Solutions

Solution 1: CCD Image Processing

a) To measure instrumental magnitude we should choose an aperture. Careful investigation of the image, shows that a 5×5 pixel aperture is enough to measure m_I for all stars. m_I can be calculated using:

$$m_I = -2.5 \log\left(\frac{\sum_{i=1}^{N} I_{i(star)} - N\bar{I}_{Sky}}{Exp}\right)$$

where $I_{i(star)}$ is the pixel value for each pixel inside the aperture, N is number of pixels inside the aperture, $\bar{I}_{Sky}$ is the average of sky value per pixel taken from dark part of image and Exp is the exposure time. Table (1.4) lists values for m_I and $Zmag$ calculated for all three identified stars.

$$\bar{I}_{Sky} = 4.42$$

$$N = 25$$

$$Exp = 450$$

Table (1.4)

Star	m_I	m_t	$Zmag$
1	-3.02	9.03	12.38
3	-5.85	6.22	12.40
4	-4.04	8.02	12.39

b) Average $Zmag$ = 12.4

c) Following part (a) for stars 2 and 5, we can calculate true magnitudes (m_t) for these stars

Table (1.5)

Star	m_I	m_t
2	-2.13	9.93
5	-0.66	11.4

d) Pixel scale for this CCD is calculated as

$$p = \frac{pixel\ size}{focal\ length} \times \frac{180 \times 3600}{\pi}$$

$$= 4.30''$$

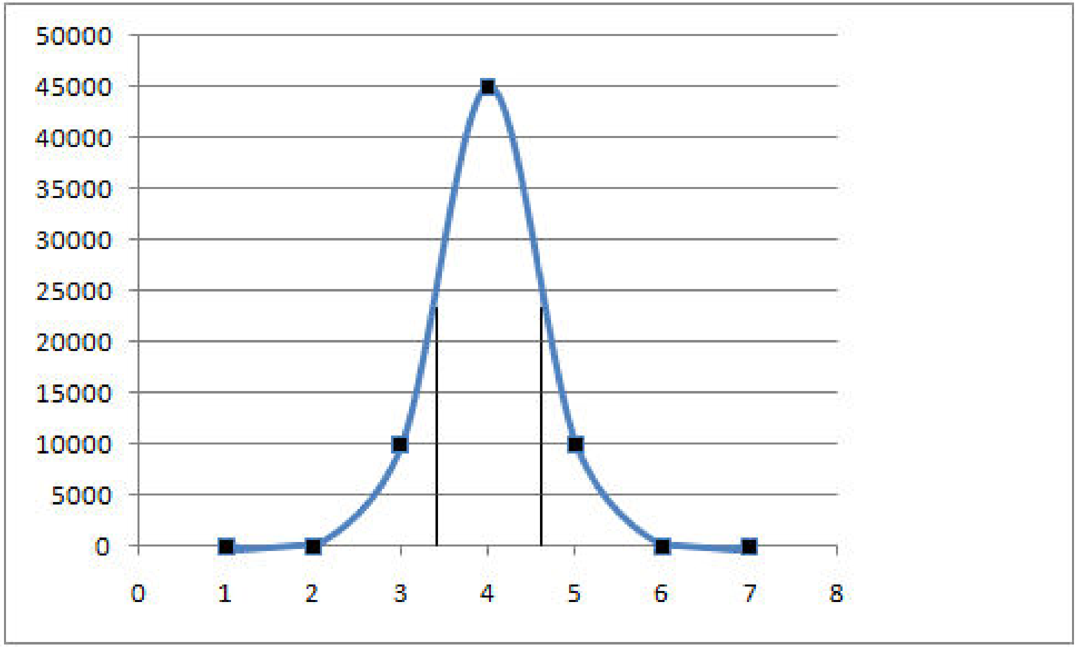
e) Average sky brightness:

$$m_{sky} = -2.5 \log \frac{\bar{I}_{\mathrm{Sky}}}{(Exp)(p)^2} + Zmag$$

$$= 20.6$$

f) To estimate astronomical seeing, first we plot pixel values in x or y direction for one of the bright stars in the image. As plot (1) shows, the FWHM of pixel values which is plotted for star 3, is 1 pixel , hence astronomical seeing is equal to

$$seeing \cong 4''$$



Plot (1)

CCD Image Problem Marking Scheme

Part	Tot. Pts.	Details	Max.	Explanation
a	10	Relation	2	Each value :+2 $\bar{I}_{sky}$ (within calculation) : +2 m_I relation (in calculation) +2
		m_I	6	
		$\bar{I}_{sky}$	2	
b	10	Z_{mag}	10	$3Z_{mag}$ and average , for each less: - 2
c	10	m_t	10	For each one:+ 5, for each numerical mistake: -2
d	10	P (pixel Scale)	10	
e	10	Relation of m_{sky}	5	
		Value of m_{sky}	5	
f	10	Seeing	10	Seeing: +4, Gaussian profile: +3, FWHM: +3

Solution 2: Venus

a) The $\angle SVE$ angle should be calculated from the phase of Venus. Figure 2.1 shows that projected area of Venus disk which is illuminated by the Sun is

$$\frac{\pi r^2}{2} + \frac{\pi r r'}{2}$$

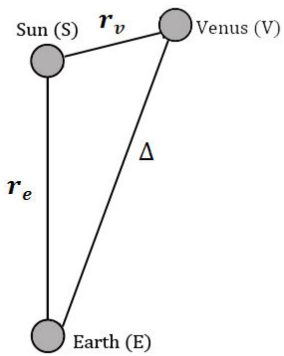
where

$$r' = r cos(\angle SVE)$$

Then,

$$Phase = (\frac{\frac{\pi r^2}{2} + \frac{\pi r^2 \cos(\angle SVE)}{2}}{\pi r^2}) \times 100 = \frac{100}{2}(1 + \cos(\angle SVE)) = 100 cos^2(\frac{\angle SVE}{2})$$

The angle $\angle SVE$ is calculated and written in table 2.2, column 2.



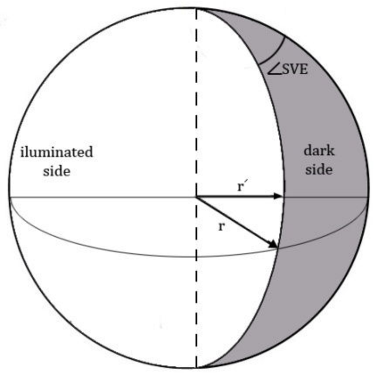


Figure 2.1

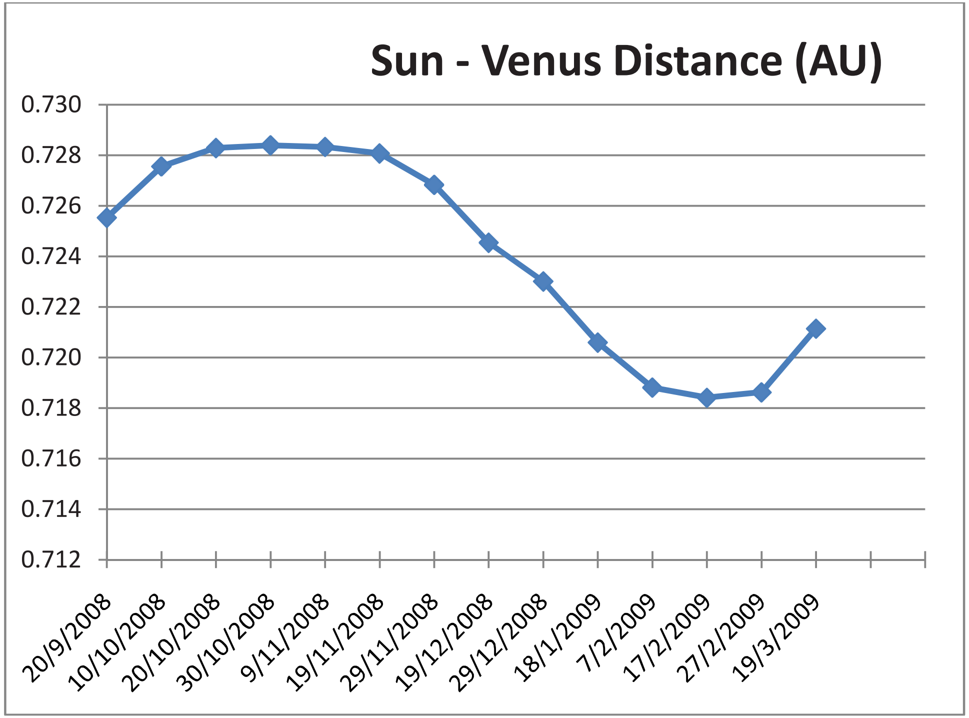
b) As in figure 2.1, in SEV triangle we have,

$$\frac{r_e}{\sin(\angle SVE)} = \frac{r_v}{\sin(\angle SEV)}$$

$$r_v = r_e \frac{\sin(\angle SEV)}{\sin(\angle SVE)}$$

where r_e and $\angle SEV$ (elongation) is given in table 2.1 then, r_v for all observing dates is calculated and written in table 2.2 column 3.

c)



Sun - Venus Distance (AU)
0.730
0.728
0.726
0.724
0.722
0.720
0.718
0.716
0.714
0.712
20/9/2008
10/10/2008
20/10/2008
30/10/2008
9/11/2008
19/11/2008
29/11/2008
19/12/2008
29/12/2008
18/1/2009
7/2/2009
17/2/2009
27/2/2009
19/3/2009

d) According to the obtained values written in table 2.2 column 3,

$$r_v^{max} = 0.728\ AU$$
$$r_v^{min} = 0.718\ AU$$

e) Semi-major axis is

$$a = \frac{(r_v^{max} + r_v^{min})}{2} = 0.723\ AU$$

f) Eccentricity could be calculated from both of aphelion and perihelion distances as

$$e = \frac{r_v^{max} - r_v^{min}}{2a} = 6.92 \times 10^{-3}$$

Table 2.2

Column 1	Column 2	Column 3
Date	SVE (°)	Sun - Venus Distance (AU)
2008-Sep-20	39.83	0.726
2008-Oct-10	47.16	0.728
2008-Oct-20	50.80	0.728
2008-Oct-30	54.55	0.728
2008-Nov-09	58.26	0.728
2008-Nov-19	62.10	0.728
2008-Nov-29	66.17	0.727
2008-Dec-19	74.81	0.725
2008-Dec-29	79.63	0.723
2009-Jan-18	90.57	0.721
2009-Feb-07	104.83	0.719
2009-Feb-17	114.08	0.718
2009-Feb-27	125.59	0.719
2009-Mar-19	157.52	0.721

Venus Problem Marking Scheme

part	Tot. Pts	Details	Max
a	16	Angle derivation	6
		Calculation of $\angle SVE$	10
b	14	Relation	4
		Sun-Venus distance	10
c	6	Plotting Sun-Venus distance	6
d	8	Perihelion	4
		Aphelion	4
e	8	a (relation)	4
		a (value)	4
f	8	e (relation)	4
		e (value)	4

Note: reported numbers in table 2 are not acceptable if they are out of 0.75 and 1.25 times of designer answer.