8th International Olympiad on Astronomy and Astrophysics
Suceava - Gura Humorului - August 2014

ALL THE CORRECT SOLUTIONS WHICH ARE DIFFERENT FROM THE AUTHOR'S SOLUTION WILL BE EVALUATED AND MARKED ACORDINGLY TO THE MARKING SCHEME.

Content

Problem 1. Lagrange Points

The *Lagrange* points are the five positions in an orbital configuration (assume circular orbits), where a small object is stationary relative to two big bodies, only gravitationally interacting with them- for example, an artificial satellite relative to Earth and Moon, or relative to Earth and Sun. In the **Figure 1** are sketched two possible locations of Lagrange points L_3 relative to relative to the Earth – Sun system . Find out which of the two locations L_3^1 and L_3^2 could be the real Lagrange point relative to the system Earth – Sun; show the reason for your

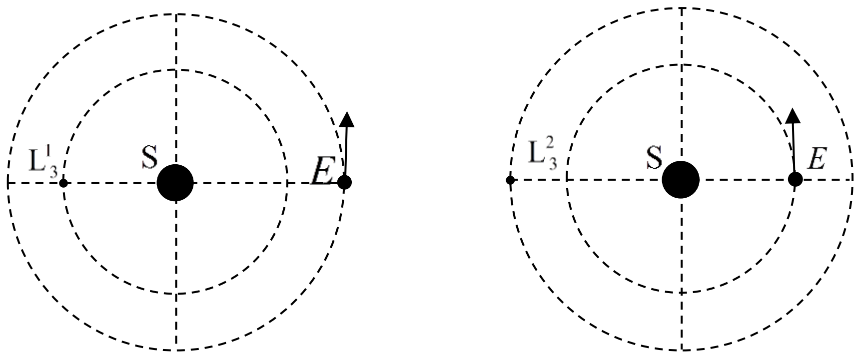


Figure 1A **Figure 1B**

answer with appropriate equations and calculate the difference between one AU and Sun - L_3 distance . You know the following data: the Earth - Sun distance $d_{ES} = 14.96 \cdot 10^7$ km and the Earth – Sun mass ratio $M_E / M_S = 1/332946$

Problem 1. Marking scheme Lagrange Point

1.

According to the notations in fig.1.1 and fig. 2.1

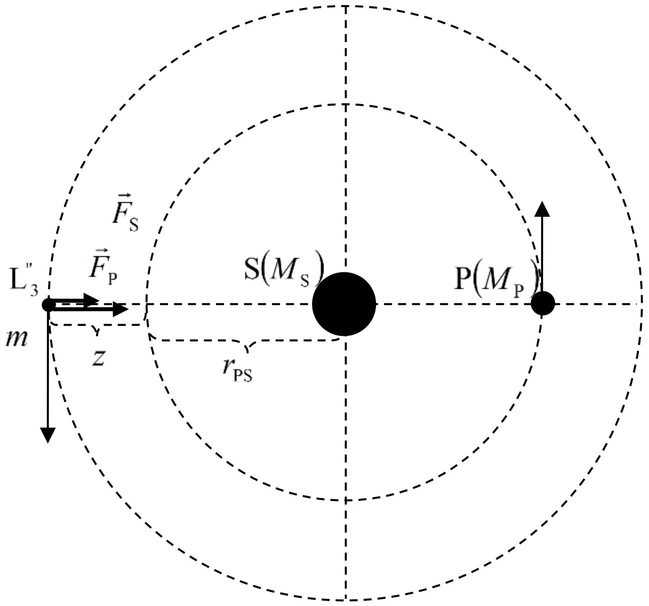


Figure 1a

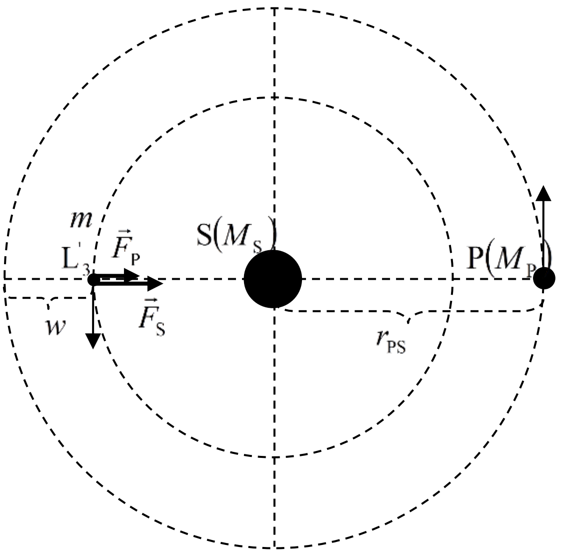


Figure 1 b

$$\vec{F}_S + \vec{F}_P = \vec{F}_{cp} = m\vec{a}_{cp}$$

$$F_S + F_P = ma_{cp} = m\omega^2(r_{PS} \pm w);$$

.... **2 points**

The sign “+” for position L_3' and „– “for L_3''

$$K\frac{mM_S}{(r_{PS} \pm w)^2} + K\frac{mM_P}{(2r_{PS} \pm w)^2} = m\omega^2(r_{PS} \pm w);$$

Using the assumption that $w << r_{PS}$

$$\left(1 \pm \frac{w}{r_{PS}}\right)^{-2} \approx 1 \mp 2\frac{w}{r_{PS}}; \quad \left(1 \pm \frac{w}{2r_{PS}}\right)^{-2} \approx 1 \mp \frac{w}{r_{PS}};$$

.... **2points**

The rotation speed

$$\omega^2 = \frac{KM_S}{r_{PS}^3}$$

.... **2p**

The final relation

$$w = \mp\frac{M_P r_{PS}}{(12M_S + M_P)}$$

The value has to be positive, thus the L''_3 is the position of one Lagrange point **2p**

$w \approx 37.44\, Km$ **2p**

Problem 2. Sun gravitational catastrophe!

In a gravitational catastrophe, the mass of the Sun mass decrease instantly to half of its actual value. If you consider that the actual Earth orbit is elliptical, its orbital period is $T_0 = 1\, year$ and the eccentricity of the Earth orbit is $e_0 = 0.0167$.

Find the period of the Earth`s orbital motion, after the gravitational catastrophe, if it occurs on: a) 3rd of July (aphelion) b) 3rd of January.

Problem 2. Marking scheme Sun gravitational catastrophe!

- Correct analyze of the initial conditions when the catastrophe occurs **(A)** **5 points**
- Correct calculations **(B)** **5 points**
  - Correct use of laws of conservation 2 points
  - Finding out that in the first case the orbit will be elliptic, relations (1) and (2) 2 points
  - Correct conduct of calculations 1 point

Detailed solution

(A) The orbit of Earth is elliptical, so the shape of the orbit after the solar catastrophe will depend on the moment when the decrees of the mass of the Sun will occur.

For following explanation

a) In 3rd July the Earth is at the aphelion. The speed of the Earth is smaller than the speed of Earth on a circular orbit with radius $r_{0,\max} = a_0(1+e_0)$.. **1p**

b) In 3rd January the Earth is at perihelion. The speed of the Earth is bigger than the speed of Earth on a circular orbit with radius $r_{0,\max} = a_0(1-e_0)$.. **1p**

According to Keppler's second law and the law of energy conservation the following relations can be written :

$$v_{0,per}\, r_{0,per} = v_{0,aph}\, r_{0,aph};$$ **1p**

$$\frac{v_{0,per}^2}{2} - K\frac{M_0}{r_{0,per}} = \frac{v_{0,aph}^2}{2} - K\frac{M_0}{r_{0,aph}}$$ **1p**

$$r_{0,\min} = r_{0,per} = a_0(1-e_0)$$

$$r_{0,\max} = r_{0,aph} = a_0(1+e_0)$$

$$KM_0 = v_0^2 r_0 = v_0^2 a_0$$

$$v_0 = \sqrt{K\frac{M_0}{r_0}} = \sqrt{K\frac{M_0}{a_0}}$$

$$\mathrm{v}_{0,per} = \mathrm{v}_0 \sqrt{\frac{1+e_0}{1-e_0}} > \mathrm{v}_0$$

$$\mathrm{v}_{0,per} > \mathrm{v}_0 \qquad \textbf{(1)}$$

$$\mathrm{v}_{0,aph} = \mathrm{v}_0 \sqrt{\frac{1-e_0}{1+e_0}}$$

$$\mathrm{v}_{0,aph} < \mathrm{v}_0 \qquad \textbf{(2)}$$

Conclusion – According to the relations (1) and (2) the new orbit of the Earth could be an elliptic one. **1p**

For the new elliptical Earth orbit:

$$r_{per} = r_{0,aph};$$

$$r_{min} = r_{per} = a(1-e);$$

$$a_0(1+e_0) = a(1-e); \; a = a_0 \frac{1+e_0}{1-e};$$

$$\mathrm{v}_{per} = \mathrm{v}_{0,aph};$$

$$\mathrm{v}_{per} = \mathrm{v}\sqrt{\frac{1+e}{1-e}},$$

Where v este is the Earth's speed on a circular orbit with the radius $r = a,$ when the mass of the Sun becomes $M = M_0/2;$

$$\mathrm{v}\sqrt{\frac{1+e}{1-e}} = \mathrm{v}_0\sqrt{\frac{1-e_0}{1+e_0}};$$

1p

$$e = 1-2e_0; \; a = a_0\frac{1+e_0}{2e_0}.$$

1p

Conclusion

$$T_0 = \frac{2\pi r_0}{\mathrm{v}_0} = \frac{2\pi a_0}{\mathrm{v}_0}$$

$$T = \frac{2\pi r}{\mathrm{v}} = \frac{2\pi a}{\mathrm{v}};$$

$$\frac{T}{T_0} = \frac{a}{a_0}\frac{\mathrm{v}_0}{\mathrm{v}} = \frac{1+e_0}{2e_0}\sqrt{\frac{2a}{a_0}} = \frac{1+e_0}{2e_0}\sqrt{2}\sqrt{\frac{1+e_0}{2e_0}};$$

1p

$$T = T_0\sqrt{2}\left(\frac{1+e_0}{2e_0}\right)^{3/2} \approx 238\,years$$

1p

b) In 3rd of January the Earth is at perihelion. In that moment the Erath speed is larger than the speed necessary for an Earth's circular orbit. Thus the trajectory of the Earth after the catastrophe will be an open trajectory, i.e. an hyperbolic or parabolic orbit.

Conclusion it is not necessary to calculate the period of revolution or could be issued as infinite

1p

Problem 3. Cosmic radiation

During studies concerning cosmic radiation, a neutral unstable particle – the π^0 meson was identified. The rest-mass of meson π^0 is much larger than the rest-mass of the electron. The studies reveal that during its flight, the meson π^0 disintegrates into 2 photons. In a particular case, one of the created photons has the maximum possible energy $E_{\max}$ and, consequently, the other one has the minimum possible energy $E_{\min}$.

Find an expression for the initial velocity of the meson π^0, as a function of $E_{\max}$ and $E_{\min}$. You may use as known **c -** the speed of light and the relation between the energy and momentum of any relativistic particles

$E^2 = p^2c^2 + m_0^2c^4$

Problem 3. Marking scheme Cosmic radiation

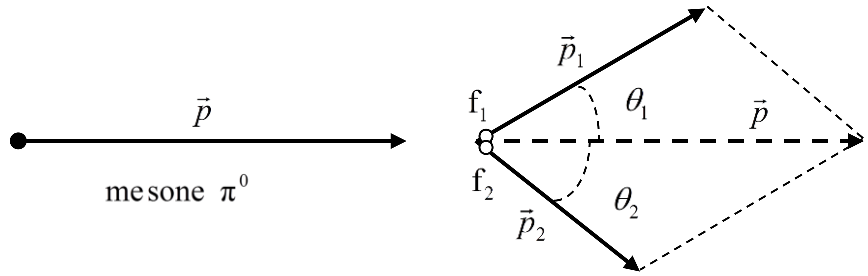
- Correct use of general laws of conservation **(A)** **5 points**
- Correct applying of the laws of conservation for the conditions stated in the problem **(B)** **4 points**
- Correct conduct of calculations and final solution (C) **1 point**

Detailed solution

(A) 5 points

In the disintegration process the laws of energy conservation and the law of the conservation of momentum are both obeyed.

In the general case the law of conservation of the momentum is represented in the down below figure.



the total initial energy of the π^0 meson is

$$E^2 = p^2c^2 + m_0^2c^4$$ 1p

And its kinetic energy is

$$E_c = E - m_0c^2$$

The expressions of the 2 conservation laws written after the disintegration are:

$$\vec{p} = \vec{p}_1 + \vec{p}_2;$$ 1p

$$E_c + m_0c^2 = E_1 + E_2,$$ 1p

The energy of the photon 1 can be calculated using the notations in the figure

$$\begin{Bmatrix} p_1 \sin\theta_1 = p_2 \sin\theta_2 \\ p = p_1 \cos\theta_1 + p_2 \cos\theta_2 \end{Bmatrix};$$

$$\frac{E_1}{c}\sin\theta_1 = \frac{E_2}{c}\sin\theta_2;$$ 1p

$$p^2c^2 = E^2 - m_0^2c^4;$$

$$E = E_1 + E_2;$$

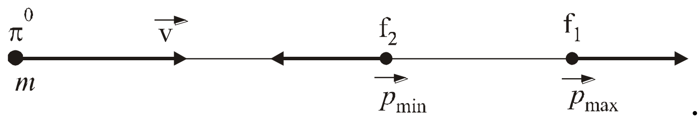
$$E_1 = \frac{m_0{}^2c^4}{2}\frac{1}{E_c + m_0c^2 - \cos\theta_1\sqrt{E_c\left(E_c + m_0c^2\right)}}.$$

Similar the second photon energy is:

$$E_2 = \frac{m_0{}^2c^4}{2}\frac{1}{E_c + m_0c^2 - \cos\theta_2\sqrt{E_c\left(E_c + m_0c^2\right)}}$$ **1p**

(B) 4 points

If one of the photon has the maximum possible energy E_{max} and consequently the other photon has the minimum possible energy E_{min} the law of momentum conservation is sketched:



. **2p**

Thus the relations become very simple:

$$mv = \frac{m_0v}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{E_{max} - E_{min}}{c};$$ 1p

$$mc^2 = \frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}} = E_{max} + E_{min}$$ 1p

$$v = c\frac{E_{max} - E_{min}}{E_{max} + E_{min}}.$$ **1p**

Problem 4. Sandra Bullock And George Cloony

An astronaut, with mass $M = 100\,\text{kg}$, gets out of the space ship for a repairing mission. He has to repair a satellite at rest relative to the space ship, at about $d = 90\,\text{m}$ away from it. After he finishes his job, he realizes that the systems designed to assure his come-back to shuttle are broken. He also observes that he has air only for 3 minutes. He also notices that he possessed a sealed cylindrical can (base section $S = 30\,\text{cm}^2$) firmly attached to his/her glove, with $m = 200\,\text{g}$ of ice inside. The can is not completely filled with ice.

Determine if the astronaut is able to return safely to the shuttle, before his air reserve is empty, if he manages to open the can in correct direction. Briefly explain your calculations. Note that he cannot throw away anything of its equipment, or touch the satellite.

You may use the following data: $T = 272\,K$/ the temperature of the ice in the can, $p_s = 550\,Pa$ - the pressure of the saturated water vapors at the temperature $T = 272\,K$; $R = 8300\,J/(kmol \cdot K)$ - the universal gas constant; $\mu = 18\,kg/kmol$ - the molar mass of the water.

Problem 4. Marking scheme The Astronaut saved by … ice from a can!

-	**A.** For the use with an adequate justify of one of the relationships (4)	3 points
-	**B. Reasoning** The student describe correctly the processes before and after the can is opened.	4points
-	**C.** Calculations according to the reasoning, and/or as support for reasoning	2 points
-	**D.** Correct result	1 point

Detailed solution

Theoretical considerations:

$$\Omega = \frac{\Delta N}{\Delta t} = \frac{\frac{N}{6}}{\Delta t} = \frac{\frac{N}{6}}{\frac{a}{\bar{v}}} = \frac{1}{6} \cdot \frac{N \bar{v}}{a},$$

$$\bar{v} = \sqrt{\frac{3RT}{\mu}},$$

$$N = n \cdot a^3,$$

$$\Omega = \frac{1}{6} \cdot \frac{n \cdot a^3 \cdot \bar{v}}{a} = \frac{1}{6} \cdot n \cdot a^2 \cdot \bar{v};$$

$$a^2 = S;$$

$$\Omega = \frac{1}{6} \cdot n \cdot S \cdot \bar{v};$$

$$\Phi = \frac{\Delta m}{\Delta t},$$

$$\Phi = \frac{\Delta m}{\Delta t} = \frac{m_0 \cdot \Delta N}{\Delta t},$$

$$\Phi = m_0 \cdot \Omega = \frac{1}{6} \cdot m_0 \cdot n \cdot S \cdot \bar{v};$$

$$m_0 \cdot n = m_0 \cdot \frac{N}{a^3} = \frac{m_0 N}{a^3} = \frac{m}{V} = \rho,$$

$$\rho = \frac{\mu p}{RT},$$

$$\Phi = \frac{1}{6} \cdot \rho \cdot S \cdot \bar{v} = \frac{1}{6} \cdot \frac{\mu p}{RT} \cdot S \cdot \sqrt{\frac{3RT}{\mu}} = \frac{1}{6} \cdot p \cdot S \cdot \sqrt{\frac{3\mu}{RT}}.$$ **3p**

B. **Reasoning**

Because the cylindrical can is not full of ice, in the empty part of it there are saturated vapors, i.e the mass flux of the molecule which sublimate is equal with mass flux of gass which transform into ice. Thus the pressure in the can is the saturated vapor pressure p_s and it has the corresponding maximum density ρ_s See figure 6.2

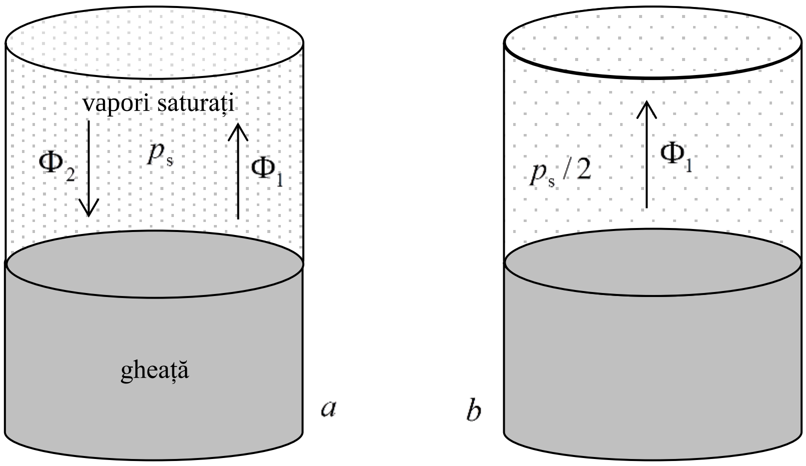


Fig. 6.2

$\Phi_1 = \Phi_2.$ 2p

$$\Phi_1 = \Phi_{\text{sub}limation} = \frac{1}{6}\cdot \rho_s \cdot S \cdot \bar{v} = \frac{1}{6}\cdot p_s \cdot S \cdot \sqrt{\frac{3\mu}{RT}};$$

$$\Phi_2 = \Phi_{solidification} = \frac{1}{6}\cdot \rho_s \cdot S \cdot \bar{v} = \frac{1}{6}\cdot p_s \cdot S \cdot \sqrt{\frac{3\mu}{RT}};$$

After the can was opened, there will be no molecules which desublimate thus the mass flux of the molecules which gather the ice become null. So the pressure becomes $(p_s/2)$. 2p

C. **Calculations according to the reasoning**, and/or as support for reasoning

$$F = \frac{p_s}{2}\cdot S,$$

$$a = \frac{F}{M} = \frac{p_s \cdot S}{2M} = \frac{550\ \text{Nm}^{-2}\cdot 30\cdot 10^{-4}\ \text{m}^2}{2\cdot 10^2\ \text{kg}} = 0{,}00825\,\text{ms}^{-2}.$$ 1p

The total time of the acceleration movement will be the total time of ice sublimation:

$$\tau = \frac{m}{\Phi_1} = \frac{m}{\frac{1}{6}\cdot p_s \cdot S \cdot \sqrt{\frac{3\mu}{RT}}} = \frac{6m}{p_s \cdot S}\cdot\sqrt{\frac{RT}{3\mu}} \approx 150\,\text{s}.$$ **1p**

D. Correct result **1p**

The travel distance in this time will be :

$$L = \frac{a\tau^2}{2} = \frac{0{,}00825\,\text{ms}^{-2}\cdot 225\cdot 10^2\ \text{s}^2}{2} \approx 93\,\text{m},$$

The astronaut could arrive safely in at the shuttle if he didn’t lose to much time by solving the problem.

Problem 5. The life –time of a main sequence star

The plot of the function $\log(L/L_S) = f(\log(M/M_S))$ for data collected from a number of stars is represented in figure 2. L and M are the luminosity and the mass of a star respectively and L_S and M_S the luminosity and the mass of the Sun respectively.

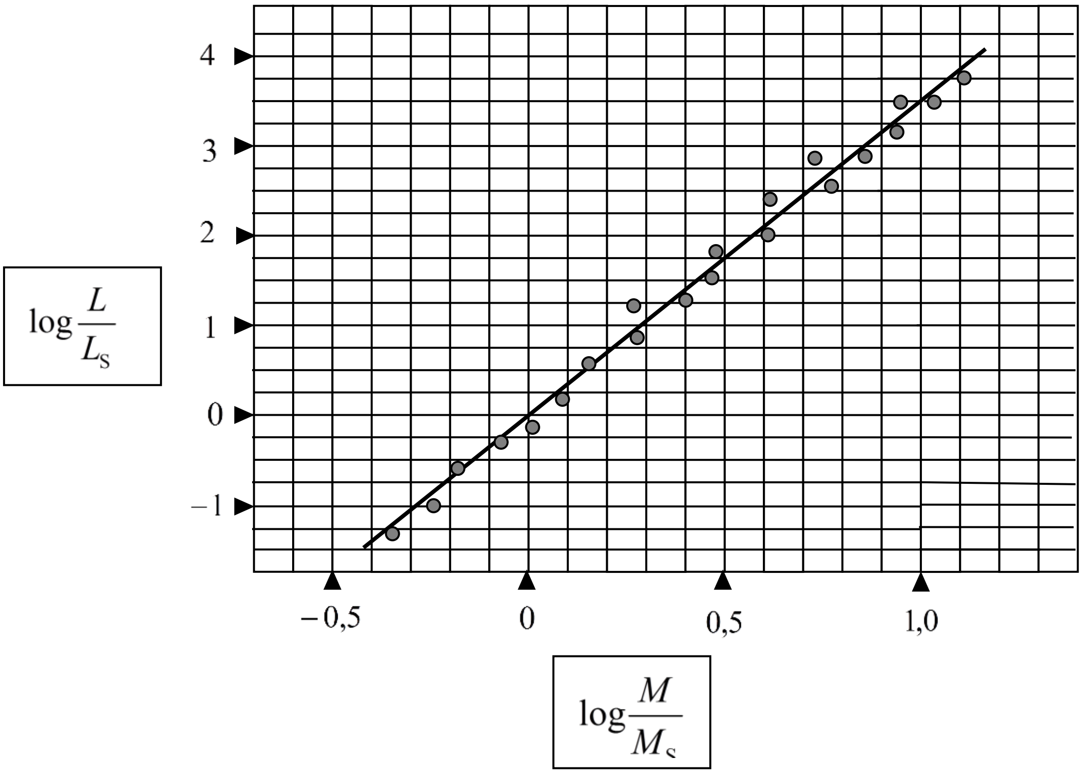


Figure 2

Find an expression for the main sequence life- time for a main sequence star from Hertzprung – Russell diagram, as a function of mass fraction converted to energy η and mass ratio to the solar mass n , Use the following assumptions: the time spent by Sun in the same Main Sequence is τ_S, for each star the mass fraction which changed into energy is η, the percent of the mass of Sun which changes into energy is η_S, the mass of each star is expressed as $n = \frac{M}{M_S}$ and assume that luminosity of the star remains constant, during its main sequence life time.

Problem 5. Marking scheme The life –time of a star from the main sequence

A. The analysis of the graph :

The graph is linear:

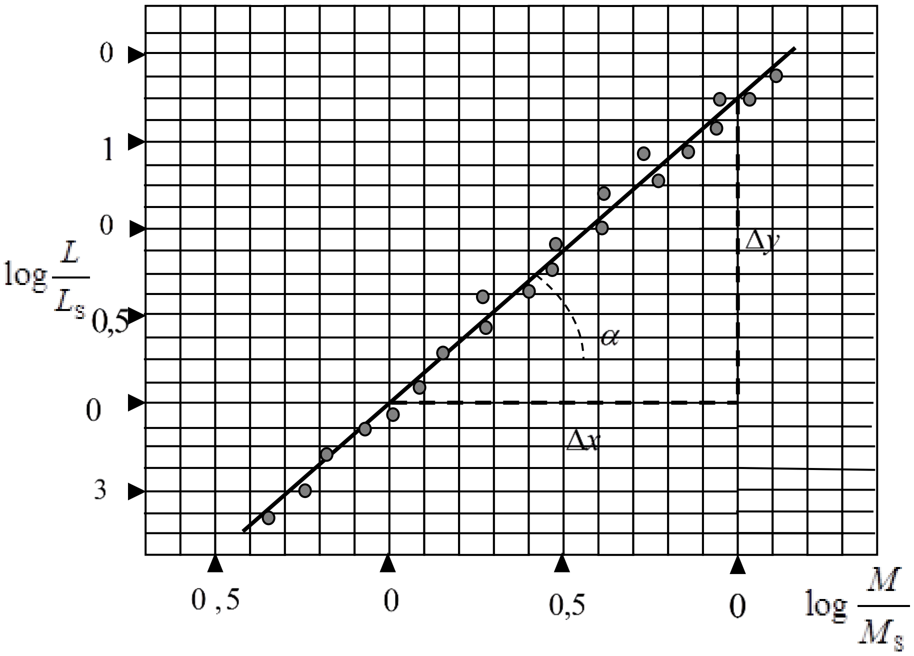
$y = ax + b = ax$ **1p**

From the graph it can be obtain the following data:

$\log \frac{L}{L_S} = a \cdot \log \frac{M}{M_S}$ **1p**

$a = \tan\alpha = \frac{\Delta y}{\Delta x} = \frac{3{,}5}{1} = 3{,}5;$ **1p**

$L \sim M^{3,5}$. **1p**



1p

The total energy of the star is:

$E = Mc^2,$

So the emitted energy due to the mass variation of the star is:

$\Delta E = c^2 \Delta M,$

According to the text

1p

$\Delta M = \eta M;$

$\Delta E = c^2 \eta M.$

By using the definition of the luminosity :

$\frac{\Delta E}{\Delta t} = L;$ **1 p**

1p

$\Delta t = \tau;$

$\frac{c^2 \eta M}{\tau} = L;$

$\tau = \frac{c^2 \eta M}{L},$ **(2)**

Which represents the life-time of the star.

1p

By using the results from the graph analysis

$L = \frac{L_S}{M_S^{3,5}} \cdot M^{3,5},$

Thus :

$$\tau = \frac{c^2 \eta M_S^{3,5}}{L_S} \cdot M^{-2,5}.$$

1p

If use the same calculations for the Sun it can be obtain

$E_S = M_S c^2;$

$$\tau_S = \frac{c^2 \eta_S M_S}{L_S},$$

Which is the life-time of the Sun

$$\tau = \frac{\eta}{\eta_S} \cdot \tau_S \cdot M_S^{2,5} \cdot M^{-2,5};$$

$$\frac{\tau}{\tau_S} = \frac{\eta}{\eta_S} \cdot \left(\frac{M}{M_S}\right)^{-2,5}; \; M = nM_S;$$

$$\tau = \frac{\eta}{\eta_S} (n)^{-2,5} \tau_S.$$

Problem 6. The effective temperature of a star

From the radiation emitted by a star, two radiations with wavelength values in a narrow range $\Delta\lambda << \lambda$ are studied, i.e. the wavelength have values between λ and $\lambda + \Delta\lambda$. According to Planck's relationship (for an absolute black body), the following relation defines, the energy emitted by star in unit time, through a unit area of its surface, per unit wavelength interval:

$$r = \frac{2\pi hc^2}{\lambda^5 \left(e^{\frac{hc}{k\lambda T}} - 1\right)}.$$

The spectral intensities of the radiation with wavelengths λ_1 and respectively λ_2, both within the range $\Delta\lambda$ measured on Earth are $I_1(\lambda_1)$ and $I_2(\lambda_2)$ respectively.

Find out the relation between wavelength λ_1 and λ_2, if $I_1(\lambda_1) = 2I_2(\lambda_2)$, when $hc << \lambda kT$.

Here: h – Planck's constant; k – Boltzmann's constant; c – speed of light in vacuum. $e^x \approx 1 + x \;\; if \;\; x << 1$

Problem 6. Marking scheme The effective temperature of a star

3points

a. We start from the definition of r:

$$r = \frac{\Delta E}{\Delta t \cdot S_{stea} \cdot \Delta\lambda} = \frac{\Delta E}{\Delta t \cdot 4\pi R^2 \cdot \Delta\lambda}$$ where R is the radius of the star

$$r = \frac{2\pi hc^2}{\lambda^5\left(e^{hc/k\lambda T} - 1\right)}$$

3 p

Considering **d** as the distance from the star to the Earth, the definition- relation of the spectral intensity can be written as follows:

$$I(\lambda) = \frac{\dfrac{\Delta E}{\Delta t \cdot \Delta\lambda}}{4\pi d^2}$$

$$I(\lambda) = \frac{2\pi hc^2 R^2}{d^2\lambda^5\left(e^{hc/\lambda kT} - 1\right)};$$

1p

Particularly for each wavelength:

$$I_1(\lambda_1) = \frac{2\pi hc^2 R^2}{d^2\lambda_1^5\left(e^{hc/\lambda_1 kT} - 1\right)}; \quad I_2(\lambda_2) = \frac{2\pi hc^2 R^2}{d^2\lambda_2^5\left(e^{hc/\lambda_2 kT} - 1\right)};$$

The ratio of the 2 above relations

$$\frac{I_1(\lambda_1)}{I_2(\lambda_2)} = \left(\frac{\lambda_2}{\lambda_1}\right)^5 \cdot \frac{e^{hc/\lambda_2 kT} - 1}{e^{hc/\lambda_1 kT} - 1} \qquad (1)$$

Represents an equation which allow to find out the temperature of star's surface **T by using spectral measurements**

1p

b. If $hc << \lambda kT$, then:

$$\frac{hc}{k\lambda_1 T} << 1; \; e^{hc/\lambda_1 kT} - 1 \approx 1 + \frac{hc}{k\lambda_1 T} - 1 = \frac{hc}{k\lambda_1 T}$$

$$\frac{hc}{k\lambda_2 T} << 1; \; e^{hc/\lambda_2 kT} - 1 \approx 1 + \frac{hc}{k\lambda_2 T} - 1 = \frac{hc}{k\lambda_2 T}$$

1p

The relation (1) becomes:

$$\frac{I_1(\lambda_1)}{I_2(\lambda_2)} = \left(\frac{\lambda_2}{\lambda_1}\right)^5 \cdot \frac{\dfrac{hc}{k\lambda_2 T}}{\dfrac{hc}{k\lambda_2 T}} = \left(\frac{\lambda_2}{\lambda_1}\right)^4$$

1p

$$I_1 = 2I_2;\ \left(\frac{\lambda_2}{\lambda_1}\right)^4 = 2;\ \lambda_2 = \lambda_1 \cdot \sqrt[4]{2} \approx 1{,}2 \cdot \lambda_1.$$

Problem 7. Pressure of light

For an observer on Earth the pressure of the radiation emitted by Sun is $p_{\text{rad,S}}$ and the pressure of the radiations emitted by a star Σ is $p_{\text{rad},\Sigma}$.

Calculate the visual apparent magnitude of the star Σ if the apparent visual magnitude of the Sun is m_S. The following assumption may be useful for solving the problem:

Generally, the pressure of the electromagnetic radiation in vacuum is equal to the volume energy density of the electromagnetic radiation $\left(p_{\text{rad}} = \frac{\Delta E}{\Delta V}\right)$.

The following data are known: M_S - the mass of the Sun, R_S - the radius of Sun, G - universal gravitational constant; σ Stefan - Boltzmann's constant ; c – speed of light in vacuum

Problem 7. Marking scheme . Pressure of light

2 p

Pogson law $\log \frac{\Phi_{\Sigma, L_{E\Sigma}}}{\Phi_{S, L_{ES}}} = -0.4(m_\Sigma - m_S)$

$$\Phi = \frac{W}{S \cdot t}$$

2p

$$\left.\begin{array}{l} Q_{planet} = \dfrac{E_e}{S_{star} \cdot t} = \sigma \cdot \mathrm{T}^4 \\ E_e = P_{emis\,star} = L_{star} \end{array}\right\} \Rightarrow L_{star} = \sigma \cdot \mathrm{T}^4\ 4\pi R^2{}_{star}$$

2 p

At distance L the energy flux of the star

$$\Phi_{star,L} = \frac{E_{star}}{S \cdot t} = 2\frac{\sigma T^4 R^2{}_{star}}{L^2}$$

2 p the pressure emitted by star

$$p=\frac{F}{S}=\frac{E_e}{\Delta V}=\frac{\sigma T^4 R_{star}^2}{c\,L^2}$$

2p

$$\log\frac{p_{star}}{p_{SUN}}=-0{,}4\left(m_\Sigma-m_S\right);$$

$$m_\Sigma=m_S-2{,}5\cdot\log\frac{p_{star}}{p_{sun}}.$$

Problem 8. Space – ship orbiting the Sun

A spherical space –ship orbits the Sun on a circular orbit, and spin around an axis of rotation that is perpendicular to the orbital plane of the space-ship. The temperature on the exterior surface of the ship is T_N. Assume the space -ship is a perfect black body and there is no activity inside it .

Find out the apparent magnitude of the Sun and the angular diameter of the Sun as seen by the astronaut on board of the space – ship. The following values are known:, T_S - the effective temperature of the Sun; R_S - the radius of the Sun; d_0 - the Earth –Sun distance; m_0 - apparent magnitude of Sun measured from Earth; R_N - the radius of the space –ship.

Problem 8. Marking scheme Space – ship orbiting the Sun

According to the Stefan – Boltzmann law, the luminosity of the Sun is:

$$L_{sun}=Q_{sun}\cdot 4\pi R_{Sun}^2=\sigma T_{Sun}^4\cdot 4\pi R_{Sun}^2,$$ **1p**

At distance d from the Sun , where the space ship is the energy which passes the unit of surface in an unit of time is:

$$\phi_{Sun,d}=\frac{L_S}{4\pi d^2}=\frac{\sigma T_S^4\cdot 4\pi R_S^2}{4\pi d^2}.$$ **1p**

The space ship receive through its entire surface, in the unit of time, the energy:

$$P_{received}=\frac{\sigma T_{Sun}^4\cdot 4\pi R_{Sun}^2}{4\pi d^2}\cdot \pi R_{ship}^2.$$

Corresponding to its temperature, T_N, according the Stefan - Boltzmann law, the emitted energy by starship through its hole surface in the unit of time :

$$P_{emis,N}=\sigma T_N^4\cdot 4\pi R_N^2.$$ **1p**

When the temperature stabilized at thermic equilibrium :

$$P_{received,N}=P_{emis,N}$$

$$\frac{\sigma T_S^4\cdot 4\pi R_S^2}{4\pi d^2}\cdot \pi R_N^2=\sigma T_N^4\cdot 4\pi R_N^2$$ **1p**

the distance of orbiting the Sun of the space ship is:

$$d=\frac{T_S^2 R_S}{2T_N^2},$$ **1p**

The angular diameter of the Sun as seen from the space ship :

$\alpha / 2 = \frac{R_S}{d}$ **1p**

$\alpha = \frac{2R_S}{d} = 4\left(\frac{T_N}{T_S}\right)^2$ **1p**

According the Pogson formula written for Sun seen from Earth and space ship the following relation occurs:

$\lg \frac{E_{S,Nava}}{E_{S,P}} = -0{,}4(m - m_0)$ **1p**

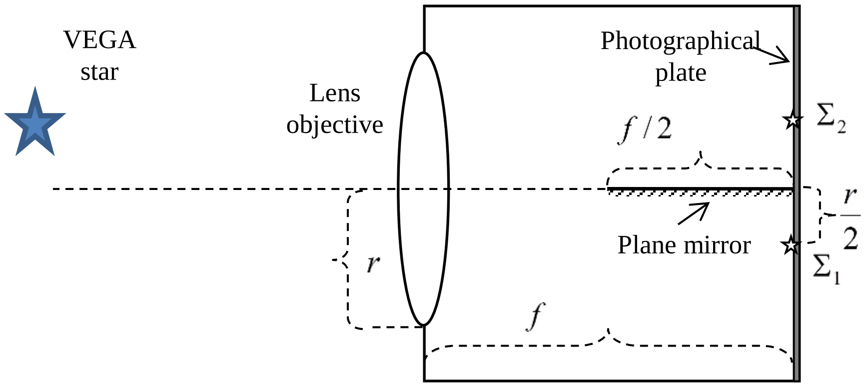
$2 \cdot \lg\left(\frac{d_0}{d}\right) = -0{,}4(m - m_0)$ **1p**

The apparent magnitude of the Sun as seen from the space ship

$m = m_0 - 5 \ \cdot \lg \frac{2d_0 T_N^2}{R_S T_S^2}$ **1p**

Problem 9. The Vega star in the mirror

Inside a camera a plane mirror is placed along the optical axis of the objective (as shown in figure). The length of the mirror is half the focal length of the objective. A photographic plate is placed at the focal plane of the camera. Two images with different brightness are captured on the photographic plate (as shown in figure). The star Vega is not on the optical axis of the lens. The distance between the optical axis and the image Σ_1 is $\frac{r}{2}$. Find the difference between the apparent photographical magnitudes of the two images of the star Vega.



Figure

Problem 9. Marking scheme The Vega star in the mirror

The light beam arriving from **Vega Star** can be considered paraxial, due to the distance from it to the observer on Earth. The explanation for the existence of two distinct images of the star is that the optical axis of the objective is not parallel with the light beam from the star.

The images on the camera plate are symmetrical placed relative to the principal optical axis.

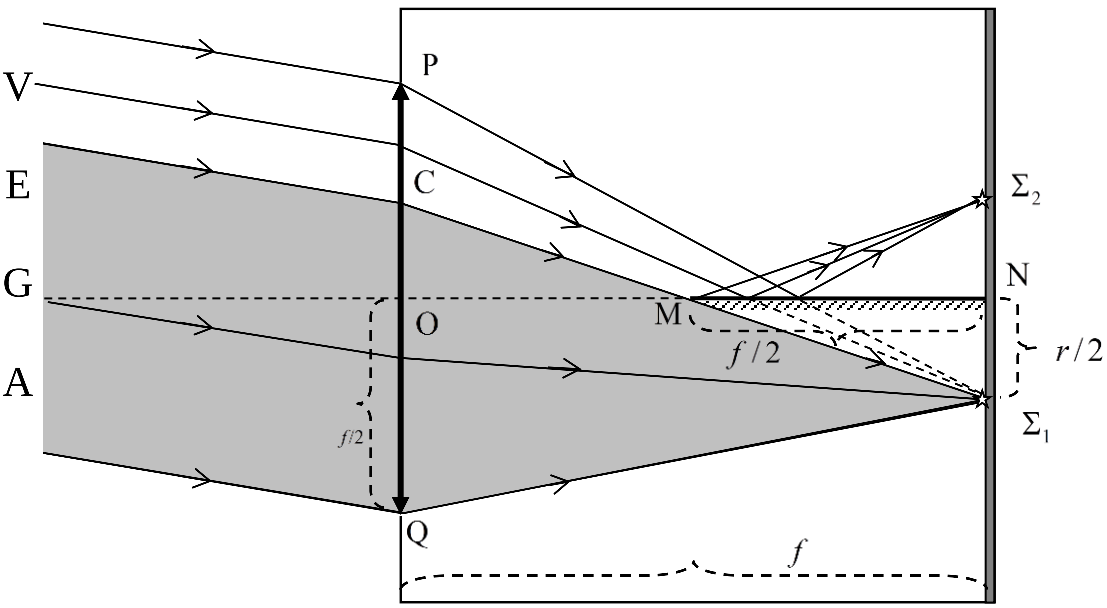


Fig. 12

2p

Each of the point images of the Vega Star Σ_1 and Σ_2 didn't concentrate the same light fluxes. In the down below figure it can be seen the sections of the lens which correspond to each image. The sector APBC is passed by the light which concentrates in the image Σ_2 and the light passing the sector ACBQ concentrates into the point image Σ_1 See the picture in figure 13 .

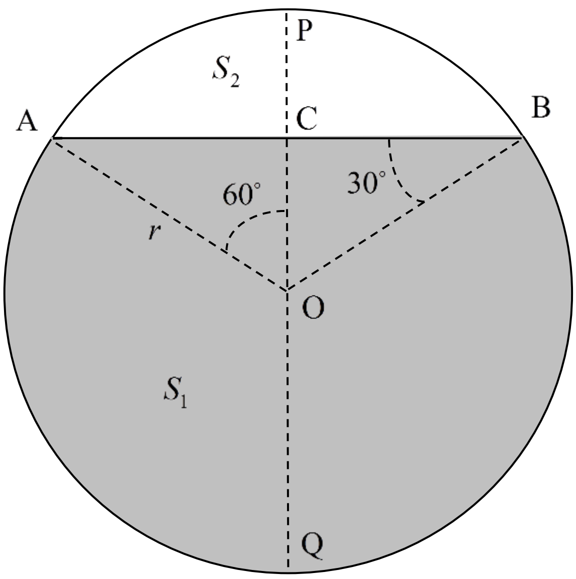


Fig.13 b **2p**

The ratio between the light fluxes concentrated into the two image points will directly depend on the ratio of the two sectors areas.

From the geometry of the figure 2 results :

$$MN = OM;\ N\Sigma_1 = OC = \frac{r}{2};$$

$$\angle(CBO) = 30^\circ;\ \angle(BOC) = 60^\circ;\ \angle(AOB) = 120^\circ;$$

$$\frac{S_1}{S_2} = \frac{8\pi + 3\sqrt{3}}{4\pi - 3\sqrt{3}} \approx 4.$$

2p

Using the Pogson formula :

$$\log\frac{E_1}{E_2} = \log\frac{\dfrac{\sigma T_V^4 \cdot 4\pi R_V^2}{4\pi d_{PV}^2}\cdot S_1}{\dfrac{\sigma T_V^4 \cdot 4\pi R_V^2}{4\pi d_{PV}^2}\cdot S_2} = -0{,}4(m_1 - m_2);$$

$$\log\frac{S_1}{S_2} = -0{,}4(m_1 - m_2);$$

$$m_2 - m_1 = 1{,}5^{m}.$$

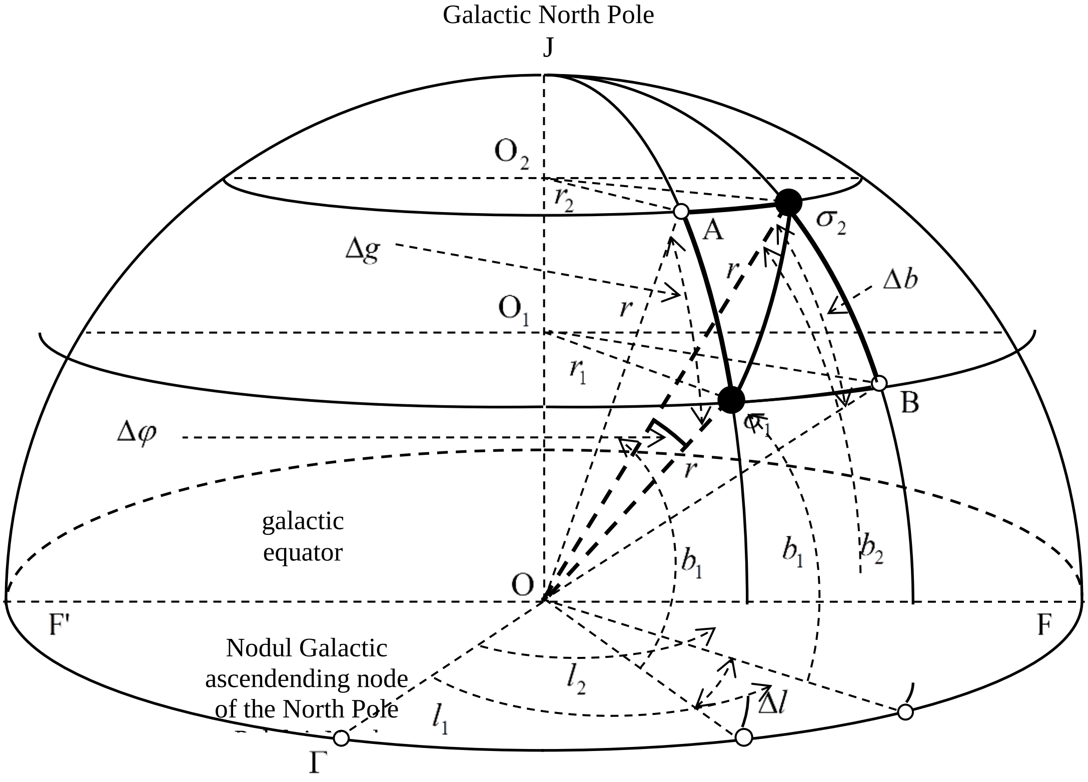
2p

Problem 10. Stars with Romanian names

Two Romanian astronomers Ovidiu Tercu and Alex Dumitriu from Galati Romania, recently discovered two variable stars. The galactic coordinates of the two stars are: Galati V1 $(l_1 = 114.371^\circ; b_1 = -11.35^\circ)$ and Galati V 2 $(l_2 = 113.266^\circ; b_2 = -16.177^\circ)$.

Estimate the angular distance between the stars Galati V1 and Galati V2

Problem 10. Marking scheme Stars with Romanian names



In the figure bellow the two stars σ_1 and σ_2, are located using the galactic coordintates $(l_1;b_1)$ and respectively $(l_2;b_2)$.on the geocentric celestial sphere. The spherical triangles $\sigma_1 \mathrm{A} \sigma_2$ $(l_1;b_1)$ and respectively may be considered rectangular plane triangles because the angles $\Delta l = l_2 - l_1$ $(l_1;b_1)$ and respectively $\Delta b = b_2 - b_1$ are very small

Thus:

$$\sigma_1\sigma_2 = \sqrt{(\sigma_1 \mathrm{A})^2 + (\sigma_2 \mathrm{A})^2},$$

or:

$$\sigma_1\sigma_2 = \sqrt{(\sigma_1 \mathrm{B})^2 + (\sigma_2 \mathrm{B})^2},$$

6p

$$\sigma_1 \mathrm{A} = r \cdot \Delta b;$$
$$\sigma_2 \mathrm{A} = r_2 \cdot \Delta l = r \cdot \cos g_2 \cdot \Delta l;$$
$$\sigma_1\sigma_2 = r \cdot \Delta\varphi,$$

Where $\Delta\varphi$ is the angular distance between two stars

$$r \cdot \Delta\varphi = \sqrt{(r \cdot \Delta b)^2 + (r \cdot \cos b_2 \cdot \Delta l)^2};$$
$$\Delta\varphi = \sqrt{(\Delta b)^2 + (\cos b_2 \cdot \Delta l)^2};$$
$$\sigma_1 \mathrm{B} = r_1 \cdot \Delta l = r \cdot \cos b_1 \cdot \Delta l;$$

$$\Delta\varphi = \sqrt{(\cos l_1 \cdot \Delta b)^2 + (\Delta l)^2};$$

2p

$$\left(l_1 = 114.371^\circ; b_1 = -11.35^\circ\right); \left(l_2 = 113.266^\circ; b_2 = -16.177^\circ\right);$$
$$\Delta l = l_2 - l_1 = -1{,}105^\circ;\ \Delta b = b_2 - b_1 = -4{,}827^\circ;$$
$$\cos l_1 = 0.98 \quad \cos l_2 = 0.96$$
$$\Delta\varphi = \sqrt{\left(-4{,}827^\circ\right)^2 + (0{,}96)^2 \cdot \left(-1{,}105^\circ\right)^2} \approx 4{,}942^\circ;$$
$$\Delta\varphi = \sqrt{(0{,}98)^2 \cdot \left(-1{,}105^\circ\right)^2 + \left(-4{,}827^\circ\right)^2} \approx 4{,}946^\circ,$$

The angular distance between Galați V1 and Galați V2. **2p**

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Problem 11. Apparent magnitude of the Moon

The apparent magnitude of the Moon as seen from the Sun is $M_M = 0.25^m$

Calculate the values of the apparent magnitudes of the Moon (as seen from the Earth) corresponding to the following Moon – phases : full-moon and the first quarter. Assume: the Moon – Earth distance - $d_{ME} = 385000\ \text{km}$, the Earth – Sun distance - $d_{ES} = 1\,\text{AU}$, the Moon –Sun distance, $d_{MS} = 1\,\text{AU}$. For terrestrial observers, following phase factor must be used to correct the lunar brightness for curvature of lunar surface and phase of the moon

$p(\Psi) = \frac{2}{3} \cdot \left[\left(1 - \frac{\Psi}{\pi}\right)\cos\Psi + \frac{1}{\pi}\sin\Psi\right]$, where Ψ is the phase angle.

Problem 11. Marking scheme Apparent magnitude of the Moon

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1. General analysis of the problem — 6 points
2. The analysis of the 2 particular situations — 4 points

The apparent magnitude of a planet from the Solar System depends on the phase angle $M = M(\Psi)$.
The apparent magnitude of the body is given by the relation:

$$m = M + 2{,}5 \cdot \log \frac{d_{C,S}^2 \cdot d_{C,O}^2}{d_0^4 \cdot p(\Psi)},$$

unde: $d_{B,S}$ – the distance between the body and the Sun; $d_{B,O}$ – distance between the body and observer; $d_0 = 1\,\text{AU}$; Ψ – the phase angle ; $p(\Psi)$ – the phase function :

$$p(\Psi) = \frac{2}{3} \cdot \left[\left(1 - \frac{\Psi}{\pi}\right)\cos\Psi + \frac{1}{\pi}\sin\Psi\right],$$

Ψ as seen in the figure bellow is given by the cosine law.

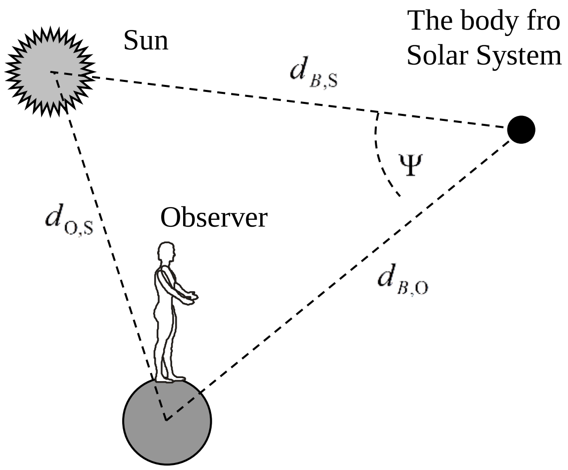
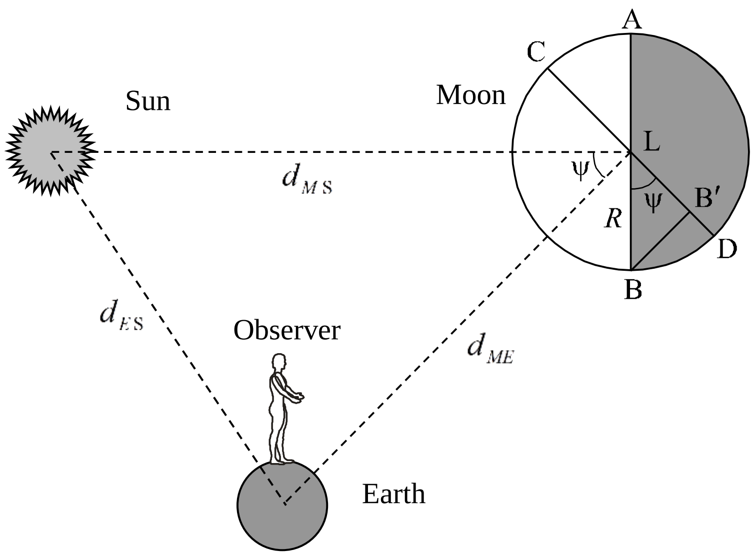


Fig.

$$\cos\Psi = \frac{d_{BO}^2 + d_{BS}^2 - d_{OS}^2}{2d_{BO}\cdot d_{BS}}.$$

In particularaly for the Moon



$$\cos\Psi = \frac{d_{ME}^2 + d_{MS}^2 - d_{ES}^2}{2d_{ME}\cdot d_{MS}};$$

$$p(\Psi) = \frac{2}{3}\cdot\left[\left(1-\frac{\Psi}{\pi}\right)\cos\Psi + \frac{1}{\pi}\sin\Psi\right];$$

$$m_M = M_M + 2{,}5 \cdot \log \frac{d_{M,S}^2 \cdot d_{ME}^2}{d_0^4 \cdot p(\Psi)}.$$

Particular cases:

1) Full moon

$$\Psi = 0;$$
$$\cos\Psi = 1;\ \sin\Psi = 0;$$
$$p(\Psi) = \frac{2}{3};$$
$$d_{MS} = 1\,\text{AU};\ d_{ME} = 385000\,\text{km} \approx 0{,}00256\,\text{AU} = 256 \cdot 10^{-5}\,\text{SU};\ d_0 = 1\,SU;$$

$$m_M = M_M - 12{,}5^{\text{m}} = 0{,}25^{\text{m}} - 12{,}5^{\text{m}} = -12{,}25^{\text{m}}.$$

2) First Quarter

$$\Psi = 90^\circ;$$
$$\cos\Psi = 0;\ \sin\Psi = 1;$$
$$p(\Psi) = \frac{2}{3\pi} \approx 0{,}2;$$
$$\frac{d_{M,S}^2 \cdot d_{ME}^2}{d_0^4 \cdot p(\Psi)} = \frac{65536 \cdot 10^{-10}}{0{,}2} = 491520 \cdot 10^{-10};$$

$$m_M = M_M - 10{,}75^{\text{m}} = 0{,}25^{\text{m}} - 10{,}75^{\text{m}} = -10{,}5^{\text{m}}.$$

Problem 12. Absolute magnitude of a cepheid

The cepheids are variable stars, whose luminosities vary due to stellar pulsations. The period of the oscillations of a cepheid star is:

$$P = 2\pi R\sqrt{\frac{R}{GM}},$$

where: R – the mean radius of the cepheid; M – the mass of the cepheid (remains constant during oscillation), you can assume that the temperature is constant during the pulsation;

Express the mean absolute magnitude of the cepheid M_{cep}, in the following form:

$$M_{cep} = -2{,}5^{\text{m}} \cdot \log k - \left(\frac{10}{3}\right)^{\text{m}} \cdot \log P,$$

where P is the period of cepheid's pulsation.

Problem 12. Marking scheme Apparent magnitude of the Moon

$$P = 2\pi R\sqrt{\frac{R}{GM}},$$

rezults

$$P^2 = \frac{4\pi^2 R^3}{KM};\ R = \sqrt[3]{\frac{KMP^2}{4\pi^2}} = \left(\frac{KM}{4\pi^2}\right)^{1/3} \cdot P^{2/3};$$

$$R^2 = \left(\frac{KM}{4\pi^2}\right)^{2/3} \cdot P^{4/3}.$$ **1p**

The absolute brightness is:

$$L_{\text{cef}} = \sigma T_{\text{cef}}^4 \cdot 4\pi R^2,$$ **1.5p**

And the apparent brightness :

$$E_{\text{cef}} = \frac{L_{\text{cef}}}{4\pi d_{\text{P,cef}}^2} = \frac{\sigma T_{\text{cef}}^4 \cdot 4\pi R^2}{4\pi d_{\text{P,cef}}^2},$$ **1.5 p**

$d_{\text{P,cef}}$ is the distance between the observer on Erath and the cepheide

$$E_{\text{cef}} = \frac{\sigma T_{\text{cef}}^4 \cdot 4\pi \cdot \left(\frac{KM}{4\pi^2}\right)^{2/3} \cdot P^{4/3}}{4\pi d_{\text{P,cef}}^2}$$ **1p**

Similarly for Sun

$$E_{\text{S}} = \frac{L_{\text{S}}}{4\pi d_{\text{PS}}^2} = \frac{\sigma T_{\text{S}}^4 \cdot 4\pi R_{\text{S}}^2}{4\pi d_{\text{PS}}^2}.$$ **1p**

By using the Pogson formula:

$$\log \frac{E_{\text{cef}}}{E_{\text{S}}} = -0{,}4(m_{\text{cef}} - m_{\text{S}});$$ **1p**

$$M_{\text{cef}} = M_{\text{S}} - 5^{\text{m}} \log \frac{|d_{\text{P,cef}}|}{|d_{\text{PS}}|} - 2{,}5 \cdot \log \frac{E_{\text{cef}}}{E_{\text{S}}}$$ **1p**

$$\frac{T_{\text{cef}}^4 \cdot \left(\frac{KM}{4\pi^2}\right)^{3/2} \cdot d_{\text{PS}}^2}{T_{\text{S}}^4 \cdot R_{\text{S}}^2 \cdot d_{\text{P,cef}}^2} = k_1 = \text{constant};$$ **1p**

$$M_{\text{cef}} = M_{\text{S}} - 5^{\text{m}} \log \frac{|d_{\text{P,cef}}|}{|d_{\text{PS}}|} - 2{,}5 \cdot \log k_1 - \frac{10}{3} \cdot \log P;$$

$$M_{\text{S}} - 5^{\text{m}} \log \frac{|d_{\text{P,cef}}|}{|d_{\text{PS}}|} - 2{,}5 \cdot \log k_1 = -2{,}5 \cdot \log k;$$

$$k = \text{constant};$$

$$M_{\text{cef}} = -2{,}5 \cdot \log k - \frac{10}{3} \cdot \log P$$ **1p**