

$$\Delta\theta \propto \frac{4GM_\odot}{dc^2}$$

The present-day accuracy of the VLBI (Very Long Baseline Interferometry) technique in the radio wavelength range is 0.1 mas (milliarcsecond). Is it possible to detect a deflection of radio photons from a quasar by (a) Jupiter, (b) the Moon? Estimate the angle of deflection in both cases and **mark "YES" or "NO" on the answer sheet**.

3. **The supermassive black hole in the centre of Milky Way Galaxy and M87** **(10 p)**

The first image of a black hole was constructed recently by the international team of the Event Horizon Telescope (EHT). The imaged area surrounds the supermassive black hole in the centre of the galaxy M87. The observations resulting in the final image were carried out at a wavelength $\lambda = 1.3$ mm where the interstellar extinction is not prohibitively large.

a) How large an instrument would be needed to resolve the shadow (in effect the photon capture radius, which is three times the size of event horizon) of a supermassive black hole in the centre of a galaxy? Express the result as a function of the distance d and the mass M of the black hole. (6 p)

b) Give the size of the instrument in units of Earth radius for

   i. the supermassive black hole in the centre of M87, (1 p)
      ($d_{\text{BH-M87}} = 5.5 \times 10^7$ ly, $M_{\text{BH-M87}} = 6.5 \times 10^9$ M$_\odot$)

   ii. and Sgr A*, the supermassive black hole of our own galaxy, Milky Way. (1 p)
      ($d_{\text{Sgr A*}} = 8.3$ kpc, $M_{\text{Sgr A*}} = 3.6 \times 10^6$ M$_\odot$)

c) What type of technology is needed for the development of such an instrument? **Mark the letter of your answer with × on the answer sheet.** (2 p)

    (A)   Gravitational lensing by dark matter
    (B)   Interferometry with an array of radio telescopes
    (C)   Photon deceleration in a dense environment
    (D)   Reducing the effect of incoming wavefront distortions
    (E)   Neutrino focusing with strong electromagnetic fields

4. **Improving a common reflecting telescope** **(10 p)**

A student has an average quality Cassegrain telescope, with primary and secondary mirrors having $\varepsilon_1 = 91$ % reflectivity aluminium layers.

a) What will be the change in the limiting magnitude of this telescope by replacing the mirror coatings with "premium" quality $\varepsilon_2 = 98$ % reflectivity ones? (5 p)

b) Assuming the student also uses a star diagonal mirror, also with reflectivity ε_1 with the original telescope - what will be the improvement if he/she also replaces this piece with an $\varepsilon_3 = 99$ % reflectivity ("dielectric" mirror) model, combined with the new ε_2 mirrors? (3 p)
(A star diagonal mirror is a flat mirror, inclined to the optical axis by 45°.)

c) Is this difference obviously detectable by the human eye? **Mark "YES" or "NO" on the answer sheet.** (2 p)

Consider the whole visual band and disregard any wavelength dependence and geometric effects.

5. **Cosmic Microwave Background Oven** **(10 p)**

Since the human body is made mostly of water, it is very efficient at absorbing microwave photons. Assume that an astronaut's body is a perfect spherical absorber with mass of $m = 60$ kg, and its average density and heat capacity are the same as for pure water, i.e. $\rho = 1000$ kg m^{-3} and $C = 4200$ J kg^{-1} K^{-1}.

a) What is the approximate rate, in watts, at which an astronaut in intergalactic space would absorb radiative energy from the Cosmic Microwave Background (CMB)? The spectral energy distribution of CMB can be approximated by blackbody radiation of temperature $T_{\text{CMB}} = 2.728$ K. (5 p)

b) Approximately how many CMB photons per second would the astronaut absorb? (3 p)

c) Ignoring other energy inputs and outputs, how long would it take for the CMB to raise the astronaut's temperature by $\Delta T = 1$ K? (2 p)

6. **The height of the chimney of Tiszaújváros power plant** **(20 p)**

The European Copernicus Earth-observation programme operates two Sentinel-2 remote sensing satellites. These satellites orbit the Earth on Sun-synchronous polar orbits at about 800 km altitude. They pass over a given area once every few days, always taking images at the same local time (accurate to within a few minutes). The cameras are sensitive to 13 different optical and near-infrared spectral bands. The resolution of the images is 10 meters.

The 3rd tallest building in Hungary is the chimney of a power plant near the town Tiszaújváros. You can see two Sentinel-2 satellite images. One is from June 29, another one is from December 16, close to the summer and winter solstices, respectively. The orientation of the images is as normal, i.e. north is up and east is to the right.

The estimated shadow lengths, based on the images above and the scales given in the lower-left corners, of them are $x_1 = 125$ m and $x_2 = 780$ m. Answer the following questions:

a) On which date do we expect the shadow to be longer? **Mark the letter of your answer with × on the answer sheet.** (1 p)

(A) on June 29
(B) on December 16

b) At which time of the day did the Sentinel-2 satellites fly over this area? **Mark the letter of your answer with × on the answer sheet.** (1 p)

(A) early morning
(B) late morning
(C) early afternoon
(D) late afternoon

c) Based on the given shadow lengths, estimate the height of this chimney. For this calculation only, assume that the satellite images were taken at local noon. (16 p)

d) What could affect the accuracy of the chimney's derived height (more than one choice is possible)? **Mark the letter of your answer with × on the answer sheet.** (2 p)

(A) The oblate spheroid shape of the Earth.
(B) Limited resolution of the satellite images and the ill-defined edge of the shadow.
(C) The elevation of the base of the tower above sea level.
(D) Seasonal variation in the tilt of the Earth's rotational axis.
(E) Taking into account the effect of atmospheric refraction.

7. Effect of sunspots on solar irradiance (20 p)

Since 1978, the solar constant has been almost continuously measured by detectors on-board artificial satellites. These accurate measurements revealed that there are seasonal, monthly, yearly, and longer timescale variations in the solar constant. While the seasonal variations have their origins in the periodically varying Earth–Sun distance, the decade-long quasi-cyclic variations mainly depend on the activity cycle(s) of the Sun.

a) Calculate the value of the solar constant at the top of the Earth's atmosphere, when the Earth is 1 au from a perfectly quiet Sun, assuming that Sun emits as a perfect black-body. (4 p)

b) Calculate the solar constant of this perfectly quiet Sun in early January and early July, and find their ratio. (4 p)

c) Calculate the solar constant again at 1 au in the presence of a near equatorial sunspot with mean temperature of $T_{sp} = 3300$ K and diameter of sunspot, $D_{sp} = 90\,000$ km. Calculate the ratio - blank Sun to Sun with sunspot. (7 p)

Assume the sunspot is circular and ignore the effects of its spherical projection. Neglect any other activity features. Also assume that the Sun is rotating fast enough, hence solar irradiance is still isotropic.

d) In reality solar irradiance is no longer isotropic. Calculate the ratio of solar irradiance for the cases when the sunspot is not visible from the Earth to the case when it is fully visible.

(5 p)

8. Amplitude variation of RR Lyrae stars (20 p)

Hungarian astronomers significantly contributed to the study of pulsating variable stars of RR Lyrae type, which show cyclic amplitude modulation in their light variation (Blazhko effect)

The light variation of an RR Lyrae star is observed at two different wavelengths: $\lambda_1 = 500$ nm and $\lambda_2 = 2000$ nm. At each wavelength, we see into the star at a different depth. We refer to the depths as layer 1 and layer 2. One can approximate the light intensity of a star at a given wavelength by the radius and the black-body radiation intensity at the appropriate depth. Moreover, the Wien approximation can be used for the black-body radiation to calculate the emitted power from unit surface area:

$$F(\lambda, T) \propto \frac{1}{\lambda^5} \exp\left(-\frac{hc}{k\lambda T}\right),$$

where h is the Planck constant, k is the Boltzmann constant, and c is the speed of light. To simplify the calculations we introduce a new constant $C_b = hc/k \approx 0.0144$ m K.

a) Assume that the temperature varies between $T_1 = 6000$ K and $T_2 = 7400$ K in each of the layers and ignoring radius variation, what is the ratio of the amplitudes of variation in magnitudes at the two wavelengths? (5 p)

b) What is the peak to peak amplitude of the light curve at λ_1? Use the magnitude scale. (3 p)

c) Ignoring temperature variation, what is the contribution of the radius variation to the light-curve amplitude for a given wavelength, if $R_{min} = 0.9 \langle R \rangle$ and $R_{max} = 1.05 \langle R \rangle$? $\langle R \rangle$ is the mean radius of the given layer. (3 p)

d) Recent observations and models show that the radius of the photosphere is only minimally modulated during the Blazhko cycle; however, the temperature variation is significant. As a result, the amplitude of light curve itself keeps varying. Let us assume that during the minimum amplitude of pulsation the temperature variation is reduced to $T_{min} = 6100$ K and $T_{max} = 6900$ K. What is the modulation amplitude at the two wavelengths? For the maximum amplitude, use the temperature values given in part a). (5 p)

e) Which statement is correct? (There can be more than one correct selection.) **Mark the letter of your answer with × on the answer sheet.** (4 p)

(A) It is easier to observe the Blazhko effect in the infrared band.
(B) The temperature variation dominates the visual light curve.
(C) If we neglect the radius variation, the amplitude is inversely proportional to the wavelength.
(D) Multi-colour observations are not useful in understanding the Blazhko effect.

9. **Distance of the Lagrangian point L_2 of the Earth–Moon system** (20 p)

On 3 January 2019, the Chinese spacecraft Chang'e-4 landed on the far side of the Moon in the area of the von Kármán crater, which was named after the world famous Hungarian-born physicist Theodore von Kármán.

As the Earth remains below the horizon of the spacecraft all the time, a relay station is also necessary for the communication with mission control on the Earth. For this purpose, The Chinese Space Agency launched a spacecraft, Queqiao, which was placed into a halo orbit around the outer Lagrangian point of the Earth–Moon system, L_2, the far side of the Moon.

Calculate the distance (h) of this satellite above the surface of the Moon. The Moon's orbit should be considered as a perfect circle with a radius of $R = 384\,400$ km. Neglect perturbations from the Sun and other planets.

Hint: You can use the following approximation: $1/(1 + x)^2 \approx 1 - 2x$, if $|x| \ll 1$.

10. **South → East → North** (20 p)

Consider the Earth as a perfect, rigid, sphere with radius $R = 6378$ km. There exist some points on the surface of the Earth from which we can travel first 6378 km South, then 6378 km East, and after that 6378 km North, and as a result return to the original point of departure. Find such points and paths. Calculate the geographic coordinates of the turning points of your solutions and draw the paths.

For the sake of simplicity measure the geographic longitude from 0° to +360° eastward from Greenwich, the geographic latitude from 0° to +90° north of the Equator and from 0° to −90° south of the Equator. Solutions resulting from rotational symmetry should not be considered to be different.

11. Identification of light curves of types of selected variable stars **(25 p)**

TESS (Transiting Exoplanet Survey Satellite) is NASA's most recent exoplanet hunter mission. It has been surveying the southern sky to locate exoplanets around the brightest and closest stars, along with a large number of time-variable phenomena (pulsating and eclipsing variable stars, supernovae, stellar flares, and asteroids among others).

On the separate sheet you will find graphs with light curve plots of 8 periodic variable stars (intrinsic, eclipsing and rotational; namely FO Eri, RW Dor, 24 Eri, TIC 147272181, ST Pic, UY Eri, VV Ori, AH Col) from the TESS target list, numbered from 1 to 8. The horizontal axis is BJD − 2 400 000 in days, and the vertical axis is V in magnitudes, where BJD is the Barycentric Julian Date, and V is the visual brightness.

a) Below is a list of variable star types. Match each light curve of the star with its variability type **by writing its number into the rectangle on the answer sheet**. (8 p)

- Heart-beat star
- RR Lyrae type (RRab subclass) pulsating variable star
- Eclipsing binary of Algol type (semi-detached) with a pulsating component
- α^2 CVn pulsating variable star
- W Vir type (Population II) Cepheid pulsating variable star
- Detached eclipsing binary with strong reflection effect
- Contact eclipsing binary of W UMa type
- Rotationally variable (spotted) star

b) Based on the light curve plots, estimate the periods of each variable star in days. **Give your answers on the answer sheet up to 2 decimal places.** Periods in the range ±5% of the true periods are acceptable. (16 p)

c) What type of astronomical object produced the TESS light curve shown in the figure below? **Mark the letter of your answer with × on the answer sheet.** (1 p)
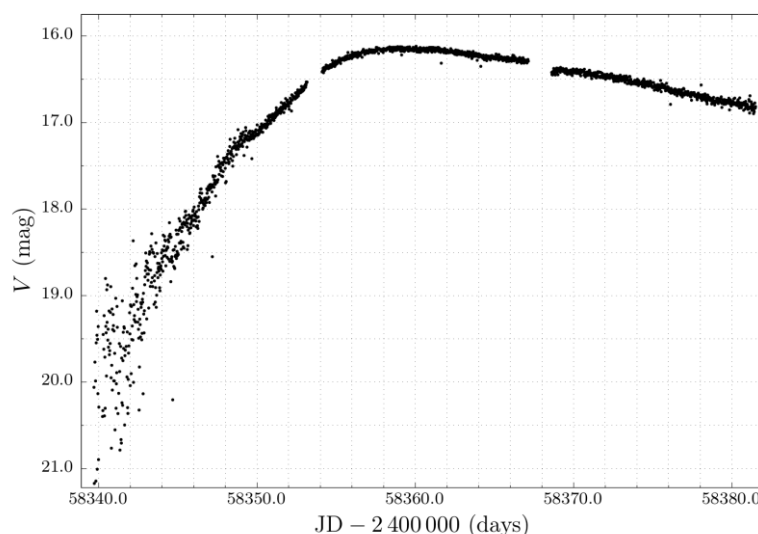
(A) Microlensing event
(B) Saturated galaxy due to the proximity of Mars
(C) Comet's coma close to the edge of the camera field
(D) Supernova in a distant galaxy
(E) Superflare on a supergiant star

12. Distance to a Near-Earth Asteroid (25 p)

Assume that a Near-Earth Asteroid is observed by two astronomers, one from Nagykanizsa, Hungary and one from Windhoek, Namibia. The longitudes of the two cities are exactly 17° east of Greenwich. They observe the asteroid when it crosses their respective meridians. The Nagykanizsa-observer sees the asteroid 25° south of his zenith at this instant, while the Windhoek-observer sees it 45° north of his zenith at the same instant. The latitudes of the two cities are 46°27' N and 22°34' S, respectively. The sites of both astronomers are at sea level.

    a) Draw a diagram of the geometric configuration. (5 p)

    b) What is the distance of this asteroid from the centre of the Earth, expressed in units of Earth-radii and the average Earth-Moon distance? Provide a solution which makes use of all available information. Neglect the effect of the atmospheric refraction. (20 p)

13. Distance to the Coma galaxy cluster (40 p)

The Coma galaxy cluster (Abell 1656) has an angular diameter on the sky of about 100 arcminutes, and contains more than 1000 individual galaxies, most of which are dwarf and giant ellipticals orbiting the common center of mass of the cluster in approximately circular orbits. The table below lists the measured radial velocities of a few individual cluster member galaxies.

No.	v_r (km/s)	No.	v_r (km/s)	No.	v_r (km/s)	No.	v_r (km/s)
1	6001	6	7116	11	7156	16	7111
2	7666	7	7004	12	7522	17	8292
3	6624	8	4476	13	7948	18	5358
4	5952	9	6954	14	4951	19	4957
5	5596	10	8953	15	7797	20	7183

    a) Derive the distance of the cluster from the mean radial velocity of the galaxies listed in the table. (8 p)

    b) Estimate the physical diameter of the cluster (in Mpc). (4 p)

    c) The virial theorem states that if the galaxy cluster is in dynamic equilibrium, then the mean kinetic energy, $\langle K \rangle$, and the mean gravitational potential energy, $\langle U \rangle$, are related by

$$-2\langle K \rangle = \langle U \rangle$$

assuming the Coma cluster is spherical.

For simplicity, assume that each galaxy has approximately the same mass, m.

Use the virial theorem to prove that, in this case, the cluster mass M (also called as the *virial mass*) can be expressed as

$$M = \frac{5R}{G} \sigma_r^2$$

where σ_r^2 is the velocity dispersion of the cluster. (10 p)

The formula for standard deviation:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}$$

d) Using the data in the table, estimate the virial mass of the Coma cluster in solar masses. (12 p)

e) The total luminosity of the Coma cluster (in solar luminosity, $L_\odot$) is $L \approx 5 \times 10^{12} L_\odot$. Calculate the mass-luminosity ratio of the cluster in solar mass per solar luminosity units. (2 p)

f) Which of the following statements is true (more than one answer is possible)? **Mark the letter of your answer with × on the answer sheet.** (4 p)

(A) The mass-luminosity ratio of the Coma cluster is much higher than that of a typical spiral galaxy, like the Milky Way.
(B) The mass-luminosity ratio of the Coma cluster is similar to that of a typical spiral galaxy.
(C) The mass-luminosity ratio of the Coma cluster is much less than that of a typical spiral galaxy.
(D) The Coma cluster contains much more dark matter than a typical spiral galaxy.
(E) The Coma cluster contains much less dark matter than a typical spiral galaxy.

14. Photographing a nanosatellite (60 p)

The very first, entirely Hungarian-made, satellite was "MASAT–1", a nanosatellite "cubesat". It was made mostly of aluminium with total mass 1 kg, sides length l = 10 cm and a longer communication aerial. It was designed and prepared at the Technical University of Budapest (BME) in 2009 by students. The launch occurred on February 13, 2012, using a Vega rocket from the Kourou launch site – together with several other cubesats from other countries. It operated successfully right up to the last minutes of its lifetime (the transmitted last data packages were captured by radio receivers on the evening of January 9, 2015, a few hours later the nanosatellite re-entered the atmosphere and disintegrated).
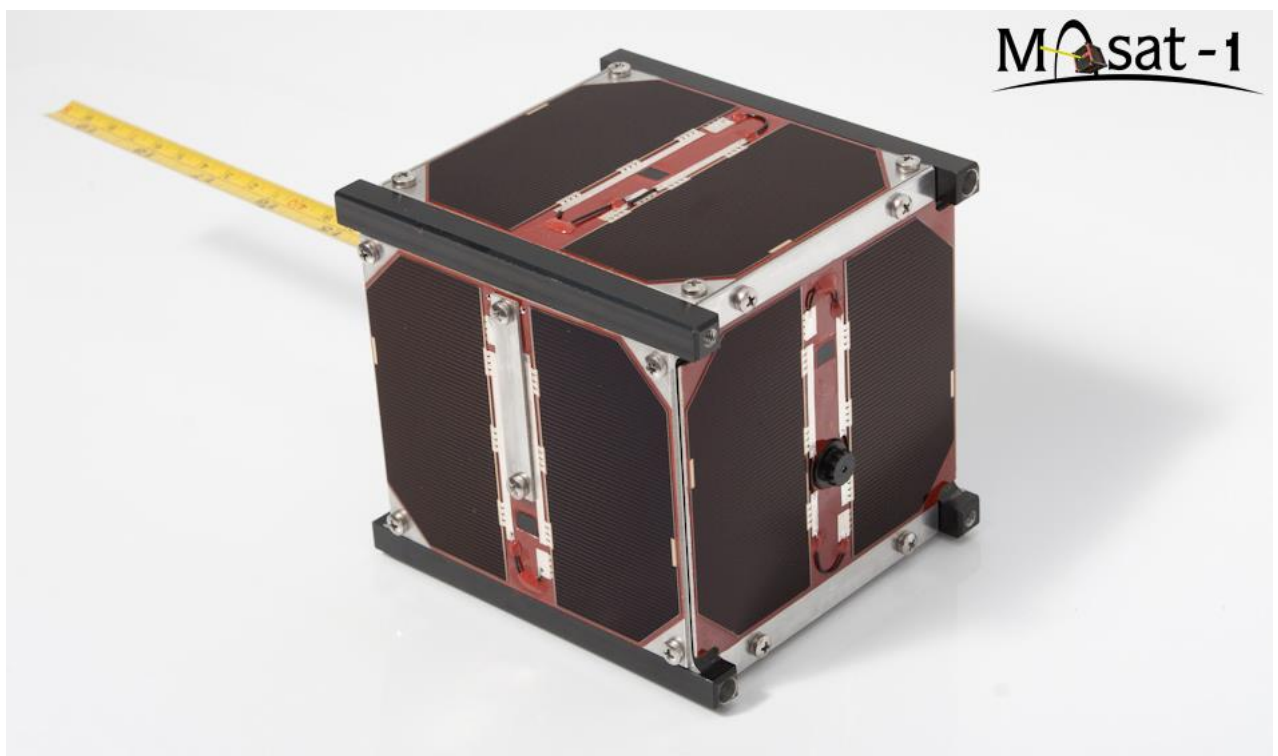
The orbital altitude of MASAT–1 changed between $h_{min} = 350$ km and $h_{max} = 1450$ km (due to its highly eccentric orbit), but in this question assume it to be in a circular orbit 900 km above the sea level.

The MASAT team wished to photograph their nanosatellite from the ground. Thus, they called the staff of Baja Astronomical Observatory (South Hungary, $\lambda_B = 19.010843°$, $\varphi_B = 46.180329°$, $h_B = 100$ m) to photograph the orbiting MASAT–1 with their telescope.

The observatory has a Ritchey-Chrétien type reflecting telescope with a diameter of 50 cm, and focal ratio of f/8.4. The CCD camera installed on the telescope had a 4096×4096 chip with 9 μm sized square pixels. The quantum efficiency of the CCD was about 70 %. The practical detection limit was about 19.5^m visual band applying a $\tau_{exp} = 2$ min exposure time. Disregard any effects of seeing. The orbital inclination of MASAT–1 was about i=70° and the direction of the orbital motion was the same as the Earth rotation. In addition, let us assume that the reflection area always is 100 cm². Throughout this event, the telescope was pointed to the local zenith, and its RA motor was tracking the stars. Consider only light from the Sun, you can ignore light reflected from Earth and Moon. Ignore the effects of atmospheric extinction.

a) Calculate the apparent visual magnitude of this cubesat under ideal observational circumstances, i.e. when it was at the local zenith of the observing site (Baja, Hungary) at midnight. Omit all atmospheric effects, and consider the Earth to be a sphere. The albedo of a specular aluminium plate is $a \approx 0.70$.   (10 p)

Hint: Use a comparison of MASAT–1 with the Full Moon.

b) What was the Observatory's answer to the MASAT team; would it ever be possible to photograph their nanosatellite with the existing equipment at the observatory? **Mark "YES" or "NO" on the answer sheet.** Support your answer with detailed calculations.   (42 p)

c) What would have been the answer if they had taken into account the blurring effect of the atmosphere, the so-called seeing? In Hungary the typical FWHM (Full Width at Half Maximum) value of the blurred stellar images – which can be approximated by a symmetric 2D Gaussian profile close to the zenith – is about 3.5″. **Mark "YES" or "NO" or "Close to the limit" on the answer sheet.** Support your answer with a short calculation.   (8 p)

Hint: Although the illumination of the seeing spot in the focal plane can be approximated by a symmetric 2D Gaussian profile, you can take it to be homogeneous in your calculation.