Instructions

1. The theoretical competition will be 5 hours in duration and is marked out of a total of 300 points.

2. There are **Answer Sheets** for carrying out detailed work/rough work. On each **Answer Sheet**, please fill in

- Student Code
- Question No.
- Page no. and total number of pages.

3. Start each problem on a separate Answer Sheet. Please write only on the printed side of the sheet. Do not use the reverse side. If you have written something on any sheet which you do not want to be marked, cross it out.

4. Please remember that the graders may not understand your language. As far as possible, write your solutions only using mathematical expressions and numbers. If it is necessary to explain something in words, please use short phrases (if possible in English).

5. You are not allowed to leave your working desk without permission. If you need any assistance (malfunctioning calculator, need to visit a restroom, need more Answer Sheets, etc.), please put up your hand to signal the proctor.

6. The beginning and end of the competition will be indicated by a long sound signal. Additionally, there will be a short sound signal fifteen minutes before the end of the competition (before the final long sound signal).

7. At the end of the competition you must stop writing immediately. Sort and put your sheets in

separate stacks,

(a) Stack 1: answer sheets of part 1
(b) Stack 2: answer sheets of part 2
(c) Stack 3: answer sheets of part 3
(d) Stack 4: question papers and paper sheets that you do not want to be graded.

8. Wait at your table until your envelope is collected. Once all envelopes are collected, your student guide will escort you out of the competition room.

9. A list of constants for this competition is given on the next page.

Point distribution of this exam

Problem number	**Points**
T1	10
T2	10
T3	10
T4	10
T5	10
T6	25
T7	25
T8	25
T9	25
T10	75
T11	75
Total	300

Table of constants

Mass ($M_{\oplus}$)	5.98×10^{24} kg	**Earth**
Radius ($R_{\oplus}$)	6.38×10^{6} m	
Acceleration of gravity (g)	$9.81\ \mathrm{ms^{-2}}$	
Obliquity of Ecliptic	23°27′	
Length of Tropical Year	365.2422 mean solar days	
Length of Sidereal Year	365.2564 mean solar days	
Albedo	0.39	
Mass ($M_{☾}$)	7.35×10^{22} kg	**Moon**
Radius ($R_{☾}$)	1.74×10^{6} m	
Mean Earth-Moon distance	3.84×10^{8} m	
Orbital inclination with the Ecliptic	5.14°	
Albedo	0.14	
Apparent magnitude (mean full moon)	−12.74	
Mass ($M_{\odot}$)	1.99×10^{30} kg	**Sun**
Radius ($R_{\odot}$)	6.96×10^{8} m	
Luminosity ($L_{\odot}$)	3.83×10^{26} W	
Absolute Magnitude	4.80 mag	
Surface Temperature	5772 K	
Angular diameter at Earth	30′	
Orbital velocity in Galaxy	$220\ \mathrm{kms^{-1}}$	
Distance from Galactic center	8.5 kpc	
1 *au*	1.50×10^{11} m	**Physical constants**
1 *pc*	206265 au	
Gravitational constant (G)	$6.67 \times 10^{-11}\ \mathrm{Nm^2kg^{-2}}$	
Planck constant (h)	6.62×10^{-34} Js	
Boltzmann constant (k_B)	$1.38 \times 10^{-23}\ \mathrm{JK^{-1}}$	
Stefan-Boltzmann constant (σ)	$5.67 \times 10^{-8}\ \mathrm{Wm^{-2}K^{-4}}$	
Hubble constant (H_0)	$67.8\ \mathrm{kms^{-1}Mpc^{-1}}$	
Speed of light in vacuum (c)	$299792458\ \mathrm{ms^{-1}}$	
Magnetic Permeability of free space (μ_0)	$4\pi \times 10^{-7}\ \mathrm{Hm^{-1}}$	
1 Jansky (Jy)	$10^{-26}\ \mathrm{W\,m^{-2}Hz^{-1}}$	

Rayleigh-Jeans law is given by $B_\nu = \frac{2k_BT}{c^2}\nu^2$, which is the power emitted per unit emitting area, per steradian, per unit frequency.

(T1) Super Luminal Galaxies **(10 points)**

Read the statements given below and state if they are true or false:

(a) For some galaxies the apparent recession speed exceeds the speed of light.

(b) The velocity – Distance relation as given by Hubble cannot allow recession velocities to exceed the speed of light.

(c) Hubble-Lemaitre's law (formerly known as Hubble's Law) does not violate special relativity.

(d) If some galaxies would have an apparent recession speed exceeding the speed of light, then the photons from those galaxies can never reach us.

(e) As the expansion of Universe is accelerating, photons emitted right now from galaxies which have apparent recession speed equal to the speed of light will never reach us.

(T2) Distance **(10 points)**

An observer measured trigonometric parallaxes of stars in a star cluster. Due to random errors, the measured parallax values are distributed symmetrically around the expected value with standard deviation equal to 0.05 mas (milliarcsec). Assume there are no systematic errors and assume all stars in the said cluster have the same luminosity. It is known that the distance of this cluster from us is R= 5 kpc.

He gave the data table to 4 of his students (A, B, C and D) and they estimated the distance to the cluster in the following ways:

A. Convert each parallax measurement into distance and then find the average distance (R_A)
B. Take the average of all parallaxes first and then convert the average parallax into distance. (R_B)

C. Convert each parallax measurement into distance and then take the median distance value. (R_C)

D. Find the median value of the parallaxes and then convert the median value into distance. (R_D)

State if the following statements are true or false. **In case a given mathematical relation is false, give the correct relation.**

(l) If the i^{th} star gave the smallest value of parallax and the j^{th} star gave the highest value of parallax, in all likelihood R_i-R > R-R_j

(m) R_A = R (i.e. there is a high chance that the distance estimated by A fairly matches the true distance)

(n) R_B = R (i.e. there is a high chance that the distance estimated by B fairly matches the true distance)

(o) R_C < R (i.e. there is a high chance that the distance estimated by C will be systematically lower than the true distance)

(p) R_D = R (i.e. there is a high chance that the distance estimated by D fairly matches the true distance)

(T3) Atmospheric Refraction **(10 points)**

Consider sunrise at Beijing (φ=40°) on the vernal equinox day.

(a) Let us say r_l, r_d, r_r and r_u are distances from the centre of the undistorted disk of the Sun to the edge of the disk towards the directions left, down, right and up respectively. What will be the hierarchical relation (<, =, >) between the four radii just after the sunrise?

(b) What is the correction in the time of rise of the top edge of the disk as compared to the case without atmosphere? You may assume that typically atmospheric refraction near the horizon is 35'. Please only consider the apparent diurnal motion.

(T4) Height of a Hill **(10 points)**

Two friends wanted to measure the height of the hill next to their village (latitude φ=40°). One of the friends climbed to the top of the hill and she agreed to send a light signal to her friend in the village as soon as she sees the sunset. On March 21, when they did this experiment, the friend in village received the light signal 4.1 minutes after the sunset from the village. Estimate the height of the hill and horizon distance for the person at the hill top. Ignore atmospheric refraction.

(T5) Sidereal Time **(10 points)**

It is very interesting to observe that on one particular calendar day each year, the mean sidereal time will twice be 00:00:00.

(a) What will be the approximate R.A. of the Sun when this event happens?

(b) Estimate the exact date in 2018 for this event.

You may assume that at the Royal Greenwich Observatory, the mean sidereal time ($GMST_0$) was 6.706h at 0h, 1st January, 2018 (JD2458119.5).

(T6) Observe the Sun with FAST **(25 points)**

The Five-hundred-meter Aperture Spherical radio Telescope (FAST) is a single-dish radio telescope located in Guizhou Province, China. The physical diameter of the dish is 500 m, but during observations, the effective diameter of the collecting area is 300 m.

Consider observations of the thermal radio emission from the photosphere of the Sun at 3.0 GHz with this telescope and a receiver with bandwidth 0.3 GHz.

(a) Calculate the total energy ($E_\odot$) that the receiver will collect during 1 hour of observation.
(b) Estimate the energy needed to turn over one page of your answer sheet (E'). Hint: the typical surface density of paper is 80 gm^{-2}.
(c) Which one is larger?

(T7) Sunspot **(25 points)**

Magnetic fields are important in the physics of stars and sunspots. As an approximation, we can model the photosphere of the Sun consisting of a plasma, which can be simply treated as a single component ideal gas, and a magnetic field (**B**), which has an associated magnetic pressure $p_B = \frac{B^2}{2\mu_0}$. It behaves like any other physical pressure except that it is carried by the magnetic field rather than by the kinetic energy of particles.

Assume that the number density of particles in the photosphere is constant everywhere, but the magnetic field inside the sunspot (B_{in}=0.1T) is much stronger than outside ($B_{out}=5\times10^{-3}$T). From the blackbody spectrum, the temperature inside the sunspot is T_{in}~4000K, while the temperature outside is T_{out}~6000K (which is why the sunspot looks darker). For the sunspot to be stable, the inside must be in equilibrium with the outside.

(a) Estimate the number density of plasma particles in the solar photosphere.

(b) Compare your answer with an estimate of the number density of particles in the atmosphere at the surface of the Earth.

(T8) A Possible Dark Matter Deficient Galaxy **(25 points)**

Earlier this year, a team of astronomers reported their discovery of a galaxy with much less dark matter than the galaxy evolution model predicted (van Dokkum et al. 2018, Nature). This galaxy, named NGC 1052-DF2, is located close to the elliptical galaxy NGC 1052 (D=20Mpc from the Sun) in the sky. The shape of NGC 1052-DF2 resembles an ellipse with semi major axis (a) of 22.6" and $\frac{b}{a} = 0.85$. Half of the total light from the galaxy comes from within this ellipse and the mean surface brightness within the ellipse is about 24.7 mag arcsec^{-2}.

(a) Calculate the total apparent magnitude of this galaxy.

(b) The team suggested the galaxy is a companion of NGC 1052. Determine the total mass of stars in NGC 1052-DF2, assuming it has a mass to light ratio $\left(\frac{M/M_\odot}{L/L_\odot}\right)$ of 2.0.

(c) The team identified 10 globular clusters in NGC 1052-DF2 with a mean galactocentric distance of 78.4". They also measured the velocity dispersion of these clusters to be not more than 8.4 km/s. Estimate the dynamical mass of this galaxy. For simplicity, assume the mass distribution in the galaxies is uniform and is spherically symmetric.

(d) This discovery was challenged by other groups (Kroupa et al., Nature, 2018, Truijlo et al., MNRAS, 2018), who claimed that NGC 1052-DF2 is not a satellite of NGC 1052, and it is located at a much smaller distance to us. Show why a smaller distance would weaken the assertion of the dark matter deficiency in NGC 1052-DF2.

(T9) Radio Galaxy **(25 points)**

An observer wants to use the Five-hundred-meter Aperture Spherical radio Telescope (FAST) in China to observe a radio galaxy at redshift of $z = 0.06$. We assume that the radio source is compact compared to the beam size of the telescope at the observing frequencies, i.e., the source is point-like as seen through the telescope. To detect a point source with FAST, it must be sufficiently strong (bright) relative to the noise level (for single polarization observations), σ, which depends on the bandwidth, $\Delta\nu$, and the integration time (the radio astronomy equivalent of exposure time), t_i, as follows:

$$\sigma = \frac{2k_B T_{sys}}{A_e\sqrt{t_i \Delta\nu}}$$

where T_{sys} is the system temperature (about 150 K in the frequency range of 0.28 GHz – 0.56 GHz and 25 K in the frequency range of 1.05 GHz – 1.45 GHz), and $A_e = 4.6\text{x}10^4$ m^2 is the effective area of the telescope taking into account the total efficiency of the instrument.

This radio galaxy has an observed continuum flux density of $f_\nu = 2.5 \times 10^{-3} Jy$ at an observing frequency of 0.4 GHz. The bandwidth $\Delta\nu$ for the continuum observation centered at 0.4 GHz is 2.8×10^8 Hz.

(a) In order to detect the continuum flux density at 0.4 GHz with a signal-to-noise ratio of 30 (a so-called 30σ detection), what is the required integration time, t_i?

(b) We want to search for the neutral Hydrogen (HI) in the galaxy using 21cm absorption line. The HI 21cm line, with rest frame frequency of 1.4204 GHz. Calculate the observed frequency (υ_{obs}) of the HI line for this galaxy.

(c) The radio continuum emission from this galaxy can be described by a power law $f_\nu \sim \nu^\alpha$, with a spectral index of $\alpha = -0.2$. Calculate the continuum flux density at ν_{obs} for this galaxy.

(d) The line width of the HI 21cm absorption line is 90 km/s. Calculate the line width in Hz at the observing frequency of ν_{obs}. According to Figure 1, the HI 21cm line absorbs 4% of the continuum flux density (on average) over the line width of 90 kms^{-1}. In order to detect the absorption line at ≥3σ in three consecutive 30 kms^{-1} channels, what is the required integration time?

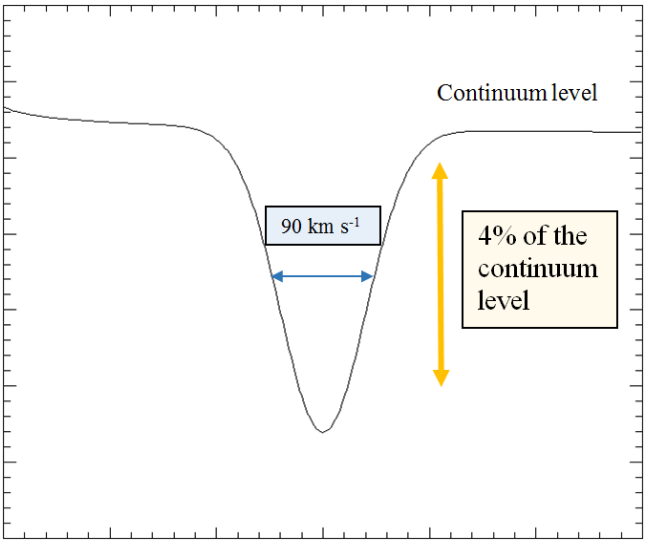

Figure 1: Spectrum of the HI 21cm absorption relative to the continuum emission in the radio galaxy

(T10) Vega and Altair **(75 points)**

As per a very famous Chinese folklore about love, Vega and Altair are two lovers. It is said that they can meet each other once every year on a bridge made up of birds over the Milky Way. The parameters of two stars are given in the table below. For the purpose of this question, assume that the coordinate frame is fixed (i.e. not affected by precession or motion of the Sun).

Star	Right Ascension (J2000.0)	Declination (J2000.0)	Parallax (mas)	Proper Motion		Radial Velocity (km/s)
				$\mu_\alpha \cos\delta$ (mas/year)	μ_δ (mas/year)	
Vega	$18^h36^m56.49^s$	+38° 47' 07.7"	130.23	+200.94	+286.23	-13.9
Altair	$19^h50^m47.70^s$	+8° 52' 13.3"	194.95	+536.23	+385.29	-26.1

Based on this data, answer the following questions：
(a) (9 points) What is the angular separation of the two stars?
(b) (6 points) Calculate the distance (in parsecs) between Vega and Altair.
(c) (3 points) Calculate position angles of the proper motion vectors of each of these two stars.

For parts d-g, assume that the angular velocity of the stars on the celestial sphere remains constant. This is not a physical situation but this is an assumption to simplify the problem.
(d) (2 points) How many common points on the celestial sphere are there which can be reached by both these stars?
(e) (20 points) Find the coordinates of the closest such point.
(Note: Drawing the situation on a celestial sphere will help you in visualising the situation)
(f) (8 points) Find when (which year) each of these stars were / will be at that point.
(g) (5 points) When Altair was / will be at that point, what would be its angular separation from Vega?
(h) (22 points) Find coordinates of any point (if it exists) in 3-D space which was /will be visited by both these stars. Do not ignore radial velocities for this part of the question.

(T11) Thermal History of the Universe **(75 points)**

Based on Einstein's general relativity, Russian physicist Alexander Friedmann derived the Friedmann Equation by which the dynamics of a homogeneous and isotropic universe can be well described. The Friedmann Equation is usually written as follows:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}(\rho_m + \rho_r) + \frac{\Lambda c^2}{3} - \frac{kc^2}{a^2}.$$

We define the Hubble parameter as $= \frac{\dot{a}}{a}$, where a is the scale factor and $\dot{a}$ is the rate of change of scale factor with time. Thus, the Hubble parameter is a function of cosmic time. In the Friedmann Equation, ρ_m is the density of matter, including dark matter and baryons, ρ_r is the density of radiation, Λ is the cosmological constant, and k is the curvature of space. Subscript 0 indicates the value of a physical quantity at present day, e.g. H_0 is the present value Hubble parameter. Also, to avoid confusion with the reduced Hubble parameter, we use the reduced Planck Constant $\hbar = h/(2\pi)$ instead of the Planck constant h.

(a) (5 points) What are the dimensions of Hubble parameter? One can define a characteristic timescale for the expansion of the Universe (i.e. Hubble time t_H) using the Hubble parameter. Calculate the present-day Hubble time t_{H0}.

(b) (5 points) Let us define the critical density ρ_c as the matter density required to explain the expansion of a flat universe without any radiation or dark energy. Find an expression of the critical density, in terms H and G. Calculate the present critical density ρ_{c0}.

(c) (6 points) It is convenient to define all density parameters in a dimensionless manner like $\Omega_i = \frac{\rho_i}{\rho_c}$, i.e. the ratio of density to critical density. The Friedmann Equation can be rewritten using these dimensionless density parameters simply as, $\Omega_m + \Omega_r + \Omega_\Lambda + \Omega_k = 1$.

Use this information to find expression for Ω_Λ and Ω_k, in terms H, c, Λ, k and a.

(d) (7 points) Another equation which is valid for matter, radiation and dark energy is often called the Fluid Equation: $\dot{\rho} + 3\frac{\dot{a}}{a}\left(\rho + \frac{p}{c^2}\right) = 0$, where p is the pressure of some component, ρ is the density and $\dot{\rho}$ is the rate of change of density over time. Radiation contains photons and massless neutrinos, and they both travel at the speed of light. The pressure exerted by these particles is 1/3 of their energy density. Show that the density of radiation $\rho_r \propto (1+z)^4$, where z is cosmological redshift. You may note that if $\frac{\dot{\rho}}{\rho} = n\,\frac{\dot{a}}{a}$, then $\rho \propto a^n$

(e) (4 points) We know that the value of the cosmological constant Λ doesn't evolve. Its equation of state has a form $p = w\rho_\Lambda c^2$, where w is an integer. Find the value of w.

(f) (13 points) Planck time, defines a characteristic timescale before which our present physical laws are no longer valid, and where quantum gravity is needed. The expression for Planck time can be written in terms of $\hbar$, G and c and non-dimensional coefficient of this expression in SI units is of the order of unity. Using dimensional analysis, find expression for Planck time and estimate its value.

(g) (7 points) Planck length defines the length scale associated with Planck time is given by $l_P = ct_P$. The minimal mass of a black hole, also called Planck mass, is defined as the mass of a black hole whose Schwarzschild radius is two times the Planck length.

Derive the Planck mass M_P and calculate $M_P c^2$ in GeV. This mass is considered to be an upper threshold for elementary particles, beyond which they will collapse to a black hole.

(h) (4 points) At the very beginning (soon after the Planck time), all the particles were in thermal equilibrium in a primordial soup. As temperature decreased, different particles then decoupled from the primordial soup one by one and could travel freely in the Universe. Photons decoupled at ~300000 years after the Big Bang. These photons emitted at that time are what constitutes the cosmic microwave background (CMB), which follows the Stefan-Boltzmann law for blackbody radiation.

$$\varepsilon_r = \frac{\pi^2}{15\hbar^3 c^3}(k_B T)^4,$$

Show that the temperature of the CMB follows $T/(1+z) = constant$.

(i) (16 points) With the expansion of the Universe, radiation density dropped more quickly than matter density, and at some epoch the matter density was equal to the radiation density. Radiation contains both photons and neutrinos. Apart from photons, neutrinos additionally contribute to the radiation energy density by 68% (i.e. $\Omega_{r0} = 1.68\Omega_{\gamma 0}$, where γ indicates photons). Estimate the redshift of matter-radiation equality z_{eq} in terms of Ω_{m0} and reduced Hubble parameter $h = \frac{H_0}{100 km\, s^{-1} Mpc^{-1}}$ You may use the current temperature of the CMB: $T_0 = 2.73K$.

(j) (8 points) The neutrinos decoupled from the primordial soup when the temperature of the universe was around 1 MeV. At this time, the radiation density in the universe was much more than all other components. Estimate the time ($t = \frac{1}{2H}$) when neutrinos decoupled, and express it in seconds since the big bang.