So

$$\Delta m = -2.5\log\frac{I_{max}}{I_{min}} = -2.5\log\frac{2\pi ab}{\pi b^2} = -2.5\log 4$$ (2 points)

$$\Delta m = -1.5$$ (2 points)

Solution 16:

a) Total energy of the projectile is

$$E = \frac{1}{2}mv_\circ^2 - \frac{GMm}{R} = -\frac{GMm}{2R} < 0$$

$E < 0$ means that orbit might be ellipse or circle. As $\theta > 0$, the orbit is an ellipse.
Total energy for an ellipse is

$$E = -\frac{GMm}{2a}$$

Then

$$a = R$$ (7 points)

b) In figure (1) we have

$$OA + O'A = 2a$$

$$O'A = a$$

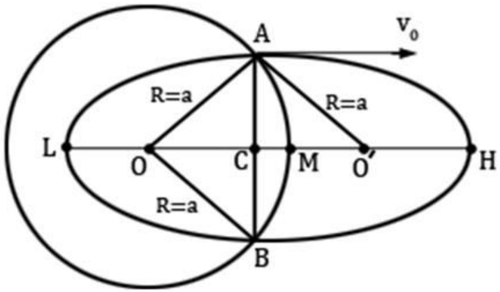


Figure (1)

In OAO' triangle it is obvious that

$$OC = CO'$$

Then C must be the center of the ellipse with the initial velocity vector $v_\circ$ parallel to the ellipse major-axis (LH).

In figure (2)

$$HM = CH - CM = a - (R - R\sin\theta) = R - R + R\sin\theta = R\sin\theta = \frac{R}{2}$$

(15 points)

c) Range of the projectile is $\widehat{AB}$

$$\widehat{AB} = 2\left(\frac{\pi}{2} - \theta\right)R = (\pi - 2\theta)R = \frac{2\pi}{3}R$$

(6 points)

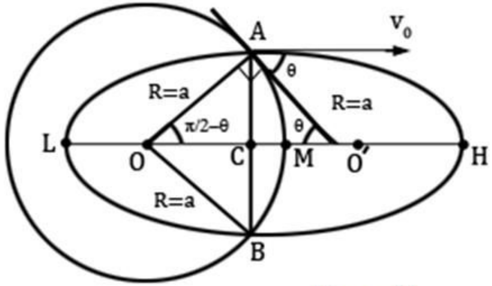


Figure (2)

d) Start with ellipse equation in polar coordinates

$$r = \frac{a(1-e^2)}{1+ecos\varphi}$$

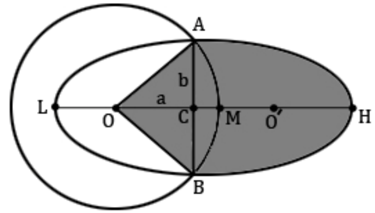
For point A

$$R = \frac{R(1-e^2)}{1-ecos(\frac{\pi}{2}+\theta)}$$

$$e = sin\theta = \frac{1}{2}$$

(5 points)

e) Using Kepler's second law



$$\frac{\Delta S}{S_0} = \frac{\Delta T}{T}$$

$$\Delta S = S_{AOBH} = S_{\Delta AOB} + \frac{S_0}{2}$$

$$= 2 \times \frac{\text{bae}}{2} + \frac{\pi\text{ab}}{2} = \text{bae} + \frac{\pi\text{ab}}{2}$$

$$\frac{\Delta S}{S} = \frac{\text{bae} + \frac{\pi\text{ab}}{2}}{\pi ab} = \frac{e + \frac{\pi}{2}}{\pi} = \frac{0.5 + \frac{\pi}{2}}{\pi}$$

Kepler's third law

$$T = \sqrt{\frac{4\pi^2 R^3}{GM}} = 84.5\, min$$

$$\Delta T = T \times \frac{0.5 + \frac{\pi}{2}}{\pi} = 55.7\, min$$

(12 points)

Solution 17:

a) Relation between the apparent and absolute magnitude is given by

$$m = M + 5log\left(\frac{d}{10}\right)$$ (3 points)

where d is in terms of parsec. Substituting $m = 18$ and $M = -0.2$, results in

(5 points)

$$d = 4.37 \times 10^4 \; pc$$

b) Adding the term for the extinction, changes the magnitude distance relation as follows

$$m = M + 0.7x + 5\log(100x)$$

where x is given in terms of kilo parsec. To have a rough value for x, after substituting m and M, this equation reduces to

$$8.2 = 0.7x + 5\log(x)$$ (6 points)

To solve this equation, we examine

$$x = 5\,, 5.5\,, 6\,, 6.5$$

where the best value is obtained roughly $x \cong 6.1\; kpc$. (8 points)

c) For a solid angle Ω, the number of observed red clump stars at the distance in the range of x and $x + \Delta x$ is given by

$$\Delta N = \Omega\, x^2\, n(x) f\, \Delta x$$

So the number of stars observed in Δx is given by

$$\frac{\Delta N}{\Delta x} = \Omega\, x^2\, n(x) f$$

(6 points)

From the relation between the distance and apparent magnitude we have

$$m_1 = M + 5log\left(\frac{x}{10}\right)$$

$$m_2 = M + 5log\left(\frac{x + \Delta x}{10}\right)$$

$$\Delta m = 5log\left(\frac{x + \Delta x}{x}\right)$$

$$\Delta m = 5log\left(1 + \frac{\Delta x}{x}\right)$$

$$\Delta m = \frac{5}{ln10} ln\left(1 + \frac{\Delta x}{x}\right)$$

$$\Delta m = \frac{5}{ln10}\left(\frac{\Delta x}{x}\right)$$

Replacing Δx with Δm, results in

$$\frac{\Delta N}{\Delta m} = \frac{\Delta N}{\Delta x} \times \frac{\Delta x}{\Delta m}$$

So the number of stars for a given magnitude is obtained by (5 points)

$$\frac{\Delta N}{\Delta m} = \frac{\Omega ln10}{5} n(x)x^3 f$$

Finally we substituting x in terms of apparent magnitude using $\mathrm{x} = 10^{\frac{\mathrm{m}+5.2}{5}}$.

In the case of no extinction, we are able to observe the Galaxy beyond the center. So $\frac{dN}{dm}$ has two terms in

$x < R_0$ and $x > R_0$. The relation between x and r for these two cases are

$$x = R_0 - r \qquad x < R_0$$

(6 points)

and

$$x = R_0 + r \qquad x > R_0$$

So in general we can write $\frac{\Delta N}{\Delta m}$ as

(6 points)

$$\frac{\Delta N}{\Delta m} = \frac{\Omega ln10}{5} n_0 \exp\left(\frac{10^{\frac{m-5.2}{5}}}{R_d}\right) \times 10^{\frac{3(m-5.2)}{5}} f \qquad x < R_0$$

$$\frac{\Delta N}{\Delta m} = \frac{\Omega ln10}{5} n_0 \exp\left(\frac{2R_0}{R_d}\right) \exp\left(-\frac{10^{\frac{m-5.2}{5}}}{R_d}\right) \times 10^{\frac{3(m-5.2)}{5}} f\, \Theta(x_0 - x) \quad x > R_0$$

where $\Theta(x)$ is the step function and x_0 is the maximum observable distance.