Theoretical Exam - Short Questions

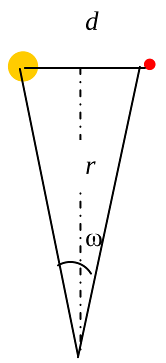
1. What would be the mean temperature on the Earth's surface if we ignore the greenhouse effect, assume that the Earth is a perfect black body and take into account its non-vanishing albedo? Assume that the Earth's orbit around the Sun is circular.

Answer : $T_{\oplus} = T_{\odot}(1-\alpha)^{1/4}\sqrt{\dfrac{R_{\odot}}{2r_{\oplus}}} = 246\ \mathrm{K}$ or $\mathbf{T_{\oplus} = -27^{o}\,C}$ **(10 Points)**

(Note: $r_{\oplus}$ = average Earth-Sun distance, the albedo, a = 0.39 is given in the *additional material*).

2. Let us assume that we observe a hot Jupiter planet orbiting around a star at an average distance d = 5 AU. It has been found that the distance of this system from us is r = 250 pc. What is the minimum diameter, D, that a telescope should have to be able to resolve the two objects (star and planet)? We assume that the observation is done in the optical part of the electromagnetic spectrum (λ~500nm), outside the Earth's atmosphere and that the telescope optics are perfect.

Answer: From the figure we have:



$$\omega(rad) \simeq \frac{d}{r} = \frac{5AU}{250\,pc} = \frac{5\times1.5\times10^{11}m}{250\times3.09\times10^{16}m} = 9.70\times10^{-8}rad$$ **(5 Points)**

Let us assume that D is the minimum diameter of our space telescope.

Its angular resolution is

$$\Delta\theta = 1.22\times\frac{\lambda}{D} \rightarrow 9.70\times10^{-8}rad = 1.22\times\frac{500\times10^{-9}m}{D} \rightarrow \boldsymbol{D = 6\ m}$$

(5 Points)

3. It is estimated that the Sun will have spent a total of about t_1 = 10 billion years on the main sequence before evolving away from it. Estimate the corresponding amount of time, t_2, if the Sun were 5 times more massive.

Answer: For the average luminosity of a main sequence star we have: $L \propto M^4$ (where M the initial mass of the star). We assume that the total energy E that the star produces is proportional to its mass $E \propto M$ **.** Therefore the amount of time that the star spends on the main sequence is approximately $t_{MS} \approx \dfrac{E}{L} \propto \dfrac{M}{M^4} \approx M^{-3}$.

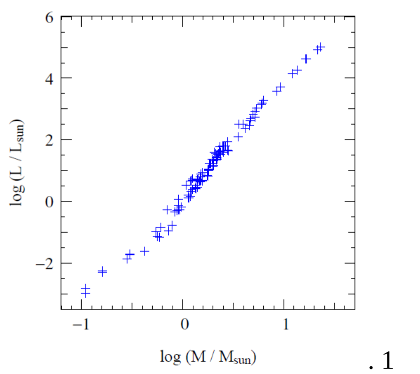


Figure 1: Mass to Luminosity relation

(5 Points)

Therefore, $\frac{t_1}{t_2} \approx \frac{1^{-3}}{5^{-3}} = 5^3 \rightarrow t_2 = \frac{1}{125} \times 10^{10}\,yr$ or $\boldsymbol{t_2 = 8\times10^7}$ **yr**. **(5 Points)**

4. Figure 2 shows the relation between absolute magnitude and period for classical cepheids. Figure 3 shows the light curve (apparent magnitude versus time in days) of a classical cepheid in a local group galaxy . (a) Using these two figures estimate the distance of the cepheid from us. (b) Revise your estimate assuming that the interstellar extinction towards the cepheid is A = 0.25 mag.

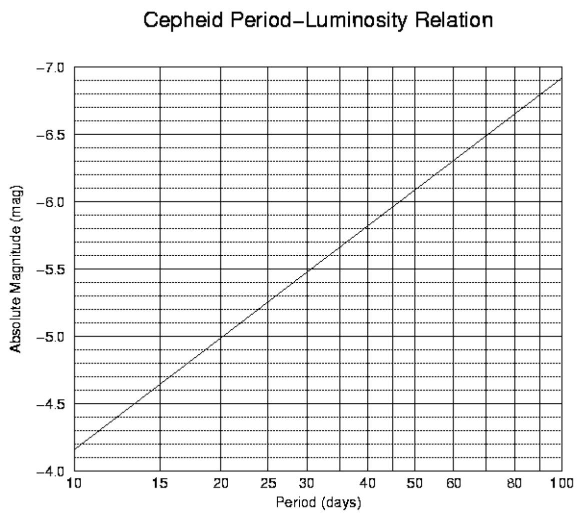


Figure 2

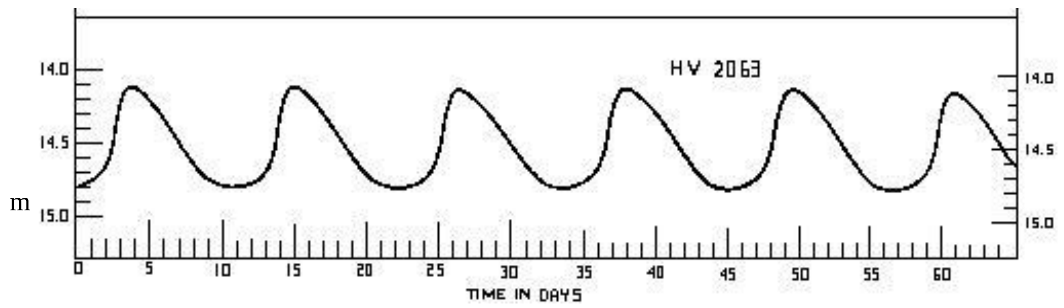


Figure 3

Answer : (a) From Figure 2, the period of the cepheid is P ~ 11 days and its average apparent magnitude is ~(14.8+14.1)/2 mag, i.e. m = 14.45 mag. **(2 points)**

[A careful student will notice that the graph is not *upside-down* symmetrical so he/she choose some value closer to the bottom; that is m=14.5 mag. **(1 point)]**

From Figure 1, we derive that for a period of 11 days the expected absolute magnitude of the cepheid is $M \approx -4.2$. **(1 point)**

[A careful student will notice that the graph is logarithmic so he/she choose a value closer to 4.3]
(1 point)

Using the formula $m - M = -5 + 5\log r$, where r is the distance of the cepheid, we get

$\log r = (14.45 + 4.2 + 5)/5 = 4.73$, thence $r = 10^{4.73} \approx 57500$ pc or $\boldsymbol{r = 57.5}$ **kpc** **(3 points)**

(b) Assuming $A = 0.25$, then $\log r = (14.45+4.2+5+0.25)/5 = 4.78$, thence $r = 10^{4.78}$ ~53000 pc or

$\boldsymbol{r = 64.5}$ **kpc.** **(2 points)**

5. The optical spectrum of a galaxy, whose distance had been measured to be 41.67 Mpc, showed the Balmer Hα line ($\lambda_o = 656.3$ nm) redshifted to $\lambda = 662.9$ nm. (a) Use this distance to calculate a value of the Hubble constant, H_o. (b) Using your results, estimate the Hubble time of the Universe.

Answer: (a) $z = \dfrac{\lambda - \lambda_o}{\lambda_o} = \dfrac{662.9 - 656.3}{656.3} \approx 0.01$. **(3 Point)**

This is small enough that we can use the classical equation for the expansion of the Universe.

$H_o = \dfrac{cz}{r} = \dfrac{3\times10^5\, km\, s^{-1}\times10^{-2}}{41.67\, Mpc}$ or $\boldsymbol{H_o} =$ $72.4\ \dfrac{km\, s^{-1}}{Mpc}$ **(4 Points)**

(b) $t_H \approx \dfrac{1}{H_o}$ or $\boldsymbol{t_H}$ **= 13.5 Gyr** **(3 Point)**

6. A star has an effective temperature $T_{eff} = 8700$ K, absolute magnitude $M = 1.6$ and apparent magnitude $m = 7.2$. Find (a) the star's distance, r, (b) its luminosity, L, and (c) its radius, R. (Ignore extinction).

Answer : (a) Its distance is calculated from equation: $m - M = 5 \log(r) - 5$,
or $7.2 - 1.6 + 5 = 5\log(r)$ → $\log(r) = 2.12$ and $\boldsymbol{r =}$ **132 pc** **(3 Points)**

(b) Its Luminosity is calculated from equation: $M_\odot - M = 2.5 \log\left(\dfrac{L}{L_\odot}\right)$ or

$4.8 - 1.6 = 2.5 \log\left(\dfrac{L}{L_\odot}\right)$ or $\log\left(\dfrac{L}{L_\odot}\right) = 1.28 \rightarrow \left(\dfrac{L}{L_\odot}\right) = 19.05$ or $L = 19.15\times L_\odot =$
$19.15\times3.9\times10^{33}$ and $\mathbf{L = 7.4\times10^{34}}$ **erg sec^{-1}** **(4 Points)**

(c) Its radius can be easily calculated from equation: $L = 4\pi\sigma R^2 T^4_{eff}$, from which

$R = \dfrac{1}{T^2_{eff}}\sqrt{\dfrac{L}{4\pi\sigma}}$ from which we get: $\mathbf{R = 1.35\times10^{11}}$ **cm** **(3 Points)**

7. A star has visual apparent magnitude m_v = 12.2 mag, parallax $\pi = 0''.001$ and effective temperature T_{eff} = 4000 K. Its bolometric correction is $B.C.$ = –0.6 mag. (a) Find its luminosity as a function of the solar luminosity. (b) What type of star is it? (i) a *red giant*? (ii) a *blue giant*? or (iii) a *red dwarf*? Please write (i), (ii) or (iii) in your answer sheet.

Answer : (a) First its bolometric magnitude is calculated from equation: $M_V - m_V = 5 - 5\log(r)$ or equivalent: $M_V - m_V = 5 + 5\log\pi \rightarrow M_V = 12.2 + 5 + 5\log(0''.001) = 12.2 + 5 - 15 = 2.2$ mag. Its barycentric correction is: $B.C. = M_{bol} - M_V$ and $M_{bol} = B.C. + M_V$ or $M_{bol} = -0.6 + 2.2$ or M_{bol} = 1.6 mag . **(4 Points)**

Then its Luminosity is calculated from:

$$M_{\odot} - M_{bol} = 2.5\log\left(\frac{L}{L_{\odot}}\right), \text{ or } 4.72 - 1.6 = 2.5\log\left(\frac{L}{L_{\odot}}\right) \text{ or } \log\left(\frac{L}{L_{\odot}}\right) = 1.25 \text{ and } \boldsymbol{L = 17.70\, L_{\odot}}$$

(2 Point)

(b) Type of star: A star with M_{bol} = 1.6 mag, $L = 17.7\, L_{\odot}$ and T_{eff} = 4000 K is much brighter and much cooler than the Sun (see *Table of constants*). Therefore it is **(i)** a red giant star. **(4 Points)**

8. A binary system of stars consists of star (*a*) and star (*b*) with brightness ratio 2. The binary system is difficult to resolve and is observed from the Earth as <u>one star</u> of 5th magnitude. Find the apparent magnitude of each of the two stars (m_a, m_b).

Answer : The apparent magnitude of star (*a*) is m_a, of star (b) is m_b and that of the system as a whole is m_{a+b}. The corresponding apparent brightnesses are ℓ_a, ℓ_b and $\ell_{a+b} = \ell_a + \ell_b$.

For star (*a*) :

$$m_{a+b} - m_a = -2.5\log\left(\frac{\ell_a + \ell_b}{\ell_a}\right) \text{ and because } \frac{\ell_b}{\ell_a} = \frac{1}{2}, \text{ we get } m_a = m_{a+b} + 2.5\log(1 + ½) \text{ or}$$

$m_a = 5 + 2.5\log(3/2)$ and finally $\boldsymbol{m_a = 5.44}$ **mag** . Similarly for star (*b*): **(5 point)**

$$m_{a+b} - m_b = -2.5\log\left(\frac{\ell_a + \ell_b}{\ell_b}\right) \text{ and because } \frac{\ell_a}{\ell_b} = 2, \text{ we get } m_b = m_{a+b} + 2.5\log(3) \text{ or}$$

$m_b = 5 + 2.5\log(3)$ and finally $\boldsymbol{m_b = 6.19}$ **mag**. **(5 Point)**

9. Find the equatorial coordinates (*hour angle* and *declination*) of a star at Madrid, geographic latitude φ = 40°, when the star has zenith angle z = 30° and azimuth A = 50° (azimuth as measured from the South)

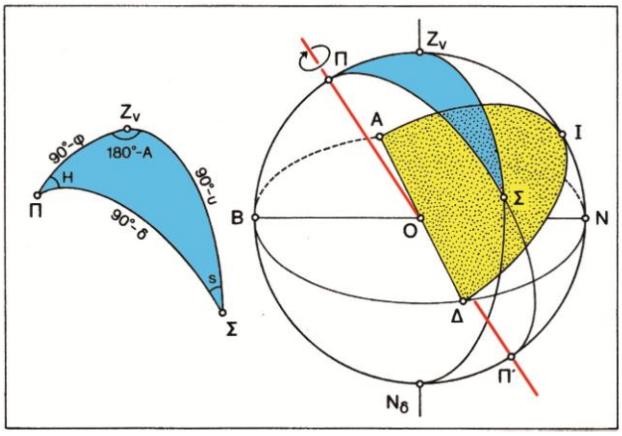


Figure 3. The position triangle

Answer : From the position triangle $\Pi Z_v \Sigma$ (Figure 3) of the star, Σ, we get, by using the *cosine law* for a spherical triangle:

cos (90 – δ) = cos(90 – *a*)×cos(90 – φ) + sin (90 – *a*)×sin(90 – φ)×cos (180 – A) **(2 Point)** where δ is the star's *declination*, *a* its *altitude* ($a = 90° - z$), φ the *geographical latitude* of the observer, H the stars *hour angle* and A the star's *azimuth*.

This can be written as:

sinδ = cos z×sin φ – sin z×cosφ×cos A or
$\sin\delta = \cos 30^o \times \sin 40^o - \sin 30^o \times \cos 40^o \times \cos 50^o$ **(2 Point)**
or
sin δ =0.866×0.643 – 0.500×0.766×0.643 = 0.311. **δ = 18° 05′** **(1 Point)**

Using the sine law for the spherical triangle, we get:

$$\frac{\sin H}{\sin(90-a)} = \frac{\sin(180^o - A)}{\sin(90^o - \delta)} \quad \text{or} \quad \frac{\sin H}{\sin z} = \frac{\sin A}{\cos\delta}$$ **(2 Point)**

$$\rightarrow \quad \sin H = \sin 50^o \times \frac{\sin 30^o}{\cos\left(18^o\, 07'\right)} = \frac{0.766 \times 0.5}{0.950}$$ **(2 Point)**

or sin H = 0.403. Therefore: H = 23° 46′ or **H = $1^h\ 35^m\ 03^s$**. **(1 Point)**

10. In the centre of our Galaxy, in the intense radio source Sgr A*, there is a black hole with large mass. A team of astronomers measured the angular distance of a star from Sgr A* and its orbital period around it. The maximum angular distance was 0.12″ (arcsec) and the period was 15 years. Calculate the mass of the black hole in solar masses, assuming a circular orbit.

Answer: $F = -\frac{GM_{BH}M_*}{R^2} = -\frac{M_* v^2}{R}$ **(2 Point)**

But $v = \frac{2\pi R}{P}$. Therefore $\frac{GM_{BH}}{R^2} = 4\pi^2 \frac{R}{P^2}$ or $GM_{BH} = 4\pi^2 \frac{R^3}{P^2}$ **(2 Points)**

Similarly:

$$GM_\odot = 4\pi^2 \frac{(1AU)^3}{(1yr)^2} \quad ή \quad G = \frac{1}{M_\odot} 4\pi^2 \frac{(1AU)^3}{(1yr)^2}$$ **(2 Point)**

From Kepler's 3rd law we get: $\frac{M_{BH}}{M_\odot} = \frac{(R/1AU)^3}{(P/1yr)^2}$ **(1 Point)**

Inserting the given data we find the distance of the star from the black hole:

$$R = \frac{0.12}{200{,}000}(8000)(3\times10^{18}cm) = 1.4\times10^{16}cm = 960AU$$ **(2 Point)**

Therefore: $\frac{M_{BH}}{M_\odot} = \frac{(960)^3}{(15)^2} = 4\times10^6$ form which we calculate the mass of the black hole:

$$M_{BH} = 4\times10^6 M_\odot$$ **(1 Point)**

11. What is the maximum altitude, a_M (max), at which the Full Moon can be observed from Thessaloniki? The geographical latitude of Thessaloniki is $\varphi_\Theta = 40°37'$. Take into account as many factors as possible.

Answer: In order to have Full Moon, the Moon should be diametrically opposite the Sun, i.e. the three bodies, Sun – Earth – Moon should be on a straight line. If the orbital plane of the Moon coincided with the ecliptic, the maximum altitude of the Full Moon would be $90° - \varphi_\Theta + 23.5°$. **(2 point)**.

Because the orbital plane of the Moon is inclined by 5.14° (5°18′) to the plane of the ecliptic, the maximum angle is larger: 90° – 40.6° + 23.5° + 5.3°, or (P)

$\boldsymbol{\alpha_M(\max) = 79.8°}$ **(3 points)**

Geocentric parallax of Moon for this situation is 0.33°, **(3 points)**

whereas refraction is only 0.2′. **(1 point)**

Final answer is therefore: 79.8° - 0.3° = **79.5°** **(1 point)**

12. *Sirius A*, with visual magnitude $m_V = -1.47$ (the brighter star on the sky) and with stellar radius $R_A = 1.7R_\odot$, is the primary star of a binary system. The existence of its companion, *Sirius B*, was deduced from astrometry in 1844 by the well known mathematician and astronomer Friedrich Bessel, before it was directly observed. Assuming that both stars were of the same spectral type and that *Sirius B* is fainter by 10 mags ($\Delta m = 10$), calculate the radius of *Sirius B*.

Answer: The distance of the two stars from our solar system is the same. Therefore

$$m_B - m_A = 2.5\ \log \frac{\frac{L_A}{4\pi r^2}}{\frac{L_B}{4\pi r^2}} = 2.5 \log \frac{L_a}{L_B}$$ **(5 Point)**

From which we get $L_A = 10^4\ L_B$. From equation $L = 4\pi R^2 \sigma T_{eff}^4$ βwe get

$$L_A/L_B = (R_A/R_B)^2\ (T_A/T_B)^4$$

Assuming that the two stars belong to the same spectral type (and therefore $T_A = T_B$) we get

$R_B = 0.01R_A = 0.01\times1.7\times696000$ km or $\boldsymbol{R_B = 1.2\ 10^4}$ **km** **(5 Point)**

13. Recently in London, because of a very thick layer of fog, the visual magnitude of the Sun, became equal to the (usual – as observed during cloudless nights) magnitude of the full Moon. Assuming that the reduction of the intensity of light due to the fog is given by an exponential equation, calculate the exponential coefficient, τ, which is usually called *optical depth*.

Answer: The absorption due to the fog in London is obviously

$A = -26.8 - (-12.74) = -14.06$ mag. **(2 Point)**

Rearranging the equation $I_\nu(r) = I_\nu(0) \times e^{-\tau}$, we get $\frac{I_\nu(0)}{I_\nu(r)} = e^{\tau}$, **(5 Points)**

or

$A = \Delta m = 2.5 \times \log(e) \times (-\tau)$ or $\tau = (-A)/(2.5 \times \log e) = 14.06/1.08$ ή $\boldsymbol{\tau = 12.9}$ **(3 Points)**

14. What is the hour angle, H, and the zenith angle, z, of *Vega* ($d = 38° 47'$) in Thessaloniki ($\lambda_1 = 1^h32^m$, $\varphi_1 = 40°37'$), at the moment it culminates at the local meridian of Lisbon ($\lambda_2 = -0^h36^m$, $\varphi_2 = +39°43'$)?

Answer: By definition at the moment when the star culminates in Lisbon, its hour angle is exactly 0°. Therefore its hour angle in Thessaloniki is $0^o + (\lambda_1 - \lambda_2)$ or $\boldsymbol{H = 02^h08^m}$. **(3 Points)**

Using the cosine law equation $\cos z = \cos(90-\varphi)\times\cos(90-d) + \sin(90-\varphi)\times\sin(90-d)\times\cos H$, **(4 Points)**

the zenith distance at Thessaloniki can be calculated to **24° 33′** **(3 Points)**

15. The Doppler shift of three remote galaxies has been measured with the help of Spectral observations:

Galaxy	Redshift, z
3C 279	0.536
3C 245	1.029
4C41.17	3.8

(a) Calculate their apparent recession velocity (1) using the classical approach, (2) using the approximate formula $v = c \ln(1+z)$, that is often used by cosmologists and (3) using the special relativistic approach.
(b) For all three formulae, at what percentage of the speed of light do they appear to recede?
(c) Which of (1) classical, (2) special relativity (3) approximate cosmological.

Answer: The recession velocity is calculated by either the classical relation, $v_c = z\times c$, or the relativistic relation, $v_r = \frac{(1+z)^2-1}{(1+z)^2+1}c$. The calculations for the three galaxies are summarized in the following Table:

Galaxy	v_c (km/s)	V_a (km/s)	v_r (km/s)	$v_r/c\times 100$
3C279	160800	128750	121390	40%
3C245	308700	212260	182740	61%
4C41.17	1140000	470580	275040	92%

(1) Table 1, Columns 2,3,4 **(5 Points)**

(2) Column 5 **(3 Points)**

(3) If the student answers "Classical" (0 Points)
If the student answers "Special Relativity" (1 Point)
If the student answers "Approximate formula" **(2 Points)**