Short Problem

Note: 10 points for each problem

1) In a binary system, the apparent magnitude of the primary star is 1.0 and that of the secondary star is 2.0. Find the maximum combined magnitude of this system.

Solution:

Let F_1, F_2, and F_0 be the flux of the first, the second and the binary system, respectively.

$$\Delta m = -2.5\lg(F_1/F_2)$$
$$(1-2) = -2.5\lg(F_1/F_2) \qquad 5$$

So, $F_1/F_2 = 10^{1/2.5} = 10^{0.4}$

$$F_0 = F_1 + F_2 = F_1(1+10^{-0.4}) \qquad 3$$

The magnitude of the binary m is:

$$m - 1 = -2.5\lg(F_0/F_1) = -2.5\lg(F_1(1+0.398)/F_1) = -0.36^m \qquad 2$$

So, $m = 0.64^m$

2) If the escape velocity from a solar mass object's surface exceeds the speed of light, what would be its radius ?

Solution:

$$\sqrt{\frac{2GM_{object}}{R_{object}}} > c$$ 4

$$R_{object} < \frac{2GM_{object}}{c^2}$$ 2

$$R_{object} < \frac{2\times 6.6726\times 10^{-11}\times 1.9891\times 10^{30}}{(2.9979\times 10^{8})^{2}}$$

$$R < 2953.6m$$ 4

3) The observed redshift of a QSO is $z = 0.20$, estimate its distance. The Hubble constant is 72 km s^{-1} Mpc^{-1}.

Solution:

Recession velocity of the QSO is

$$\frac{v}{c} = \frac{(z+1)^2 - 1}{(z+1)^2 + 1} = 0.18$$ 4

According to the Hubble's law,

$$v = H_0 D$$ 2

The distance of the QSO is

$$D = v / H_0 = 0.18c / 72 = 750Mpc,$$ 4

Remarks： if the student calculate the distance using cosmological formula and arrive at the answer $D = 735\,Mpc$,assuming $\Omega_0 = 1.0$ will get the full mark.

4) A binary system is 10 pc away, the largest angular separation between the components is 7.0", the smallest is 1.0". Assume that the orbital period is 100 years, and that the orbital plane is perpendicular to the line of sight. If the semi-major axis of the orbit of one component corresponds to 3.0", that is a_1=3.0", estimate the mass of each component of the binary system, in terms of solar mass.

Solution:

The semi-major axis is

$a = 1/2 \times (7+1) \times 10 = 40\,AU$ 2

From Kepler's 3rd law,

$$M_1 + M_2 = \frac{a^3}{p^2} = \frac{(40)^3}{(100)^2} = 6.4 M_{sun}$$ 4

since $a_1 = 3'', a_2 = 1''$,then

$$\frac{m_1}{m_2} = \frac{a_2}{a_1}$$ 2

$m_1 = 1.6 M_{sun}, m_2 = 4.8 M_{sun}$ 2

5) If 0.8% of the initial total solar mass could be transformed into energy during the whole life of the

Sun, estimate the maximum possible life time for the Sun. Assume that the solar luminosity remains constant.

Solution:

The total mass of the Sun is

$$m \approx 1.99 \times 10^{30} kg$$

0.8% mass transform into energy:

$$E = mc^2 \approx 0.008 \times 2 \times 10^{30} \times (3 \times 10^8)^2 = 1.4 \times 10^{45} J$$ 5

Luminosity of the Sun is

$$L_{sun} = 3.96 \times 10^{26} W$$

Sun's life would at most be:

$$t = E / L_{sun} = 3.6 \times 10^{18} s \approx 10^{11} years$$ 5

6) A spacecraft landed on the surface of a spherical asteroid with negligible rotation, whose diameter is 2.2 km, and its average density is 2.2g/cm^3. Can the astronaut complete a circle along the equator of the asteroid on foot within 2.2 hours? Write your answer "YES" or "NO" on the answer sheet and explain why with formulae and numbers.

Solution:

The mass of the asteroid is

$$m_1 = \frac{4}{3}\pi r^3 \rho = 1.23 \times 10^{13} kg$$ 2

Since $m_2 << m_1$, m_2 can be omitted,

Then $v = \sqrt{\frac{Gm_1}{r}} = 0.864 m/s$ 3

It is the first cosmological velocity of the asteroid.
If the velocity of the astronaut is greater then v, he will escape from the asteroid.

The astronaut must be at v_2 if he wants to complete a circle along the equator of the asteroid on foot within 2.2 hours, and

$$v_2 = \frac{2\pi \times (2200/2)m}{2.2 \times 3600s} = 0.873 m/s$$ 3

Obviously $v_2 > v$

So the answer should be "NO". 2

7) We are interested in finding habitable exoplanets. One way to achieve this is through the dimming of the star, when the exoplanet transits across the stellar disk and blocks a fraction of the light. Estimate the maximum luminosity ratio change for an Earth-like planet orbiting a star similar to the Sun.

Solution：

The flux change is proportional to the ratio of their surface areas, i.e.,

$$F_e / F_{sun} = (R_e / R_{sun})^2$$ 5

$$(R_e / R_{sun})^2 = 8.4 \times 10^{-5} \approx 10^{-4} \quad 5$$

Obviously this difference is extremely small.

8) The Galactic Center is believed to contain a super-massive black hole with a mass $M=4 \times 10^6 M_{\odot}$. The astronomy community is trying to resolve its event horizon, which is a challenging task. For a non-rotating black hole, this is the Schwarzschild radius, $R_s = 3(M/M_{\odot})$ km. Assume that we have an Earth-sized telescope (using Very Long Baseline Interferometry). What wavelengths should we adopt in order to resolve the event horizon of the black hole? The Sun is located at 8.5 kpc from the Galactic Center.

Solution:

Observationally, the diameter of the Galactic black hole at the distance of $L = 8.5 kpc$ has the angular size,

$$\theta_{BH} = 2R_s / L \quad 2$$

On the other hand, an Earth-sized telescope ($D = 2R_e$) has the resolution,

$$\theta_{tel} = 1.22\lambda / (2R_e) \quad 2$$

In order to resolve the black hole at Galactic center, we need to have $\theta_{BH} \geq \theta_{tel}$,which marginally we consider $\theta_{BH} = \theta_{tel}$

This leads to,

$$\lambda = 4R_e R_s/(1.22L)$$ 4

Taking the values, we have

$$\lambda \approx 0.9mm$$ 2

This means that we need to observe at least at near sub-mm frequencies, which is in radio or far-infrared band.

9) A star has a measured I-band magnitude of 22.0. How many photons per second are detected from this star by the Gemini Telescope(8m diameter)? Assume that the overall quantum efficiency is 40% and the filter passband is flat.

Filter	$\lambda_0(nm)$	$\Delta\lambda(nm)$	$F_{VEGA}(Wm^{-2}nm^{-1})$
I	8.00×10^2	24.0	8.30×10^{-12}

Solution:

The definition of the magnitude is:

$$m_I = -2.5\lg F_I + const$$

Where F_I is the flux received from the source. Using the data above, we can obtain the constant:

$$0.0 = -2.5\lg(0.83\times10^{11}) + const$$ Thus,

$$const = -27.7$$

$$m_I = -2.5\lg F_I - 27.7$$

$$F_I = 10^{\frac{m_I + 27.7}{-2.5}} = 1.3\times10^{-20} Wm^{-2}nm^{-1}$$ 4

For our star, at an effective wavelength $\lambda_0 = 800nm$

using this flux, the number of photons detected per unit wavelength per unit area is the flux divided by the energy of a photon with the effective wavelength:

$$N_I = \frac{1.3\times10^{-20}}{hc/\lambda_0} = 5.3\times10^{-2}\ photonss^{-1}m^{-2}nm^{-1}$$ 3

Thus the total number of photons detected from the star per second by the 8m Gemini telescope over the I band is

$$N_I(total) = (tel.collectingarea)\times QE\times Bandwidth\times N_I$$

$$= (\pi\times4^2)\times0.4\times24\times N_I$$ 3

$$= 26\,photons/s \approx 30\,photons/s$$

10) Assuming that the G-type main-sequence stars (such as the Sun) in the disc of the Milky Way obey a vertical exponential density profile with a scale height of 300pc, by what factor does the density of these stars change at 0.5 and 1.5kpc from the mid-plane relative to the density in the mid-plane?

Solution:

Since $h_z = 300pc$, we can substitute this into the vertical(exponential)disc equation:

$$n(0.5kpc) = n_0 \exp(-|500\ pc|/300\ pc) \approx 0.189\,n_0$$ 5

In other words, the density of G-type MS stars at 0.5kpc above the plane is just under 19% of its mid-plane value.

For $z = 1.5kpc$,this works out as 0.007 . 5

11) Mars arrived at its great opposition at UT 17^h56^m Aug.28, 2003. The next great opposition of Mars will be in 2018, estimate the date of that opposition. The semi-major axis of the orbit of Mars is 1.524 AU.

Solution:

$$T_M = \sqrt{\frac{R_M^3}{R_E^3}} T_E = 1.881 years \qquad 2$$

$$\frac{1}{T_s} = \frac{1}{T_E} - \frac{1}{T_M}$$

$$T_s = \frac{T_E \times T_M}{(T_M - T_E)} = \frac{1.881}{0.881} \times 365.25 = 779.8 days \qquad 3$$

That means there is an opposite of the Mars about every 780 days.

If the next great opposite will be in 2018, then

$15 \times 365 + 4 = 5479\ days$

$5479 / 779.8 = 7.026$

It means that there will have been 7 opposites before Aug.28, 2018, 3

So the date for the great opposite should be

$5479 - 7 \times 779.8 = 20.4 days$, i.e.

20.4days before Aug.28,2018, 2

It is on Aug .7, 2018.

12) The difference in brightness between two main sequence stars in an open cluster is 2 magnitudes. Their effective temperatures are 6000K and 5000K respectively. Estimate the ratio of their radii.

Solution:

$$L_1 = 4\pi R_1^2 \sigma T_{\max}^4$$ 3

$$L_2 = 4\pi R_2^2 \sigma T_{\min}^4$$

$$\Delta m = -2.5\lg(L_{\min} / L_{\max}) = -5\lg(R_{\min} / R_{\max}) - 10\lg(T_{\min} / T_{\max})$$ 3

$$\lg(R_{\min} / R_{\max}) = -0.2\Delta m - 2\lg(T_{\min} / T_{\max}) = -0.24$$ 2

So,

$$R_{\min} / R_{\max} = 0.57$$ 2

13) Estimate the effective temperature of the photosphere of the Sun using the naked eye colour of the Sun.

Solution:

The Wien law is

$$\lambda_{\max} = \frac{0.29}{T}(cm)$$ 5

So the temperature is

$$T = \frac{0.29}{550 \times 10^{-9}} = 5272 \approx 5300K$$ 5

Or

$$T = \frac{0.29}{500 \times 10^{-9}} = 5800K$$

Note:5200~6000K all full mark

14) An observer observed a transit of Venus near the North Pole of the Earth. The transit path of Venus is shown in the picture below. A, B, C, D are all on the path of transit and marking the center of the Venus disk. At A and B, the center of Venus is superposed on the limb of the Sun disk; C corresponds to the first contact while D to the fourth contact, $\angle AOB = 90^\circ$, MN is parallel to AB. The first contact occured at 9:00 UT. Calculate the time of the fourth contact.

$$T_{venus} = 224.70 days, T_{earth} = 365.25 days, a_{venus} = 0.723AU, r_{venus} = 0.949 r_{\oplus}$$

Solution:

Since the observer is at the pole, the affect of the earth's rotation on the transit could be neglected.

then the Sun's angle at the earth extends as $\theta_0 = \arcsin(\frac{2r_{sun}}{1AU}) \approx 32.0'$;

the angular velocity of the Venus around the Sun, respected to the earth is ω_1,

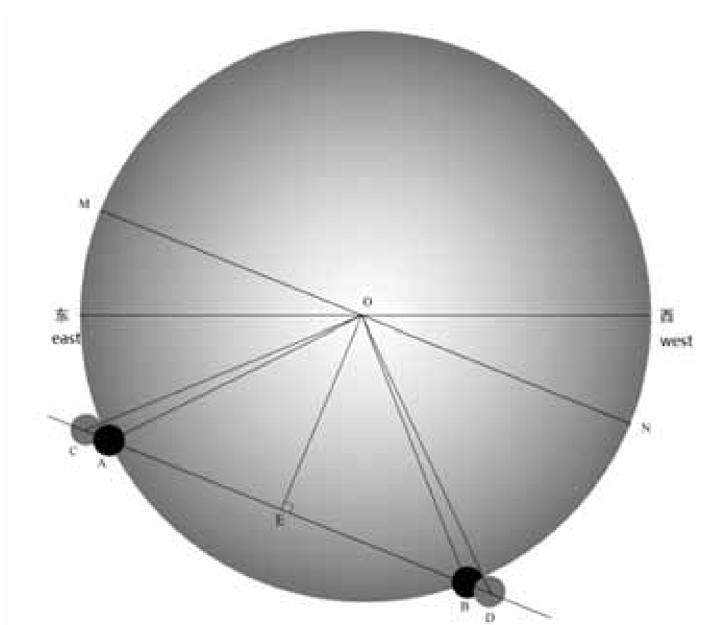
$$\omega_1 = \omega_{venus} - \omega_{earth} = \frac{2\pi}{T_{venus}} - \frac{2\pi}{T_{earth}} \approx 4.29 \times 10^{-4} ('/s) \qquad 2$$

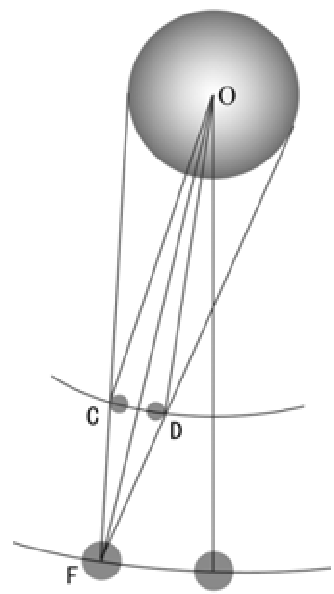
For the observer on earth, Venus moved θ during the whole transit ，

Let OE be perpendicular to AB,

OA=16 ' ∠AOB=90°, MN // AB，

So $OE = 11.3'$， $OC = \frac{d'_{venus}}{2} + \frac{r'_{sun}}{2}$， d'_{venus} is the angular size of Venus seen from Earth.





3

$$d'_{venus} = \frac{2\times 0.949\times 6378}{(1-0.723)\times 1AU} \approx 1',$$

$OC \approx 16.5'$, $CD \approx 24.0'$,

$$CE = \sqrt{OC^2 - OE^2} \approx 12.0'$$

$$CD = 2CE = 24.0'$$

So, $\theta = \angle CFD = 24.0'$, 3

As shown on the picture,

$\theta' = \angle COD$ is the additional angle that Venus covered during the transit,

$$\frac{tg\frac{\theta}{2}}{tg\frac{\theta'}{2}} = \frac{0.723}{(1-0.723)}, \quad tg\frac{\theta}{2} = tg12', \theta' = 9.195';$$ 3

$$t_{transit} = \frac{\theta'}{\omega_1} = \frac{9.195'}{4.29\times 10^{-4}\,'/s}\times\cos\varepsilon$$, that is $5^h 56^m 36^s$,

So the transit will finish at about $14^h 57^m$. 2

15) On average, the visual diameter of the Moon is slightly less than that of the Sun, so the frequency of annular solar eclipses is slightly higher than total solar eclipses. For an observer on the Earth, the

longest total solar eclipse duration is about 7.5 minutes, and the longest annular eclipse duration is about 12.5 minutes. Here, the longest duration is the time interval from the second contact to the third contact. Suppose we count the occurrences of both types of solar eclipses for a very long time, estimate the ratio of the occurrences of annular solar eclipses and total solar eclipses. Assume the orbit of the Earth to be circular and the eccentricity of the Moon's orbit is 0.0549. Count all hybrid eclipses as annular eclipses.

Solution

the semi-major axis of Moon's orbit is a; its eccentricity is e; T is the revolution period; apparent radius of the Moon is r; the distance between Earth and Moon is d; the angular radius of the Sun is R_o

When the Moon is at perigee, the total eclipse will be longest.

$$\omega_1=v_1/d_1,\ t_1=2\ (r_1-R)/\omega_1$$

Here, ωis the angular velocity of the moon, and v is its linear velocity; t2 is the during time of total solar eclipse; r1 is the angular radius of the Moon when it's at perigee.

When the Moon is at apogee, the annular eclipse will be longest.

$$\omega_2=v_2/d_2,\ t_2=2(R-r_2)/\omega_2$$

Since $v_2/v_1=d_1/d_2=(1-e)/(1+e)$,we get:

$$\frac{t_2}{t_1}=\frac{R-r_2}{r_1-R}\times\left(\frac{1+e}{1-e}\right)^2 \qquad (1)$$

3

Moon orbits the Earth in a ellipse. Its apparent size r varies with time. When r>R, if there occurred an center eclipse, it must be total solar eclipse. Otherwise when r<=R, the center eclipse must be annular.

We need to know that, in a whole moon period, what's the time fraction of r>R and r<=R. r ∝1/d.

But it's not possible to get d by solving the Kepler's equation. Since e is a small value, it would be reasonable to assume that d changes linearly with t. So, r also changes linearly with t. Let the moment when the Moon is at perigee be the starting time (t=0), in half a period, we get:

$$r = r_2 + kt = r_2 + \frac{2(r_1 - r_2)}{T} \cdot t, \quad 0 \le t < T/2$$

Here, $k = 2(r_1 - r_2)/T$=constant.

When r=R, we get a critical t :

$$t_R = \frac{R - r_2}{k} = \frac{(R - r_2)}{2(r_1 - r_2)} \cdot T \qquad (2)$$ 2

During a Moon period, if $t \in (t_R, T - t_R)$, then r>R, and the central eclipses occurred are total solar eclipses. The time interval from t_R to $T - t_R$ is $\Delta t_T = T - 2t_R$. If $t \in [0, t_R]$ & $t \in [T - t_R, T]$, then r≤R, and the central eclipses occurred are annular eclipses. The time interval is $\Delta t_A = 2t_R$.

4

The probability of occurring central eclipse at any t is the same. Thus the counts ratio of annular eclipse and total eclipse is:

$$\frac{f_A}{f_T} = \frac{\Delta t_A}{\Delta t_T} = \frac{2t_R}{T - 2t_R} = \frac{R - r_2}{r_1 - R} = \frac{t_2}{t_1} \Big/ \left(\frac{1+e}{1-e}\right)^2 \approx \frac{4}{3}$$ 1