Table of Constants

(All constants are in SI)

Parameter	Symbol	Value
Gravitational constant	G	$6.67 \times 10^{-11} N\ m^2\ kg^{-2}$
Plank constant	h	6.63×10^{-34}J s
	$\hbar$	$1.05 \times 10^{-34}\ J\ s$
Speed of light	c	$3.00 \times 10^{8}\ m\ s^{-1}$
Solar Mass	$M_{\odot}$	$1.99 \times 10^{30}\ kg$
Solar radius	$R_{\odot}$	$6.96 \times 10^{8}\ m$
Solar luminosity	$L_{\odot}$	$3.83 \times 10^{26}\ W$
Apparent solar magnitude (V)	$m_{\odot}$	-26.8
Solar constant	$b_{\odot}$	$1.37 \times 10^{3}\ W\ m^{-2}$
Mass of the Earth	$M_{\oplus}$	$5.98 \times 10^{24}\ kg$
Radius of the Earth	$R_{\oplus}$	$6.38 \times 10^{6}\ m$
Mean density of the Earth	$\rho_{\oplus}$	$5 \times 10^{3}\ kg\ m^{-3}$
Gravitational acceleration at sea level	g	$9.81\ m\ s^{-2}$
Tropical year		365.24 days
Sidereal year		365.26 days
Sidereal day		86164 s
Inclination of the equator with respect to the ecliptic	ε	23.5°
Parsec	pc	$3.09 \times 10^{16}\ m$
Light year	ly	$9.46 \times 10^{15}\ m$
Astronomical Unit	AU	$1.50 \times 10^{11}\ m$
Solar distance from the center of the Galaxy	R	$8 \times 10^{3} pc$
Hubble constant	H	$75\ kms^{-1}\ M\ pc^{-1}$
Mass of electron	m_e	$9.11 \times 10^{-31}\ kg$
Mass of proton	m_p	$1.67 \times 10^{-27}\ kg$
Central wavelength of V-band	λ	$550\ n\ m$
Refraction of star light at horizon		$34'$
	π	3.1416

Useful mathematical formula:

$$\ln(1+x) \sim x \qquad for\ \ x \to 0$$

Problem 16: High Altitude Projectile (45 points)

A projectile which starts from the surface of the Earth at the sea level is launched with the initial speed of $v_\circ = \sqrt{(GM/R)}$ and with the projecting angle (with respect to the local horizon) of $\theta = \frac{\pi}{6}$. M and R are the mass and radius of the Earth respectively. Ignore the air resistance and rotation of the Earth.

a) Show that the orbit of the projectile is an ellipse with a semi-major axis of $a = R$.
b) Calculate the highest altitude of the projectile with respect to the Earth surface (in the unit of the Earth radius).
c) What is the range of the projectile (distance between launching point and falling point) in the units of the earth radii?
d) What is eccentricity (e) of this elliptical orbit?
e) Find the time of flight for the projectile.

Problem 17: Apparent number density of stars in the Galaxy (45 points)

Let us model the number density of stars in the disk of Milky Way Galaxy with a simple exponential function of $n = n_0 \exp(-\frac{r-R_0}{R_d})$, where r represents the distance from the center of the Galaxy, R_0 is the distance of the Sun from the center of the Galaxy, R_d is the typical size of disk and n_0 is the stellar density of disk at the position of the Sun. An astronomer observes the center of the Galaxy within a small field of view. We take a particular type of Red giant stars (red clump) as the standard candles for the observation with approximately constant absolute magnitude of $M = -0.2$,

a) Considering a limiting magnitude of $m = 18$ for a telescope, calculate the maximum distance that telescope can detect the red clump stars. For simplicity we ignore the presence of interstellar medium so there is no extinction.
b) Assume an extinction of $0.7\ mag/kpc$ for the interstellar medium. Repeat the calculation as done in the part (a) and obtain a rough number for the maximum distance these red giant stars can be observed.
c) Give an expression for the number of these red giant stars per magnitude within a solid angle of Ω that we can observe with apparent magnitude in the range of m and $m + \Delta m$, (i.e. $\frac{\Delta N}{\Delta m}$). Red giant stars contribute fraction f of overall stars. In this part assume no extinction in the interstellar medium as part (a). Assume the size of the disk is larger than maximum observable distance.