Solving

a)

$$T^2 = \frac{4\pi^2}{KM} \cdot a^3;$$

$$a = \sqrt[3]{\frac{T^2 KM}{4\pi^2}};$$

$$T = 4{,}2 \text{ ani} = 4{,}2 \cdot 365 \cdot 24 \cdot 3600 \text{ s};$$

$$a = \sqrt[3]{\frac{(4{,}2 \cdot 365 \cdot 24 \cdot 36)^2 \cdot 10^4 \cdot 6{,}67 \cdot 10^{-11} \cdot 9{,}1 \cdot 10^{30}}{4 \cdot (3{,}14)^2}} \text{ m};$$

$$a = \sqrt[3]{\frac{(4{,}2 \cdot 365 \cdot 24 \cdot 36)^3 \cdot 10^4 \cdot 6{,}67 \cdot 10^{-11} \cdot 9{,}1 \cdot 10^{30}}{4{,}2 \cdot 365 \cdot 24 \cdot 36 \cdot 4 \cdot (3{,}14)^2}} \text{ m};$$

$$a = 4{,}2 \cdot 365 \cdot 24 \cdot 36 \cdot \sqrt[3]{\frac{10^4 \cdot 6{,}67 \cdot 10^{-11} \cdot 9{,}1 \cdot 10^{30}}{4{,}2 \cdot 365 \cdot 24 \cdot 36 \cdot 4 \cdot (3{,}14)^2}} \text{ m}; \ (3{,}14)^2 \approx 10;$$

$$a \approx 4{,}2 \cdot 365 \cdot 24 \cdot 36 \cdot \sqrt[3]{\frac{6{,}67 \cdot 9{,}1 \cdot 10^{22}}{4{,}2 \cdot 365 \cdot 24 \cdot 36 \cdot 4}} \text{ m};$$

$$a \approx 1324512 \cdot \sqrt[3]{\frac{60697000 \cdot 10^{15}}{5298048}} \text{ m};$$

$$a \approx 1324512 \cdot \sqrt[3]{11{,}4564836} \cdot 10^5 \text{ m}; \ \sqrt[3]{11{,}4564836} \approx 2{,}25;$$

$$a \approx 1324512 \cdot 2{,}25 \cdot 10^5 \text{ m} = 2980152 \cdot 10^5 \text{ m};$$

$$a \approx 3 \cdot 10^{11} \text{ m} = 3 \cdot 10^8 \text{ km};$$

$$e = \sqrt{1 - \frac{b^2}{a^2}}; \ \ b = a \cdot \sqrt{1 - e^2};$$

$$b = 3 \cdot 10^8 \cdot \sqrt{1 - (0{,}15)^2} \text{ km}; \ b \approx 2{,}96 \cdot 10^8 \text{ km};$$

$$c = \sqrt{a^2 - b^2} \approx 0{,}488 \cdot 10^8 \text{ km};$$

$$r_{\min} = a - c = 2{,}512 \cdot 10^8 \text{ km};$$

$$r_{\max} = a + c = 3{,}488 \cdot 10^8 \text{ km}.$$

b.

$$Q_{\text{Soare}} = \frac{E_{\text{emis,Soare}}}{tS_{\text{Soare}}} = \sigma T_S^4,$$

unde σ este o constantă;

$$\frac{E_{\text{emis,Soare}}}{t} = P_{\text{emis,Soare}};\ \sigma T_S^4 = \frac{P_{\text{emis,Soare}}}{4\pi R_S^2};$$

$$P_{\text{emis,Soare}} = \sigma T_S^4 4\pi R_S^2.$$

Densitatea fluxului energetic al Soarelui, la distanţa r_{AS} faţă de acesta (acolo unde se află Asteroidul), înseamnează energia tuturor radiaţiilor emise de Soare, care traversează unitatea de arie a unei suprafeţe, sub incidenţă normală, în unitatea de timp, adică:

$$\phi_{\text{Soare},r_{AS}} = \frac{E_{\text{emis,Soare}}}{St} = \frac{\frac{E_{\text{emis,Soare}}}{t}}{S} = \frac{P_{\text{emis,Soare}}}{S} = \frac{P_{\text{emis,Soare}}}{4\pi r_{AS}^2};$$

$$\phi_{\text{Soare},r_{AS}} = \frac{\sigma T_S^4 4\pi R_S^2}{4\pi r_{AS}^2}.$$

Semisfera asteroidului expusă radiaţiilor solare este echivalentă cu un disc plan circular, având raza R_A şi aria suprafeţei πR_A^2, aşezat perpendicular pe direcţia Soare – Asteroid, astfel încât fluxul radiaţiilor solare incidente, F, la nivelul Asteroidului (adică energia solară incidentă le nivelul Asteroidului, în unitatea de timp), este:

$$F_{\text{incident}} = \phi_{\text{Soare},r_{AS}} \cdot \pi R_A^2 = P_{\text{incident}} = P_{\text{emis,Soare}};$$

$$F_{\text{incident}} = \frac{\sigma T_S^4 4\pi R_S^2}{4\pi r_{AS}^2} \cdot \pi R_A^2 = P_{\text{incident}};$$

$$\alpha = \frac{P_{\text{reflectat,Asteroid}}}{P_{\text{incident}}};$$

$$P_{\text{reflectat,Asteroid}} = \alpha P_{\text{incident}} = \alpha \frac{\sigma T_S^4 4\pi R_S^2}{4\pi r_{AS}^2} \cdot \pi R_A^2.$$

În acord cu desenul din figura alăturată, ecuația bilanțului energetic al procesului analizat este:

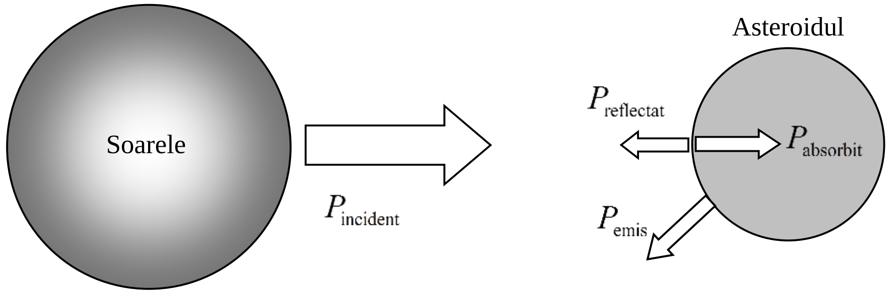


Fig.

$$P_{\text{incident,Asteroid}} = P_{\text{reflectat,Asteroid}} + P_{\text{absorbit,Asteroid}};$$

$$\frac{\sigma T_S^4 4\pi R_S^2}{4\pi r_{AS}^2} \cdot \pi R_A^2 = \alpha \frac{\sigma T_S^4 4\pi R_S^2}{4\pi r_{AS}^2} \cdot \pi R_A^2 + P_{\text{absorbit,Asteroid}}.$$

$$P_{\text{emis,Asteroid}} = \sigma T_{\text{A}}^4 \cdot 4\pi R_{\text{A}}^2.$$

Stationary

$$P_{\text{emis,Asteroid}} = P_{\text{absorbitAsteroid}} = \sigma T_{\text{A}}^4 \cdot 4\pi R_{\text{A}}^2,$$

$$\frac{\sigma T_{\text{S}}^4 4\pi R_{\text{S}}^2}{4\pi r_{\text{AS}}^2} \cdot \pi R_{\text{A}}^2 = \alpha \frac{\sigma T_{\text{S}}^4 4\pi R_{\text{S}}^2}{4\pi r_{\text{AS}}^2} \cdot \pi R_{\text{A}}^2 + \sigma T_{\text{A}}^4 \cdot 4\pi R_{\text{A}}^2;$$

$$\frac{T_{\text{S}}^4 R_{\text{S}}^2}{4 r_{\text{AS}}^2} = \alpha \frac{T_{\text{S}}^4 R_{\text{S}}^2}{4 r_{\text{AS}}^2} + T_{\text{A}}^4;$$

$$(1-\alpha)\frac{T_{\text{S}}^4 R_{\text{S}}^2}{4 r_{\text{AS}}^2} = T_{\text{A}}^4;$$

$$T_{\text{A}} = T_{\text{S}} \cdot \sqrt[4]{1-\alpha} \cdot \sqrt{\frac{R_{\text{S}}}{2 r_{\text{AS}}}},$$

Rezults

$$T_{\text{A,max}} = T_{\text{S}} \cdot \sqrt[4]{1-\alpha} \cdot \sqrt{\frac{R_{\text{S}}}{2 r_{\text{AS,min}}}};$$

$$T_{\text{S}} = 6000\,\text{K};\ R_{\text{S}} \approx 7 \cdot 10^5\ \text{km};\ r_{\text{AS,min}} = 2{,}512 \cdot 10^8\ \text{km};\ \alpha \approx 0{,}2;$$

$$T_{\text{A,max}} = 6000\,\text{K} \cdot \sqrt[4]{1-0{,}2} \cdot \sqrt{\frac{7 \cdot 10^5\ \text{km}}{2 \cdot 2{,}512 \cdot 10^8\ \text{km}}};$$

$$\sqrt[4]{1-0{,}2} = \sqrt[4]{0{,}8} \approx 0{,}945;$$

$$6000 \cdot \sqrt{\frac{7 \cdot 10^5}{2 \cdot 2{,}512 \cdot 10^8}} = \sqrt{\frac{36 \cdot 7 \cdot 10^3}{2 \cdot 2{,}512}} \approx 223{,}96;$$

$$T_{\text{A,max}} = 223{,}96 \cdot 0{,}945\,\text{K} \approx 211{,}6422\,\text{K},$$

When the astheroid is at perihelium ;

$$T_{\text{A,min}} = T_{\text{S}} \cdot \sqrt[4]{1-\alpha} \cdot \sqrt{\frac{R_{\text{S}}}{2 r_{\text{AS,max}}}};$$

$$T_{\text{S}} = 6000\,\text{K};\ R_{\text{S}} \approx 7 \cdot 10^5\ \text{km};\ r_{\text{AS,max}} = 3{,}488 \cdot 10^8\ \text{km};\ \alpha \approx 0{,}2;$$

$$T_{\text{A,min}} = 6000\,\text{K} \cdot \sqrt[4]{1-0{,}2} \cdot \sqrt{\frac{7 \cdot 10^5\ \text{km}}{2 \cdot 3{,}488 \cdot 10^8\ \text{km}}};$$

$$\sqrt[4]{1-0{,}2} = \sqrt[4]{0{,}8} \approx 0{,}945;$$

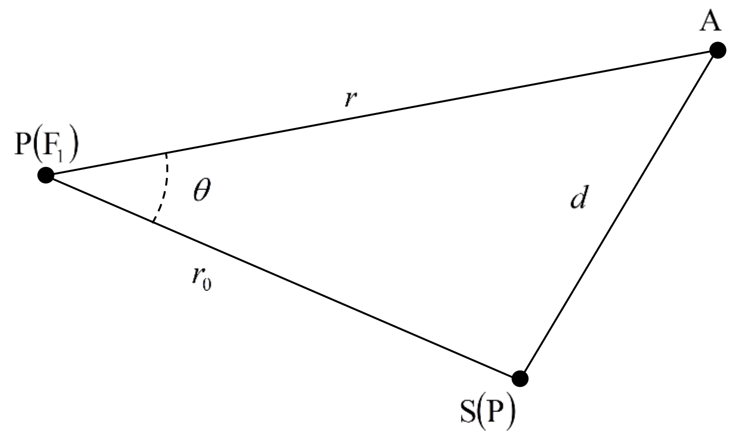
$$6000 \cdot \sqrt{\frac{7 \cdot 10^5}{2 \cdot 3{,}488 \cdot 10^8}} = \sqrt{\frac{36 \cdot 7 \cdot 10^3}{2 \cdot 3{,}488}} \approx 190{,}06;$$

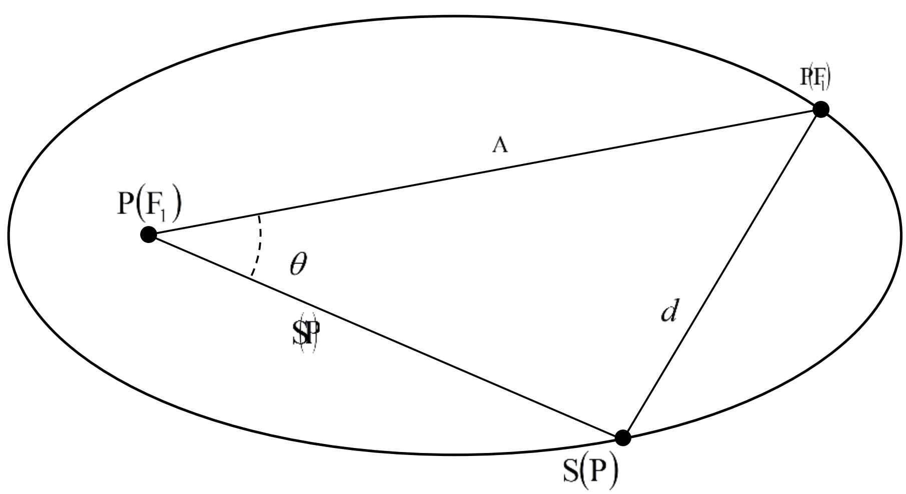
$$T_{\text{A,min}} = 190{,}06 \cdot 0{,}945\,\text{K} \approx 179{,}60\,\text{K},$$

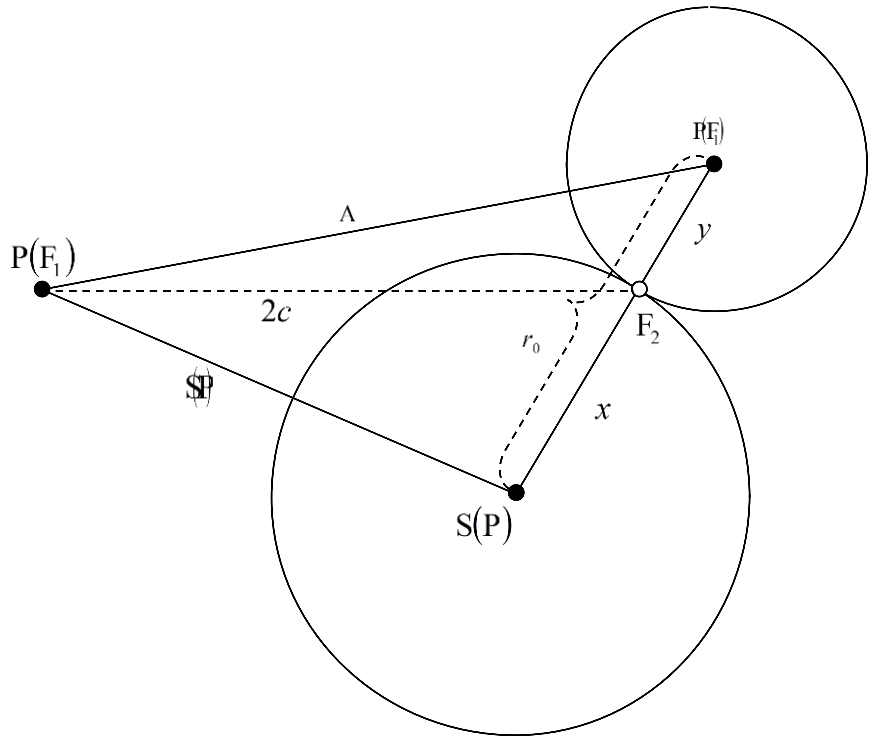
When the astheroid is at aphelium.

A.

a)







$$r_0 + x = 2a;\ r + y = 2a;$$

$$r_0 + x = r + y;$$

$$x - y = r - r_0 = \Delta r;$$

$$x + y = d;$$

$$x = \frac{1}{2}(d + \Delta r);\ y = \frac{1}{2}(d - \Delta r);$$

$$a = \frac{1}{2}(r_0 + x) = \frac{1}{2}\left[r_0 + \frac{1}{2}(d + \Delta r)\right] = \frac{1}{2}\left[r_0 + \frac{1}{2}(d + r - r_0)\right];$$

$$a = \frac{1}{2}\left[r_0 + \frac{1}{2}(d + r) - \frac{r_0}{2}\right] = \frac{1}{4}(r_0 + r + d);$$

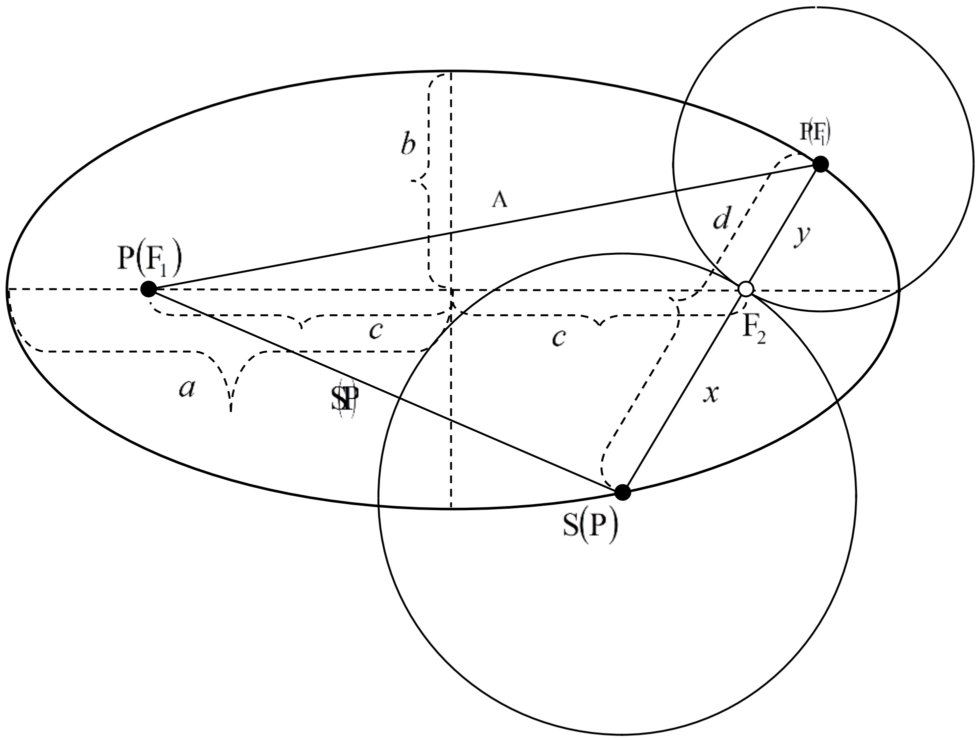
$$2a = \frac{1}{2}(r_0 + r + d);$$

$$F_1F_2 = 2c;$$

$$c = \sqrt{a^2 - b^2};\ b = \sqrt{a^2 - c^2};$$

$$e = \sqrt{1 - \frac{b^2}{a^2}};\ e = \frac{r_{max} - r_{min}}{r_{max} + r_{min}};$$

$$r_{min} = a(1 - e);\ r_{max} = a(1 + e);\ r_{min} + r_{max} = 2a.$$



Din măsurători efectuate pe desen și din calcule, rezultă:

$$r_0 = 133\ \text{mm};\ r = 153\ \text{mm};\ \ d = 115\ \text{mm};$$

$$\Delta r = r - r_0 = 20\ \text{mm};$$

$$x = 67{,}5\ \text{mm};\ \ y = 47{,}5\ \text{mm};$$

$$2a = 200\ \text{mm};\ a = 100\ \text{mm};$$

$$2b = 148\ \text{mm};\ b = 74\ \text{mm};$$

$$2c = 134\ \text{mm};\ c = 67\ \text{mm};$$

$$e \approx 0{,}67;\ r_{\text{min}} = 33\ \text{mm};\ r_{\text{max}} = 167\ \text{mm};$$

$$r_{\text{real}} = 30000\ \text{km};\ r = 153\ \text{mm};$$

$$S = \frac{r_{\text{real}}}{r} = \frac{30000}{153}\frac{\text{km}}{\text{mm}};$$

$$r_{0,\text{real}} = r_0 S \approx 26078{,}43\ \text{km};\ d_{\text{real}} = dS \approx 22549\ \text{km};$$

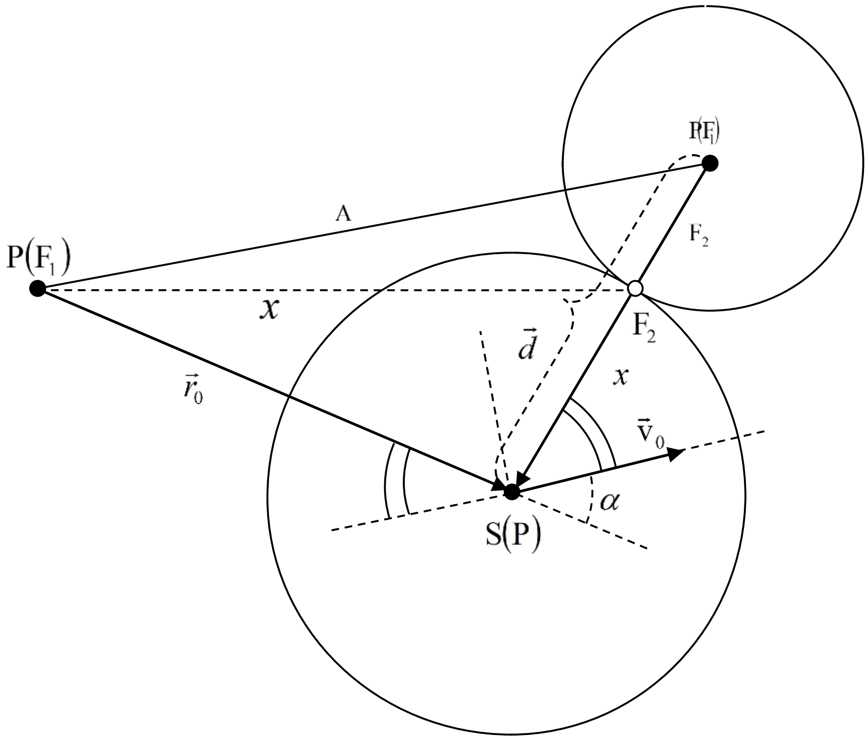
$$x_{\text{real}} = xS \approx 13235{,}3\ \text{km};\ \ y_{\text{real}} = yS \approx 9313{,}7\ \text{km};$$

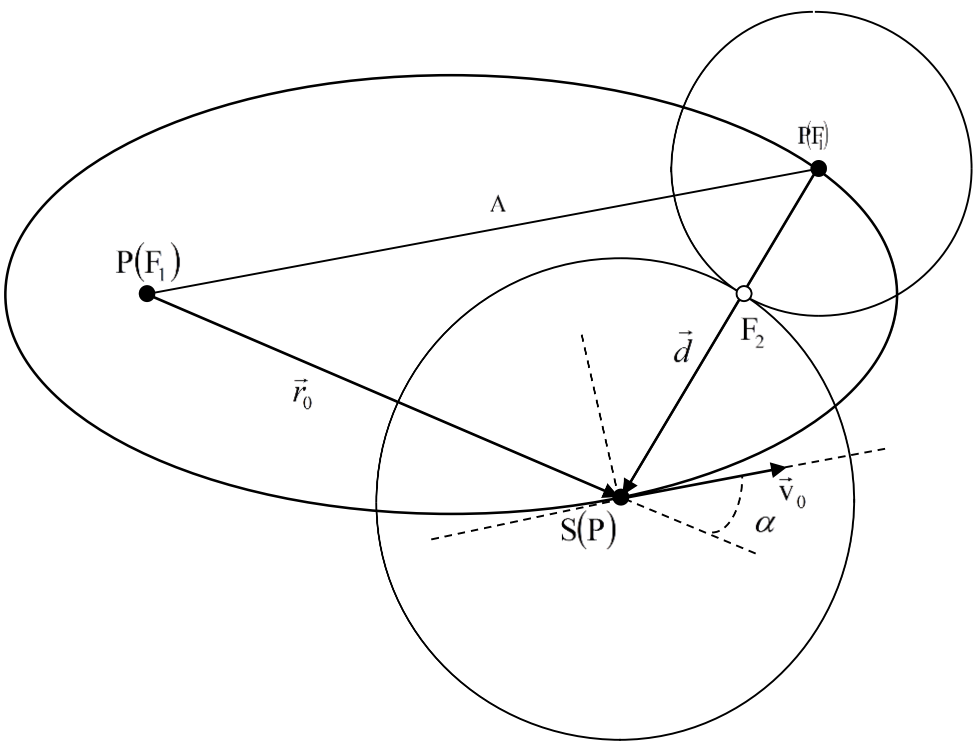
$$a_{\text{real}} \approx 19607{,}84\ \text{km};\ b_{\text{real}} \approx 14509{,}80\ \text{km};\ c_{\text{real}} \approx 13137{,}25\ \text{km};$$

$$r_{\text{min, real}} \approx 6470{,}58\ \text{km};\ r_{\text{max, real}} \approx 32745{,}09\ \text{km}.$$

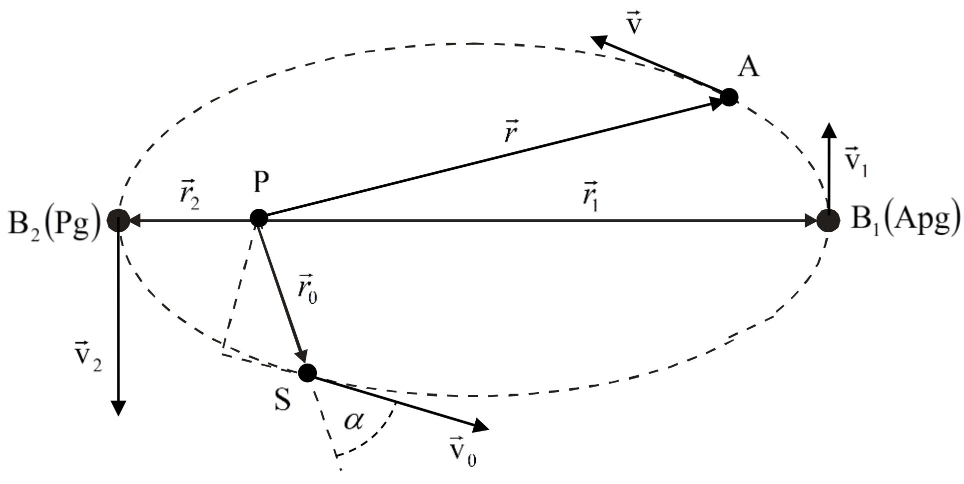
b)
:

$$\alpha \approx 54^0.$$





c)



:

$$\frac{v_{1,2}^2}{2} - K\frac{M}{r_{1,2}} = \frac{v_0^2}{2} - K\frac{M}{r_0};$$

$$v_{1,2}\, r_{1,2} = v_0\, r_0 \sin\alpha;$$

$$\left(\frac{v_0^2}{2} - K\frac{M}{r_0}\right) r_{1,2}^2 + KMr_{1,2} - \frac{1}{2} v_0^2\, r_0^2 \sin^2\alpha = 0.$$

r_1 Apogee
r_2, Perigee,:

$$a = \frac{1}{2}(r_1 + r_2),$$

$$a = \frac{KMr_0}{2KM - r_0 v_0^2},$$

relație independent of α relationship for $\vec{v}_0$

$$v_0 = \sqrt{KM\frac{2a_{real} - r_{0,real}}{a_{real} r_{0,real}}} = \sqrt{\frac{KM}{r_{0,real}} \cdot \frac{2a_{real} - r_{0,real}}{a_{real}}},$$

$$S = \frac{30000\ \text{km}}{153\ \text{mm}};$$

$$r_{0,real} = 133\ \text{mm} \cdot \frac{30000\ \text{km}}{153\ \text{mm}} \approx 26078\ \text{km};$$

$$M = 6 \cdot 10^{24}\ \text{kg};\quad K = 6{,}67 \cdot 10^{-11}\ \text{Nm}^2\text{kg}^{-2};$$

$$2a_{\text{real}} = 200\ \text{mm}\cdot S;$$

$$\text{v}_0 = \sqrt{\frac{6{,}67\cdot10^{-11}\ \text{Nm}^2\text{kg}^{-2}\cdot 6\cdot10^{24}\ \text{kg}}{26078\cdot10^3\ \text{m}}\cdot\frac{200-133}{100}};$$

$$\text{v}_0 = \sqrt{\frac{6{,}67\cdot10^{-11}\cdot 6\cdot10^{24}}{26078\cdot10^3}\cdot\frac{200-133}{100}}\ \frac{\text{m}}{\text{s}} \approx 3200\ \frac{\text{m}}{\text{s}} = 3{,}2\ \frac{\text{km}}{\text{s}}.$$

d)

$$T^2 = \frac{4\pi^2}{K(M+m)}\cdot a_{\text{real}}^3;\ m << M;\ T^2 = \frac{4\pi^2}{KM}\cdot a_{\text{real}}^3;$$

$$M = 6\cdot10^{24}\ \text{kg};\quad K = 6{,}67\cdot10^{-11}\ \text{Nm}^2\text{kg}^{-2};$$

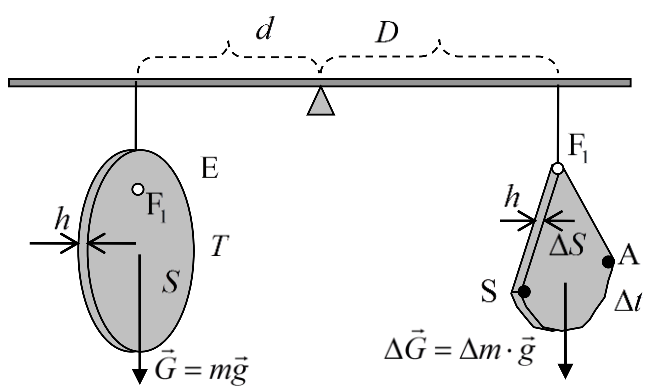
$$S = \frac{30000\ \text{km}}{153\ \text{mm};}$$

$$a_{\text{real}} = 100\ \text{mm}\cdot S \approx 19608\ \text{km};$$

$$T = 2\pi\sqrt{\frac{a_{\text{real}}^3}{KM}} = 2\cdot 3{,}14\sqrt{\frac{19608^3\cdot10^9}{6{,}67\cdot10^{-11}\cdot6\cdot10^{24}}}\ \text{s} \approx 27240\ \text{s};$$

$$T \approx 454\ \text{min} \approx 7{,}56\ \text{h}.$$

The system has to be done



$$G\cdot d = \Delta G\cdot D;\ mg\cdot d = \Delta m\cdot g\cdot D;$$

$$m\cdot d = \Delta m\cdot D;\ \rho\cdot V\cdot d = \rho\cdot\Delta V\cdot D;$$

$$V\cdot d = \Delta V\cdot D;\ S\cdot h\cdot d = \Delta S\cdot h\cdot D;$$

$$S\cdot d = \Delta S\cdot D;\ \frac{\Delta S}{S} = \frac{d}{D}.$$

According to the second Keppler law:

$$T\ldots\ldots\ldots\ldots\ldots\ldots S;$$

$$\Delta t \ldots\ldots\ldots\ldots\ldots\ldots\ldots \Delta S;$$

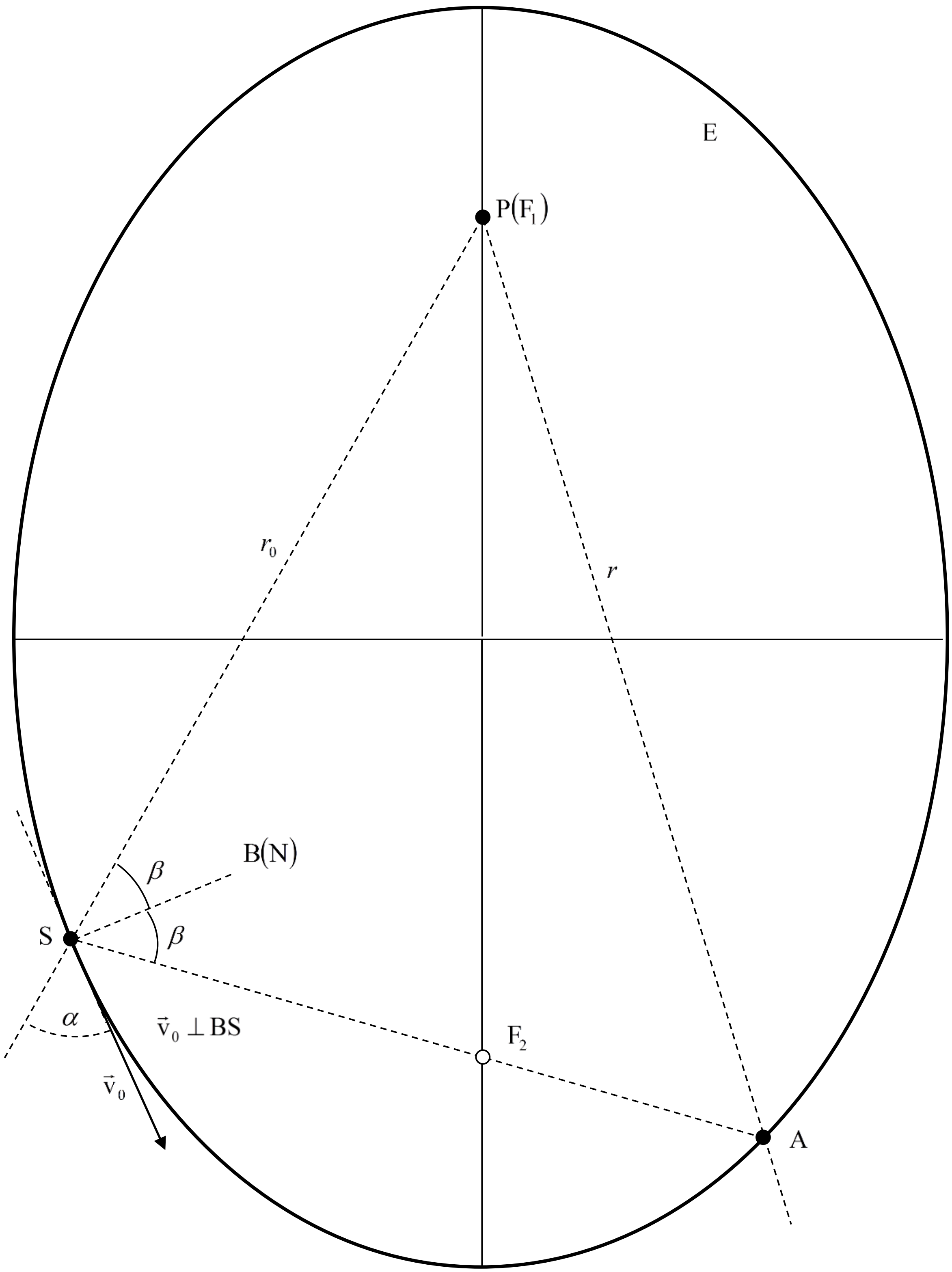
$$\Delta t = \frac{\Delta S}{S} \cdot T = \frac{d}{D} \cdot T,$$

D	*d*	*T*	Δt
26 cm	11 cm	7,56 h	3,19 h

e) Measurements ao a wire along the elipse sector between S and A:

$$l_{\mathrm{SA}} = 135\,\mathrm{mm};$$

$$l_{\mathrm{SA,real}} = l_{\mathrm{SA}} \cdot S = 135\,\mathrm{mm} \cdot \frac{30000\,\mathrm{km}}{153\,\mathrm{mm}} \approx 26470{,}58\,\mathrm{km}.$$

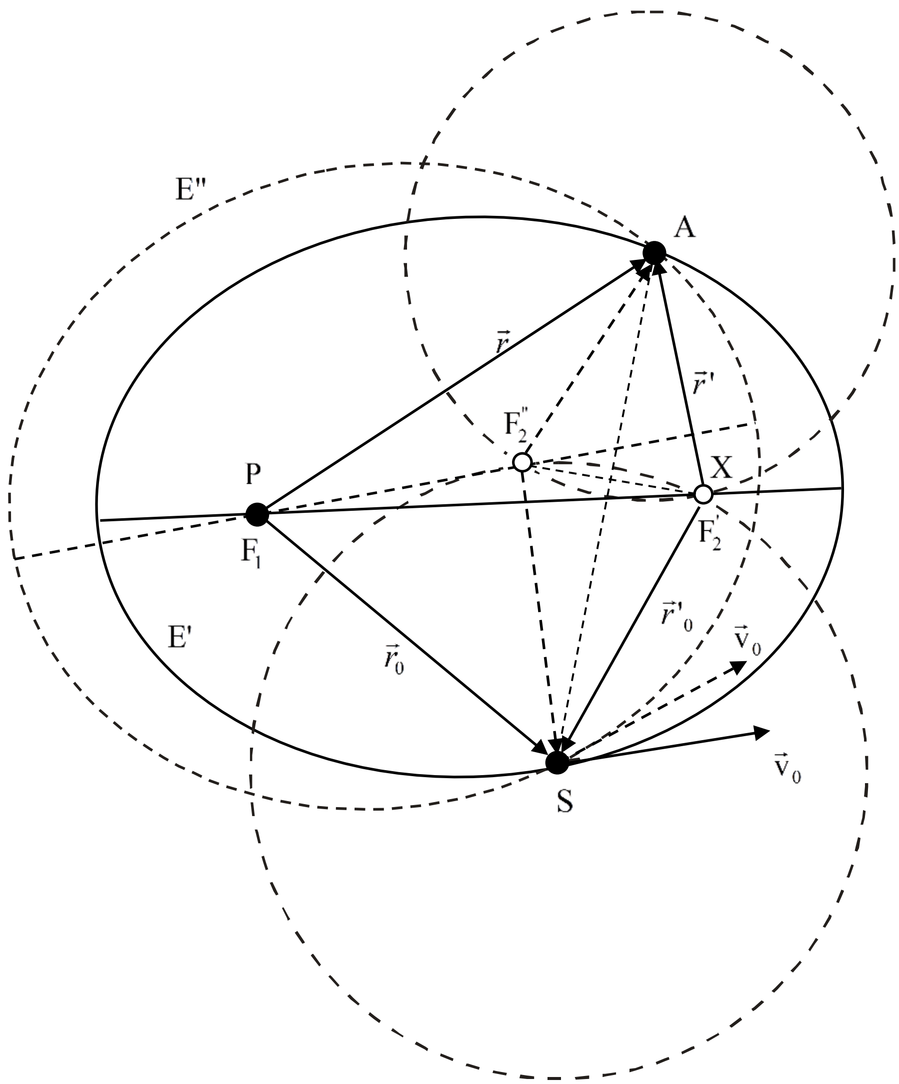

E
P(F1)
r0
r
B(N)
β
β
S
α
v0 ⊥ BS
v0
F2
A

B.

a) The 2 elipses have a common focar so the injection point will go in the same point $\vec{r}_0$, with initial velocity, v_0. **The both elipses semiaxes are identical**:

$$a = \frac{KMr_0}{2KM - r_0 v_0^2}.$$

$2c_1 \neq 2c_2;,\ 2b_1 \neq 2b_2;\ e_1 \neq e_2.$



$$PS + SX = PA + AX = 170\text{ mm},$$

$X \equiv F_2'.$

AAfter localizing the focuses of the elipses the axes can be drawn.

$$r_0 + r_0^{'} = 2a;\ r + r' = 2a.$$

Measuring r_0 and $r_0^{'}$, or r and r', can be calculated:

$$S = \frac{30000\ \text{km}}{107\ \text{mm}};$$

$$r_0 = 95\ \text{mm}\cdot S;\ r_0^{'} = 75\ \text{mm}\cdot S;$$

$$r = 107\ \text{mm}\cdot S;\ r' = 63\ \text{mm}\cdot S;$$

$$2a = 170\ \text{mm}\cdot S \approx 47664\ \text{km};$$

$$a = \frac{1}{2}\left(r_0 + r_0^{'}\right) = \frac{1}{2}\left(r + r'\right) = 85\ \text{mm}\cdot S = 85\ \text{mm}\cdot\frac{30000\ \text{km}}{107\ \text{mm}} \approx 23832\ \text{km}.$$

$$\text{F}_1\text{F}_2^{'} = 2c_1;\ b_1 = \sqrt{a^2 - c_1^2};$$

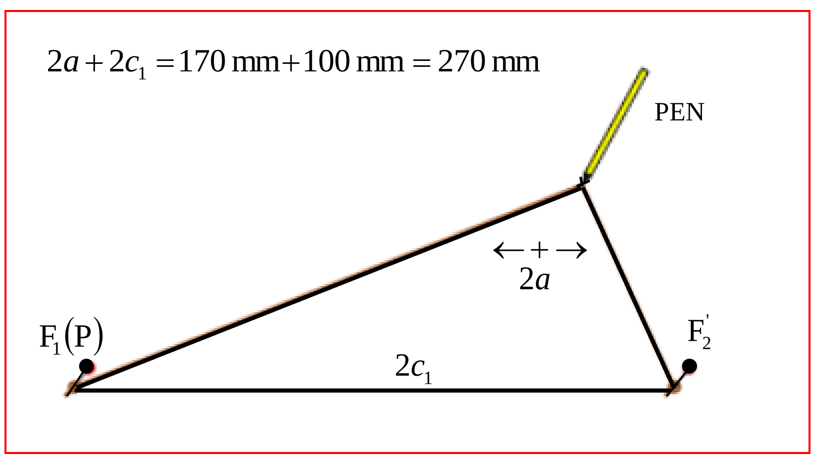
$$2c_1 = 100\ \text{mm}\cdot S;\ c_1 = 50\ \text{mm}\cdot S;$$

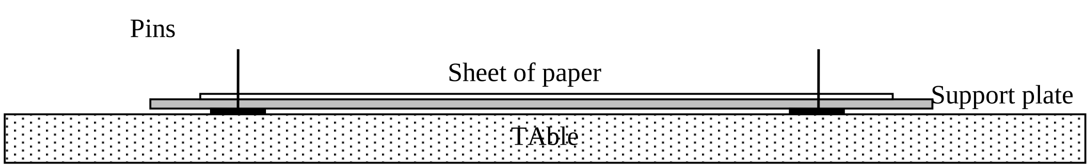
$$b_1 \approx 69\ \text{mm}\cdot S \approx 19346\ \text{km};$$

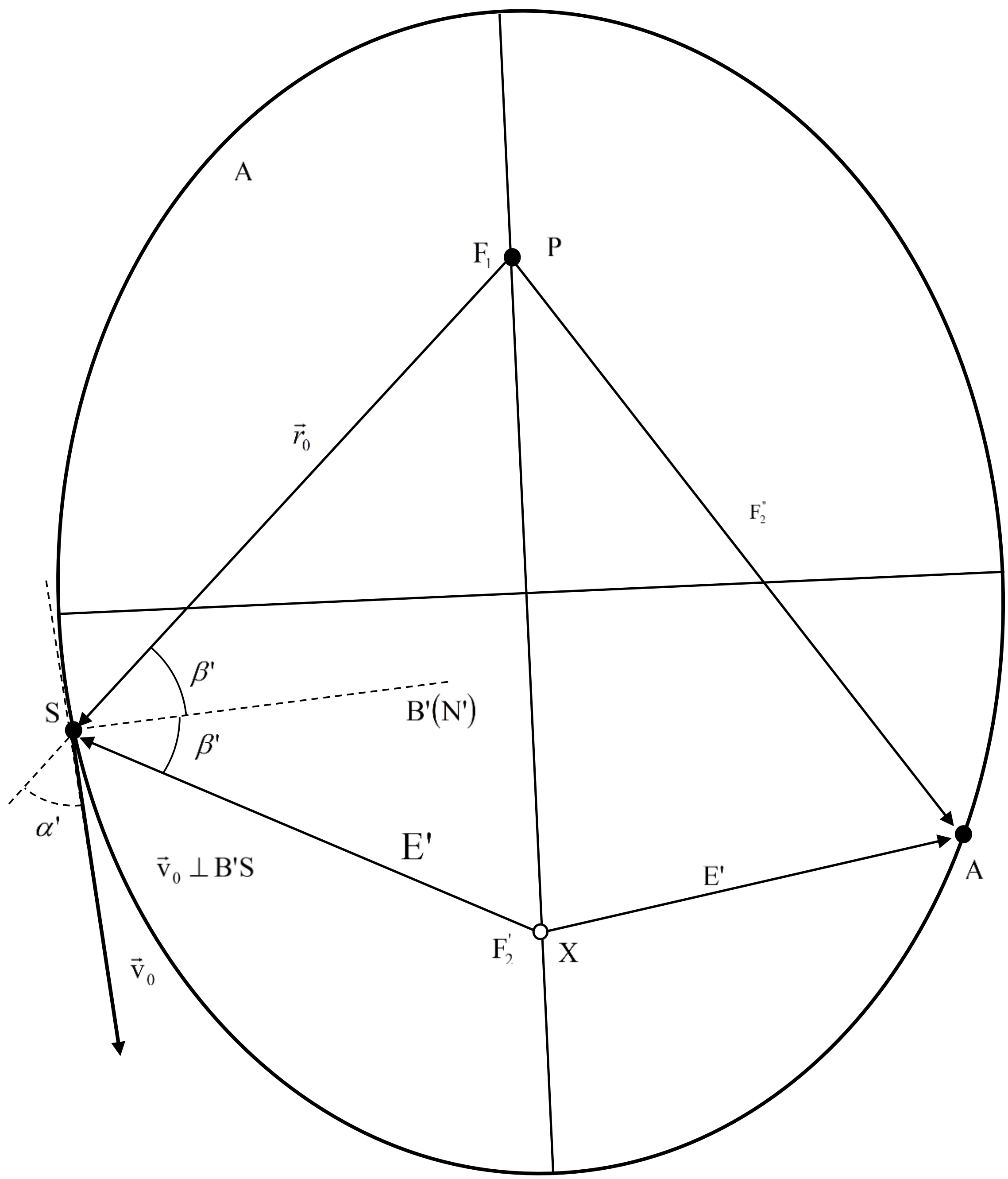
$$\text{F}_1\text{F}_2^{''} = 2c_2;\ b_2 = \sqrt{a^2 - c_2^2};$$

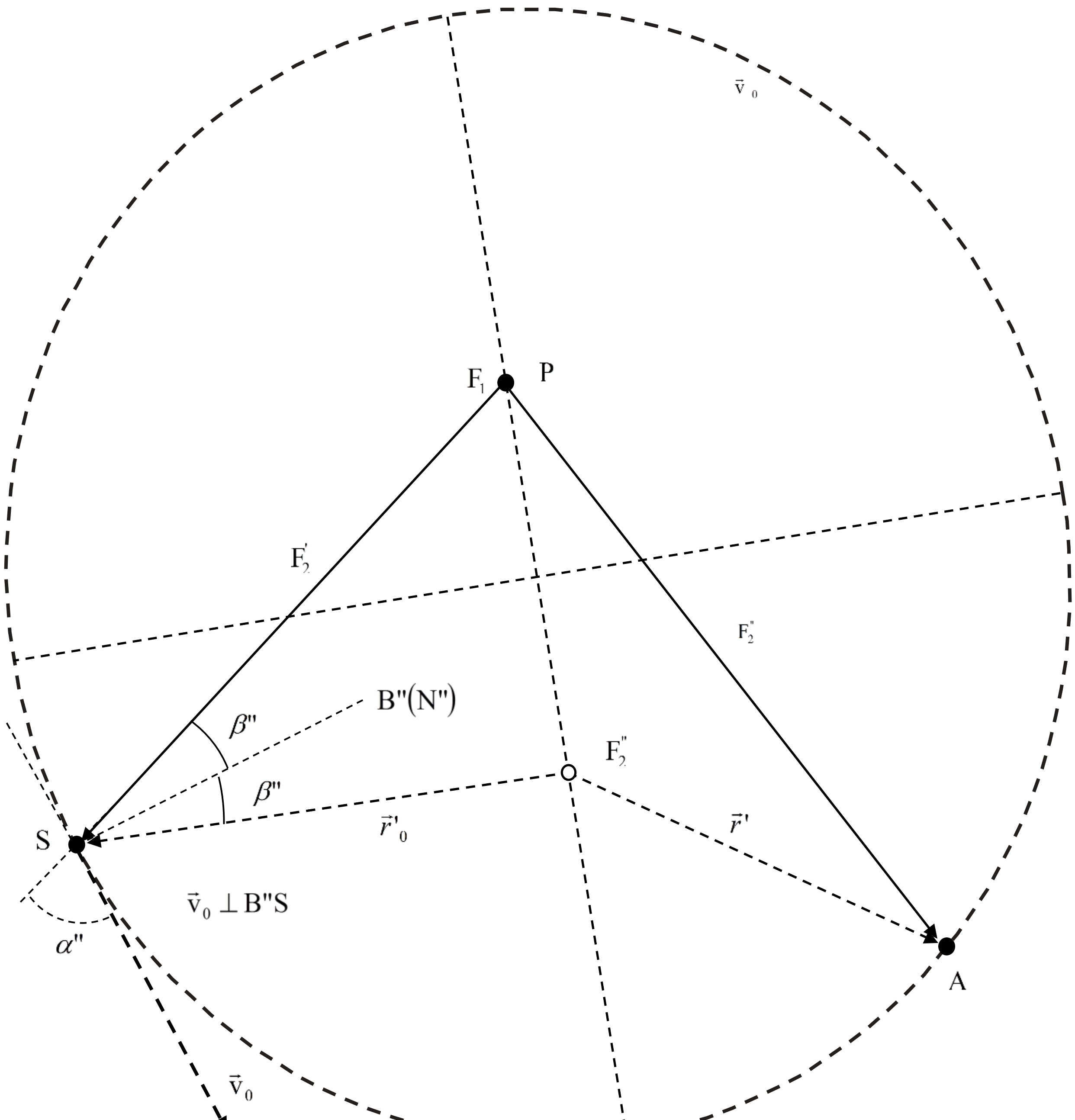
$$2c_2 = 60\ \text{mm}\cdot S;\ c_2 = 30\ \text{mm}\cdot S;$$

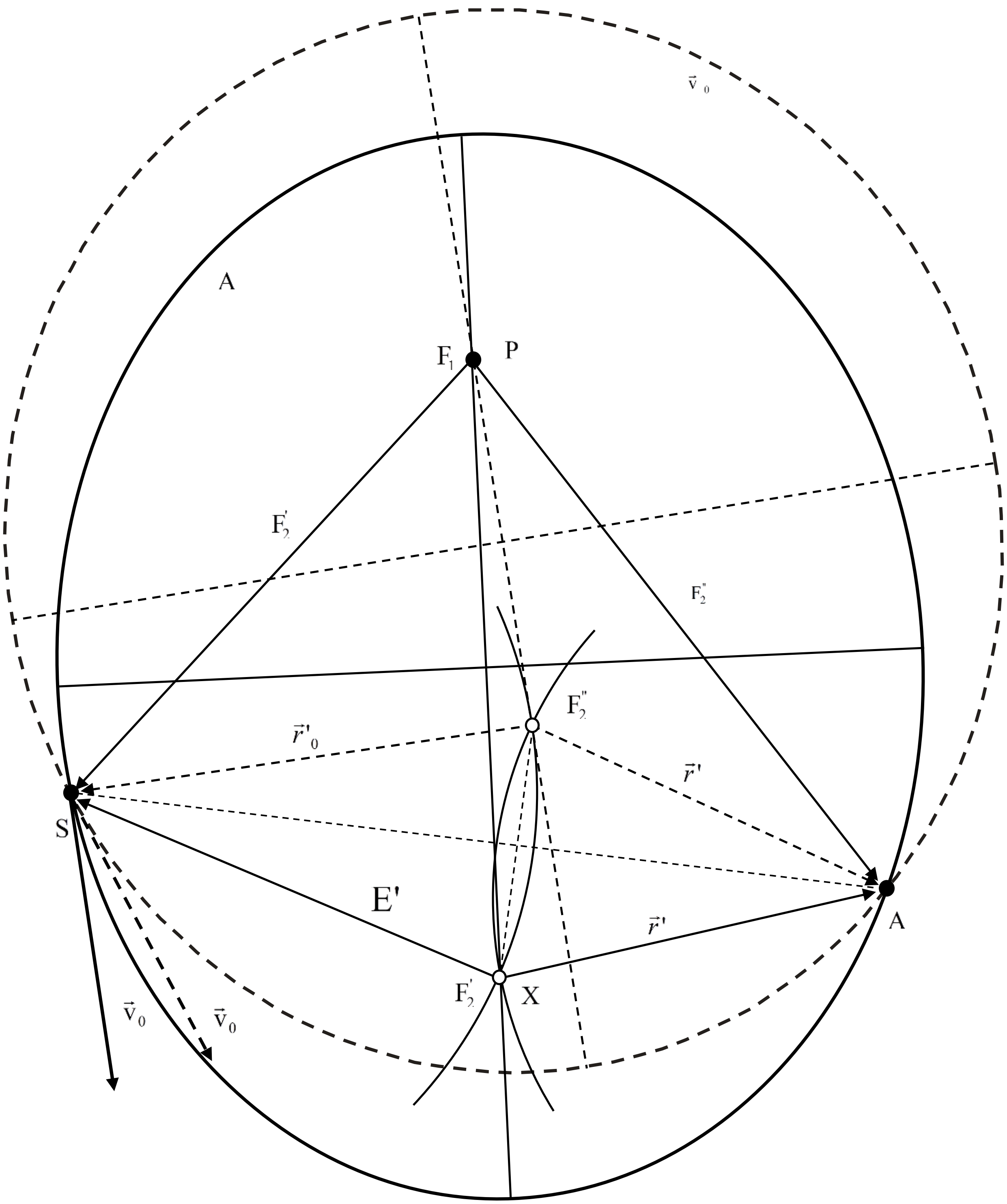
$$b_2 \approx 80\ \text{mm}\cdot S \approx 22430\ \text{km}.$$






A
F1
P
r0
F2"
β'
S
B'(N')
β'
α'
E'
v0 ⊥ B'S
E'
A
F2'
X
v0


$\vec{v}_0$
F_1
P
F_2'
F_2''
$B''(N'')$
β''
β''
F_2''
S
$\vec{r}'_0$
$\vec{r}'$
$\vec{v}_0 \perp B''S$
α''
A
$\vec{v}_0$


A
F1
P
F2'
F2''
F2''
r'0
r'
S
E'
r'
A
F2'
X
v0
v0
v0

b)

$$\alpha' \approx 50^0; \quad \alpha'' \approx 70^0.$$

c)

$$\mathrm{v}_0 = \sqrt{KM\frac{2a_{\text{real}} - r_{0,\text{real}}}{a_{\text{real}} r_{0,\text{real}}}} = \sqrt{\frac{KM}{r_{0,\text{real}}} \cdot \frac{2a - r_0}{a}};$$

$$r_0 = 95\ \text{mm} \cdot S;\ S = \frac{30000\ \text{km}}{107\ \text{mm}};$$

$$r_{0,\text{real}} \approx 26636\ \text{km};\ 2a_{\text{real}} = 170\ \text{mm} \cdot S;$$

$$a = 85\ \text{mm} \cdot S;\ \ a_{\text{real}} = 23831{,}77\ \text{km};$$

$$2a = 170\ \text{mm};\ \ 2a_{\text{real}} = 47663{,}55\ \text{km};$$

$$\mathrm{v}_0 = \sqrt{\frac{6{,}67 \cdot 10^{-11}\ \text{Nm}^2\text{kg}^{-2} \cdot 6 \cdot 10^{24}\ \text{kg}}{26636 \cdot 10^3\ \text{m}} \cdot \frac{170 - 95}{85}};$$

$$\mathrm{v}_0 = \sqrt{\frac{6{,}67 \cdot 10^{-11} \cdot 6 \cdot 10^{24}}{26636 \cdot 10^3} \cdot \frac{170 - 95}{85}}\ \frac{\text{m}}{\text{s}} \approx 3640\ \frac{\text{m}}{\text{s}} \approx 3{,}6\ \frac{\text{km}}{\text{s}}.$$

d)

$$T^2 = \frac{4\pi^2}{K(M+m)} \cdot a_{\text{real}}^3;\ m << M;\ T^2 = \frac{4\pi^2}{KM} \cdot a_{\text{real}}^3;$$

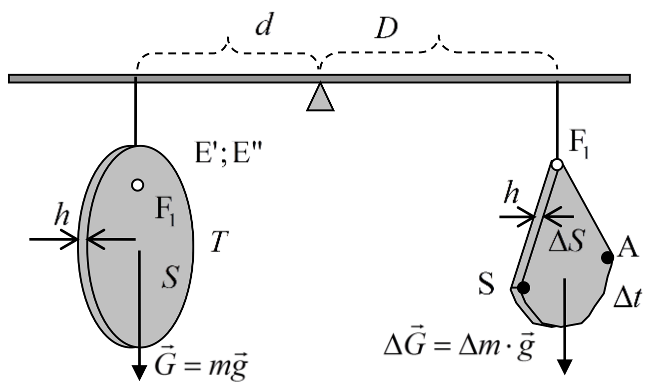
$$M = 6 \cdot 10^{24}\ \text{kg};\ \ K = 6{,}67 \cdot 10^{-11}\ \text{Nm}^2\text{kg}^{-2};$$

$$a_{\text{real}} = 23832\ \text{km};$$

$$T = 2\pi\sqrt{\frac{a_{\text{real}}^3}{KM}} = 2 \cdot 3{,}14\sqrt{\frac{23832^3 \cdot 10^9}{6{,}67 \cdot 10^{-11} \cdot 6 \cdot 10^{24}}}\ \text{s} \approx 36523\ \text{s};$$

$$T \approx 608\ \text{min} \approx 10\ \text{h}.$$

For each of two elipses the device has to be used



$$G \cdot d = \Delta G \cdot D;\ mg \cdot d = \Delta m \cdot g \cdot D;$$
$$m \cdot d = \Delta m \cdot D;\ \rho \cdot V \cdot d = \rho \cdot \Delta V \cdot D;$$
$$V \cdot d = \Delta V \cdot D;\ S \cdot h \cdot d = \Delta S \cdot h \cdot D;$$
$$S \cdot d = \Delta S \cdot D;\ \frac{\Delta S}{S} = \frac{d}{D}.$$

$$T \ldots\ldots\ldots\ldots\ldots\ldots\ldots S;$$
$$\Delta t \ldots\ldots\ldots\ldots\ldots \Delta S;$$
$$\Delta t = \frac{\Delta S}{S} \cdot T = \frac{d}{D} \cdot T;$$

For the projectile on elipse E', from measurements :

$$d = 12{,}8\,\text{cm};\ D = 23\,\text{cm};\ T = 10\,\text{h};$$
$$\Delta t = \frac{d}{D} \cdot T = \frac{12{,}8\,\text{cm}}{23\,\text{cm}} \cdot 10\,\text{h} \approx 5{,}56\,\text{h};$$
$$d = 10{,}7\,\text{cm};\ D = 25\,\text{cm};$$
$$\Delta \tau = \frac{10{,}7\,\text{cm}}{25\,\text{cm}} \cdot 10\,h \approx 4{,}28\,\text{h};$$
$$\Delta t = T - \Delta \tau = 5{,}72\,\text{h};$$
$$\Delta t = \frac{5{,}56\,\text{h} + 5{,}72\,\text{h}}{2} = 5{,}64\,\text{h}.$$

For the projectile on elipse E'', from measurements:

$$d = 8{,}5\,\text{cm};\ D = 22{,}3\,\text{cm};\ T = 10\,\text{h};$$
$$\Delta t = \frac{d}{D} \cdot T = \frac{8{,}5\,\text{cm}}{22{,}3\,\text{cm}} \cdot 10\,\text{h} \approx 3{,}81\,\text{h};$$
$$d = 13{,}5\,\text{cm};\ D = 23\,\text{cm};$$
$$\Delta \tau = \frac{13{,}5\,\text{cm}}{23\,\text{cm}} \cdot 10\,h \approx 5{,}86\,\text{h};$$

$$\Delta t = T - \Delta \tau = 4{,}14\ \text{h};$$

$$\Delta t = \frac{3{,}81\ \text{h} + 4{,}14\ \text{h}}{2} = 3{,}975\ \text{h}.$$

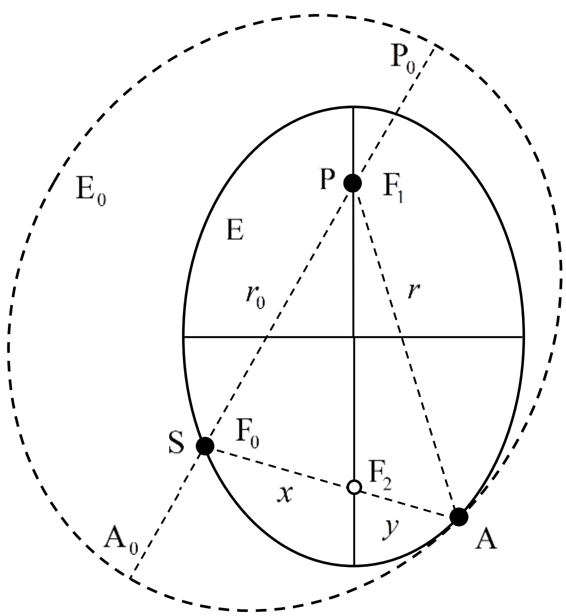
e) Modeling one wire on each sector SA:

$$l'_{\text{SA}} = 183\ \text{mm};\ l'_{\text{SA,real}} = l'_{\text{SA}} S = 183\ \text{mm} \cdot \frac{30000\ \text{km}}{107\ \text{mm}} \approx 51308{,}41\ \text{km};$$

$$l''_{\text{SA}} = 156\ \text{mm};\ l''_{\text{SA,real}} = l''_{\text{SA}} S = 156\ \text{mm} \cdot \frac{30000\ \text{km}}{107\ \text{mm}} \approx 43738{,}31\ \text{km}.$$

C.

a)



$$\text{PA} + \text{AS} = 2a_0;$$

$$r + x + y = 2a_0;$$

$$r_0 + x = 2a;\ r + y = 2a:$$

$$2a + 2a - r_0 = 2a_0;$$

$$2a_0 = 4a - r_0,$$

$$2a_0 = 222\text{ mm} - 74\text{ mm} = 148\text{ mm};$$

$$S = \frac{30000\text{ km}}{85\text{ mm}};$$

$$2a_{0,\text{real}} = 2a_0 S = 148\text{ mm}\cdot\frac{30000\text{ km}}{85\text{ mm}} \approx 52235{,}29\text{ km},$$

a-A:

$$2a_0 = 400\text{ mm} - 133\text{ mm} = 267\text{ mm};$$

$$S = \frac{30000\text{ km}}{153\text{ mm}};$$

$$2a_{0,\text{real}} = 2a_0 S = 267\text{ mm}\cdot\frac{30000\text{ km}}{153\text{ mm}} \approx 52352{,}94\text{ km};\; a_{0,\text{real}} = 26176{,}47\text{ km}.$$

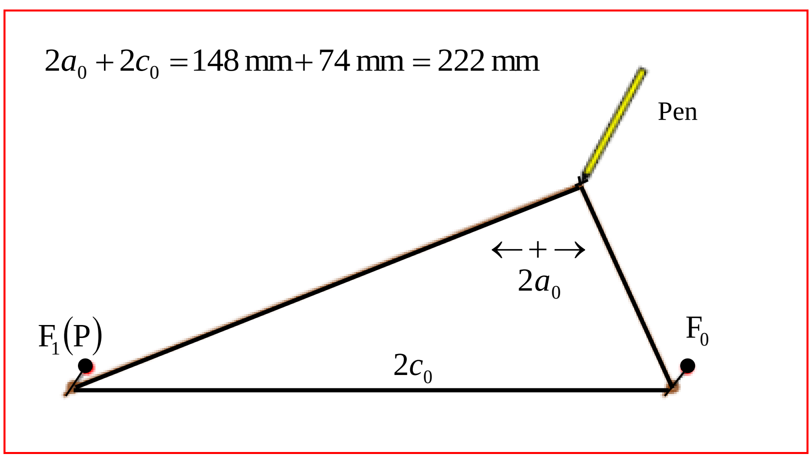
Security elipse .

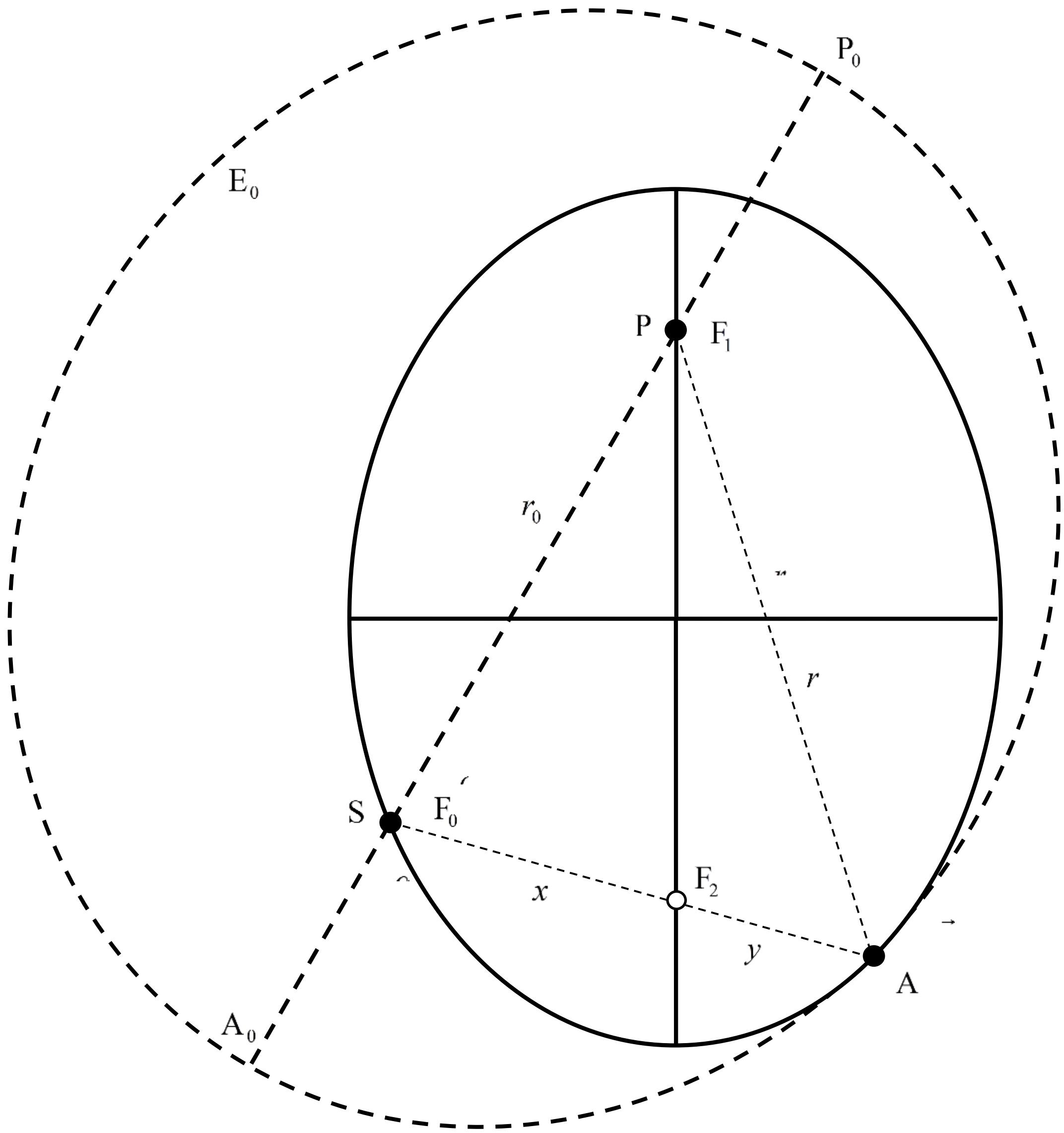
$$2c_{0,\text{real}} = r_{0,\text{real}} = 26078\text{ km};\; c_{0,\text{real}} = 13039\text{ km};$$

$$b_{0,\text{real}} = \sqrt{a_{0,\text{real}}^2 - c_{0,\text{real}}^2} \approx 22697{,}84\text{ km};$$

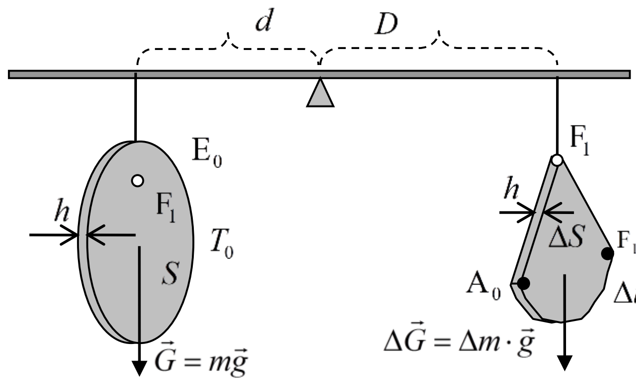
$$e_0 = \sqrt{1 - \frac{b_{0,\text{real}}^2}{a_{0,\text{real}}^2}} \approx 0{,}5.$$

Conturul elipsei de siguranță se trasează așa cum indică desenul din figura alăturată.





b)
c)



$$G \cdot d = \Delta G \cdot D;\ mg \cdot d = \Delta m \cdot g \cdot D;$$
$$m \cdot d = \Delta m \cdot D;\ \rho \cdot V \cdot d = \rho \cdot \Delta V \cdot D;$$
$$V \cdot d = \Delta V \cdot D;\ S \cdot h \cdot d = \Delta S \cdot h \cdot D;$$
$$S \cdot d = \Delta S \cdot D;\ \frac{\Delta S}{S} = \frac{d}{D}.$$

$$T_0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots S;$$
$$\Delta t \ldots\ldots\ldots\ldots\ldots\ldots \Delta S;$$
$$\Delta t = \frac{\Delta S}{S} \cdot T_0 = \frac{d}{D} \cdot T_0;$$
$$T_0^2 = \frac{4\pi^2}{KM} a_{0,\text{real}}^3;\ a_{0,\text{real}} = 26176{,}47\ \text{km};$$
$$M = 6 \cdot 10^{24}\ \text{kg};\quad K = 6{,}67 \cdot 10^{-11}\ \text{Nm}^2\text{kg}^{-2};$$
$$T_0 = 2\pi \cdot a_{0,\text{real}} \cdot \sqrt{\frac{a_{0,\text{real}}}{KM}} \approx 42042{,}42\ \text{s};$$
$$T_0 \approx 700{,}7\ \text{min} \approx 11{,}67\ \text{h}.$$

$$d = 8{,}5\ \text{cm};\ D = 23\ \text{cm};\quad T = 11{,}67\ \text{h};$$
$$\Delta\tau = \frac{d}{D} \cdot T = \frac{8{,}5\ \text{cm}}{23\ \text{cm}} \cdot 11{,}67\ \text{h} \approx 4{,}31\ \text{h};$$
$$\Delta t = T_0 - \Delta\tau = 7{,}36\ \text{h},$$