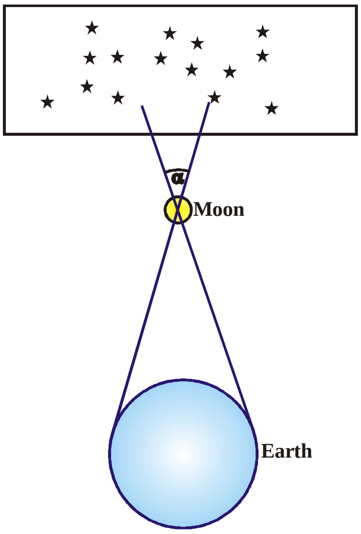
THEORETICAL PART 1

1. Two persons, on the equator of the Earth separated by 180° in longitude, observe the Moon's position with respect to the background star field at the same time. If the declination of the Moon is zero, sketch the situation and calculate the difference in apparent right ascension seen by those two persons.

Solution:



(25%)

From the Astronomical and Physical constant:

Mean distance Earth to Moon: $r = 384399$ km

Diameter of the Earth: $D = 12742$ km

$r = \frac{D}{\alpha}$, α in radian ($\alpha <<$) **(25%)**

In arcminutes,

$$\alpha = 3438' \times \frac{12742}{384399} = 113.96'$$

$= 7.60$ minutes **(50%)**

Thus, the difference in apparent right ascension is 7.60 minutes

2. On April 2, 2008 a telescope (10 cm diameter, $f/10$) at the Bosscha Observatory was used to observe the Sun and found an active region 0987 (based on the NOAA number) at 8° South and 40° West from the center of the solar disk. The region was recorded with a CCD SBIG ST-8 Camera (1600 × 1200 pixels, 9 μm/pixel) and its size was 5 × 4 pixels. According to the Astronomical Almanac, the solar diameter is 32'. How large is the corrected area of the active region in unit of millionth of solar hemisphere (msh)?

Solution :

Pixel size (″) = 206265×pixel size (μm)/(1000 × focal length (mm))

= 206265 × 9/(1000 × 1000) = 1.86" (30%)

Solar diameter = $32 \times 60''/1.86''/\text{pixel} = 1034$ pixels = 9306 μm

Solar radius, R = 4653 μm (25%)

If A_s is the measured area, then

$A_s = 5 \times 4 \times 81\ \mu m^2 = 1620\ \mu m^2$ (15%)

The corrected area due to the projection effect is

$A_m = A_s \cdot 10^6/(2\,\pi\, R^2 \cos B \cos L)$

$= 1620 \times 10^6/\, (2 \times 3.14 \times 4653^2 \times \cos 8° \times \cos 40°)$

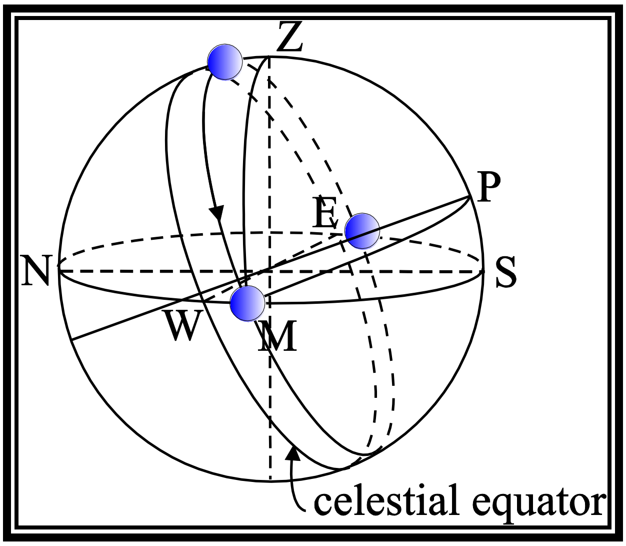
$= 15.7$ msh (30%)

3. A full moon occurred on June 19, 2008 at $00^h\,30^m$ West Indonesian Time (local civil time for western part of Indonesia with reference to geographic longitude of 105° E). Calculate the extreme values of duration of the Moon above the horizon for observers at Bosscha Observatory (longitude: 107° 35' 00″.0 E, latitude: 6° 49' 00″.0 S, Elevation: 1300.0 m). Time zone = UT $+7^h\,00^m$.

Solution:

The position of the Sun on June 19, 2008: $\delta \approx +23^{\circ}.5$, $\alpha \approx 6^h = 90^{\circ}$

At full moon phase, the declination of the Moon is between $\delta \approx -18^{o}.5$ and $\delta \approx -28^{o}.5$ depending on the position of node. (20%)



Spherical triangle: ZPM, Z = zenith, P = south celestial pole and M = moon.

$ZP = m = (90^o - 6^o\,49') = 83^o\,11'$; $ZM = p = 90^o$ and,

$PM = z = 90^o - 23^o\,30' = 66^o\,30'$

The greatest possible value of $PM = z = 90^{o} - 18^{o}\ 30' = 71^{o}\ 30'$ and the smallest possible value *of* $PM = z = 90^{o} - 28^{o}\ 30' = 61^{o}\ 30'$

$\cos p = \cos m \cos z + \sin m \sin z \cos \angle P$ (20%)

The maximum value of PM

$\cos 90^{o} = \cos 83^{o}\ 11' \cos 71^{o}\ 30' + \sin 83^{o}\ 11' \sin 71^{o}\ 30' \cos \angle P$

$\cos \angle P = -\cot 83^{o}\ 11' \cot 71^{o}\ 30' = -0.1195 \times 0.3346$

$= -0.0401$

$\angle P = 92^{o}\ 17'\ 53'' = 6^{h}\ 9^{m}\ 11^{s}$

$2\angle P = 12^{h}\ 18^{m}\ 22^{s}$ (30%)

The minimum value of PM

$\cos 90^{o} = \cos 83^{o}\ 11' \cos 61^{o}\ 30' + \sin 83^{o}\ 11' \sin 61^{o}\ 30' \cos \angle P$

$\cos \angle P = -\cot 83^{o}\ 11' \cot 61^{o}\ 30' = -0.1195 \times 0.5430$

$= -0.0649$

$\angle P = 93^{o}\ 43'\ 16'' = 6^{h}\ 14^{m}\ 53^{s}$

$2\angle P = 12^{h}\ 29^{m}\ 46^{s}$ (30%)

4. Suppose a star has a mass of 20 $\mathcal{M}_{\odot}$. If 20% of the star's mass is now in the form of helium, calculate the helium-burning lifetime of this star. Assume that the luminosity of the star is 100 $L_{\odot}$, in which 30% is contributed by helium burning. The carbon mass, ^{12}C, is 12.000000 amu.

Solution:

The mass of three helium nuclei = 3 × 4.002603 amu = 12.007809 amu (20%)

The mass converted to energy is the difference between the sum of the masses of three helium nuclei and the mass of the resulting carbon nucleus:

$$12.007809 \text{ amu} - 12.000000 \text{ amu} = 7.81 \times 10^{-3} \text{ amu}. \quad (15\%)$$

This represents a fractional loss of mass of $\dfrac{7.81\times10^{-3}}{12.007809} = 6.5 \times 10^{-4}$ of the original mass. (15%)

The stellar mass $\boldsymbol{m} = 20\times\left(1.99\times10^{33}\right) = 3.98\times10^{34}\,\text{g}$

Thus 6.5×10^{-4} of 20% of the star's mass will be converted into energy during triple-α process, or

$E = mc^2 = 0.2(3.98\times10^{34})(6.5\times10^{-4})(3\times10^{10})^2 = 4.66\times10^{51}\,\text{erg}$ (25%)

The helium-burning lifetime is

$$t = \frac{E}{0.3L} = \frac{4.66\times10^{51}}{0.3(100)(3.96\times10^{33})} = 3.92\times10^{16}\,s = 1.24\times10^{9}\,\text{years}$$ (25%)

5. The average temperature of the Cosmic Microwave Background (CMB) is currently T = 2.73 K, and it yields the origin of CMB to be at redshift z_{CMB} =1100. The densities of the Dark Energy, Dark Matter, and Normal Matter components of the Universe as a whole are, respectively, ρ_Λ = 6.7 × 10^{-30} g/cm^3, $\rho_{\rm DM}$ = 2.4 × 10^{-30} g/cm^3, and $\rho_{\rm NM}$ = 0.5 × 10^{-30} g/cm^3. What is the ratio between the density of Dark Matter to the density of Dark Energy at the time CMB was emitted, if we assume that the dark energy is vacuum energy?

Solution:

- Universe change in size and redshift relation:

$$1+z = \frac{\text{size}_{\text{ now}}}{\text{size}_{\text{ at z}}} \Longrightarrow \quad size_{\text{at z}} = \frac{1}{1+z}\text{size}_{\text{ now}} = \frac{1}{1+1100}\text{size}_{\text{ now}}$$

$$= 0.00091\,\text{size}_{\text{ now}}$$

so that the size of the Universe at the time of CMB emission was 0.00091 of its $\text{size}_{\text{ now}}$. (50%)

- If $A = \dfrac{V_{CMB\ at\ emission time}}{V_{Now}} = (0.00091)^3 = 7.5\times10^{-10}$, then the density of the Dark Matter at CMB emission time

$$\rho_{DM\ at\ CMB} = \frac{\rho_{DM}}{A} = \frac{2.4\times10^{-30}}{7.5\times10^{-10}} = 3.2 \times 10^{-21}\ \text{g/cm}^3.$$ (25%)

The ratio between the density of Dark Matter to the density of Dark Energy at the time CMB was emitted = $\dfrac{\rho_{DM\ at\ CMB}}{\rho_\Lambda} = \dfrac{3.2\times10^{-21}}{6.7\times10^{-30}} = 4.8\times10^{8}$ (25%)

6. Radio wavelength observations of gas cloud swirling around a black hole in the center of our galaxy show that radiation from the hydrogen spin-flip transition (rest frequency = 1420.41 MHz) is detected at a frequency of 1421.23 MHz. If this gas cloud is located at a distance of 0.2 pc from the black hole and is orbiting in a circle, determine

the speed of this cloud and whether it is moving toward or away from us and calculate the mass of the black hole.

Solution:

Rest frequency $\upsilon_o = 1420.41$ MHz

Detected frequency $\upsilon = 1421.23$ MHz

$\Delta\upsilon = \upsilon_o - \upsilon = 1420.41 - 1421.23 = -0.82$ MHz (25%)

Using Doppler's shift, the speed of the cloud is :

$$v = \frac{\Delta\upsilon}{\upsilon_o} c = \frac{-0.82}{1421.23}\left(3\times10^{10}\right) = -1.73\times10^{7}\, cm/s \qquad (15\%)$$

Since v is negative, then the cloud is moving toward us. (10%)

(Since $\upsilon > \upsilon_o$, then the cloud is moving toward us)

If $\mathcal{M}$ is mass of black hole, v is the speed of the cloud and R is the orbital radius of cloud, then

$$\mathcal{M} = \frac{Rv^2}{G} \qquad (25\%)$$

R = 0.2 pc = 0.2 × (3.086 × 10^{18} cm) = 6.17 × 10^{17} cm, and from the Table of Astronomical and Physical constants: $G = 6.67 \times 10^{-8}$ cm^3/s^2 g

Then, $\mathcal{M} = \frac{Rv^2}{G} = \frac{\left(6.17\times10^{17}\right)\left(-1.73\times10^{7}\right)^2}{6.67\times10^{-8}} = 2.78\times10^{39}\, gr = 1.40\times10^{6}\, \mathcal{M}_\odot$ (25%)

7. A main sequence star at a distance 20 pc is barely visible through a certain space-based telescope which can record all wavelengths. The star will eventually move up along the giant branch, during which time its temperature drops by a factor of 3 and its radius increases 100-fold. What is the new maximum distance at which the star can still be (barely) visible using the same telescope?

Solution:

L = Luminosity of the giant stars in the giant branch

L_{MS} = Luminosity of the main sequence star

F = Flux of the giant stars in the giant branch

F_{MS} = Flux of the main sequence star

T = Temperature of the giant star

T_{MS} = Temperature of the main sequence star

R = Radius of the giant star

R_{MS} = Radius of the main sequence star

Use the radius-luminosity-temperature relation

$$\frac{L}{L_{MS}} = \frac{R^2T^4}{R_{MS}^2T_{MS}^4} = \left(\frac{R}{R_{MS}}\right)^2\left(\frac{T}{T_{MS}}\right)^4 = (100)^2\left(\frac{1}{3}\right)^4 = 123.5 \qquad (50\%)$$

$$F = \frac{L}{4\pi d^2}$$

$$\frac{F}{F_{MS}} = \frac{L}{L_{MS}}\left(\frac{d_{MS}}{d}\right)^2$$

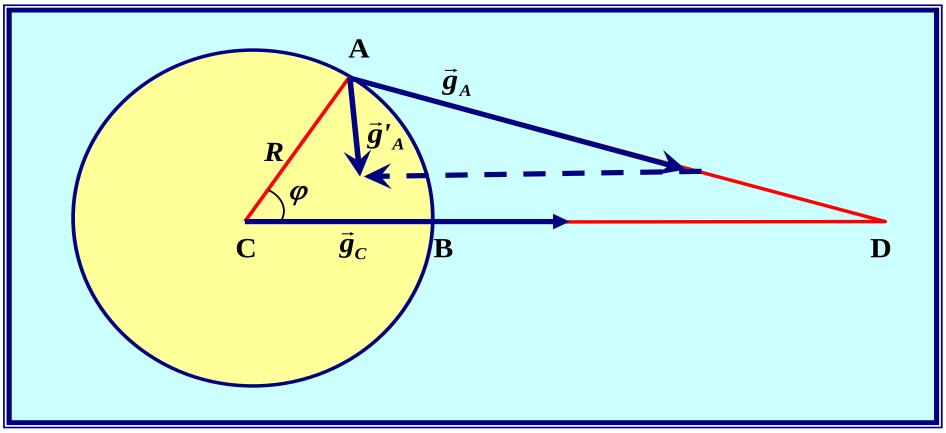
that is, the star becomes 123.5 times brighter when it becomes a giant star. To be just barely visible using the same telescope, we push the star farther so that it becomes 123.5 times dimmer (flux is inversely proportional to a square of the distance).

$$\frac{1}{123.5} = \left[\frac{1/d}{1/d_{MS}}\right]^2 \Longrightarrow \frac{d}{d_{MS}} = 11.11 \Longrightarrow d = 11.11\, d_{MS}$$

$$= 11.11\ (20)\ \text{pc} = 222.20\ \text{pc} \qquad (50\%)$$

8. Gravitational forces of the Sun and the Moon lead to raising and lowering of sea water surfaces. Let φ be the difference in longitude between points A and B, where both points are at the equator and A is on the sea surface. Derive the horizontal acceleration of sea water at position A due to Moon's gravitational force at the time when the Moon is above point B according to observers on the Earth (express it in φ, the radius $\boldsymbol{R}$ of Earth, and the Earth-Moon distance r).

Solution:



(10%)

D in the figure is the center of the Moon. The Moon's gravitational acceleration at A and at center of the Earth C are

$$\vec{g}_A = -\frac{G\mathcal{M}}{r_A^{\ 2}}\hat{r}_A$$

$$\vec{g}_C = -\frac{G\mathcal{M}}{r_C^{\ 2}}\hat{r}_C = -\frac{G\mathcal{M}}{r^{\ 2}}\hat{r}_C$$

Their directions are toward the Moon. $\mathcal{M}$ is the mass of the Moon.

r_A = AD and r_C = CD =r. $\hat{r}_A$, $\hat{r}_C$ are unit vectors from the center of the Moon toward A and C, respectively (20%)

The Moon's gravitational acceleration at A according to observers is

$$\vec{g}'_A = \vec{g}_A - \vec{g}_C = -\left(\frac{G\mathcal{M}}{r_A^{\ 2}}\hat{r}_A - \frac{G\mathcal{M}}{r^{\ 2}}\hat{r}_C\right) = -\left(\frac{G\mathcal{M}}{r_A^{\ 3}}\vec{r}_A - \frac{G\mathcal{M}}{r^{\ 3}}\vec{r}_C\right)$$

Since $\vec{r}_A = \vec{r}_C + \vec{R}$, $\vec{R}$ is the position of A with respect to the Earth's center of mass

$$\vec{g}'_A = \vec{g}_A - \vec{g}_C = -\frac{G\mathcal{M}}{r_A^{\ 3}}\vec{R} - \left(\frac{G\mathcal{M}}{r_A^{\ 3}} - \frac{G\mathcal{M}}{r^{\ 3}}\right)\vec{r}_C \qquad (20\%)$$

The first term is in radial direction. The tangential or horizontal component of the second term is equal to the horizontal component of the acceleration of sea water at A:

$$g'_A(\text{horizontal}) = \left(\frac{G\mathcal{M}}{r^{\ 3}} - \frac{G\mathcal{M}}{r_A^{\ 3}}\right) r\ \sin\varphi$$

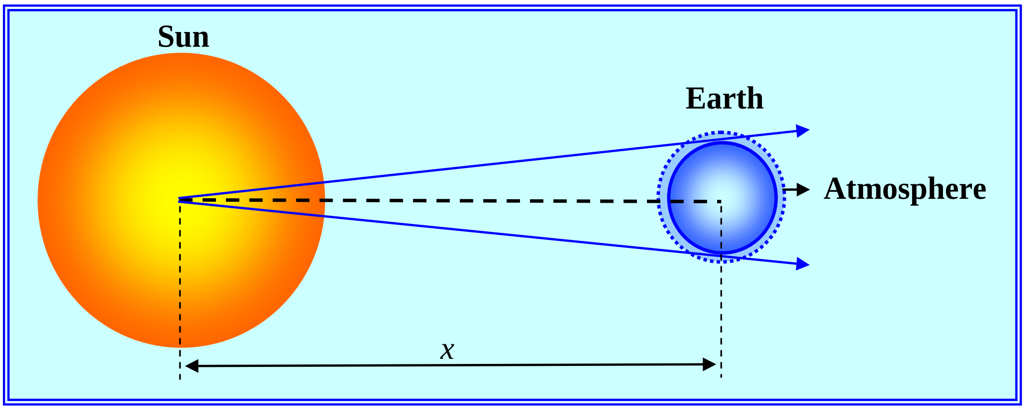
$$= G\mathcal{M}\left(\frac{1}{r^{\ 3}} - \frac{1}{(r^{\ 2} + R^2 - 2Rr\ \cos\varphi)^{3/2}}\right) r\ \sin\varphi$$

(50%)

$$= \frac{G\mathcal{M}}{r^2}\left(1 - \frac{1}{[1+(R/r)^2 - 2(R/r)\cos\varphi]^{3/2}}\right)\sin\varphi$$

9. The radiation incoming to the Earth from the Sun must penetrate the Earth's atmosphere before reaching the earth surface. The Earth also releases radiation to its environment and this radiation must penetrate the Earth's atmosphere before going out to the outer space. In general, the transmittance of the Sun radiation during its penetration to the Earth's atmosphere (t_1) is higher than that of the radiation from the Earth (t_2). Let $T_{\text{eff}\odot}$ be the effective temperature of the Sun, $R_\odot$ the radius of the Sun, $r_\oplus$ the radius of the Earth, and x the distance between the Sun and the Earth. Derive the temperature of the Earth's surface as a function of the aforementioned parameters.

Solution:



Power of radiation received by the Earth from the Sun is

$$P_m = \sigma T_{eff\odot}^4 4\pi R_\odot^2 \frac{\pi r_\oplus^2}{4\pi x^2} t_1 \qquad (40\%)$$

Power radiated by the Earth and reach the outer space

$$P_{out} = \sigma T_\oplus^4 4\pi r_\oplus^2 t_2 \qquad (30\%)$$

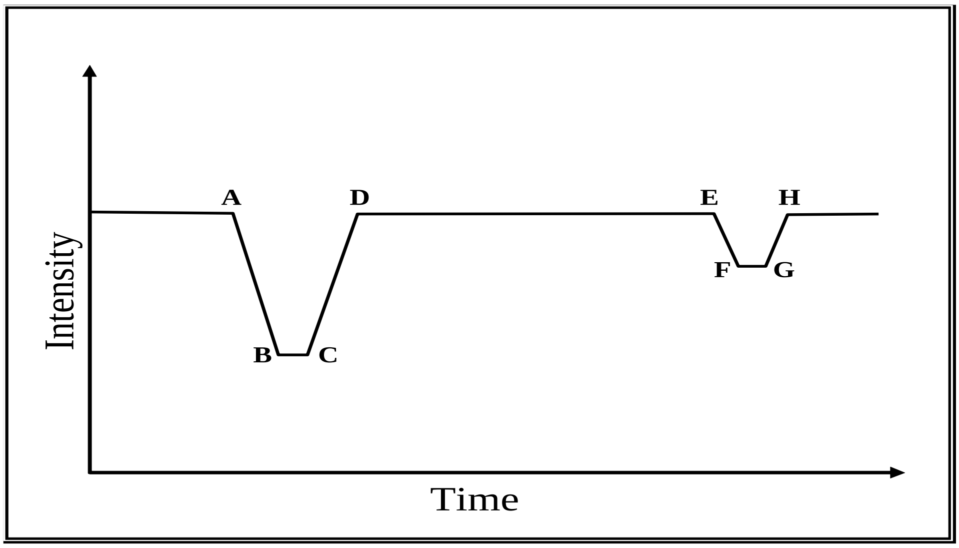
In equilibrium $P_{in} = P_{out}$ (5%)

$$\sigma T_{\text{eff}\odot}^4 \frac{\pi r_\oplus^2}{4\pi x^2} 4\pi R_\odot^2 t_1 = \sigma T_\oplus^4 4\pi r_\oplus^2 t_2$$

$$T_\oplus^4 = \frac{T_{\text{eff}\odot}^4 t_1 R_\odot^2}{4 t_2 x^2}$$

$$T_{\oplus} = \sqrt[4]{\frac{T_{\text{eff}\odot}^4 t_1 R_{\odot}^2}{4 t_2 x^2}} = T_{\text{eff}\odot} \sqrt[4]{\frac{t_1}{4t_2}\left(\frac{R_{\odot}}{x}\right)^2} \qquad (25\%)$$

10. An eclipsing binary star system has a period of 30 days. The light curve in the figure below shows that the secondary star eclipses the primary star (from point A to point D) in eight hours (measured from the time of first contact to final contact), whereas from point B to point C, the total eclipse period is one hour and eighteen minutes. The spectral analysis yields the radial velocity of the primary star to be 30 km/s and of the secondary star to be 40 km/s. If we assume that the orbits are circular and has an inclination of i = 90°, determine the radii and the masses of both stars in unit of solar radius and solar mass.



Solution:

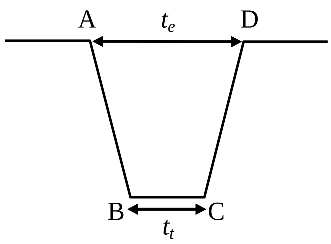
Binary Periode : P = 30 days = 0.0821 years

Time of eclipse: t_e = 8 hours = 0.0009 years

Time of total eclipse : t_t = 1 hour 18 minute = 0.0001 years

Radial velocity of primary star :V_{r1} = 30 km/s = 6.3285 AU/year

The velocity of secondary star :V_{r2} = 40 km/s = 8.4380 AU/year



For circular orbit $V_r = \frac{2\pi a}{P} \longrightarrow a = \frac{PV_r}{2\pi}$ (a = radius orbit)

For primary star *:* (10%)

$$a_1 = \frac{PV_{r1}}{2\pi} = \frac{(0,0821)(6.3285)}{2\pi} = 0.0827 \text{ AU} = 1.24 \times 10^7 \text{ km}$$

For secondary star *:* (10%)

$$a_2 = \frac{PV_{r2}}{2\pi} = \frac{(0.0821)(8.4380)}{2\pi} = 0.1103 \text{ AU} = 1.65 \times 10^7 \text{ km}$$

$a = a_1 + a_2 = 0.0827 + 0.1103 = 0.1930$ AU $= 2.89 \times 10^7$ km

The radius of primary star can be determined from the equation: (10%)

$$\frac{(t_e + t_t)}{P} = \frac{4R_1}{2\pi a} \quad \text{or} \quad R_1 = \frac{2\pi a}{4P}(t_e + t_t) = \frac{2\pi(0.1930)}{4(0.0821)}(0.0009 + 0.0001)$$

$$= 0.0039 \text{ AU} = 5.86 \times 10^5 \text{ km} = 0.84\ R_\odot \quad (20\%)$$

The radius of secondary star can be determined from the equation: (10%)

$$\frac{(t_e - t_t)}{P} = \frac{4R_2}{2\pi a} \quad \text{or} \quad R_2 = \frac{2\pi a}{4P}(t_e - t_t)$$

$$= \frac{2\pi(0.1930)}{4(0.0821)}(0.0009 - 0.0001)$$

$$= 0.0028 \text{ AU} = 4.22 \times 10^5 \text{ km} = 0.61\ R_\odot \quad (15\%)$$

From the Kepler's third law : $\frac{a^3 \sin^3 i}{P^2} = (\mathcal{M}_1 + \mathcal{M}_2)$

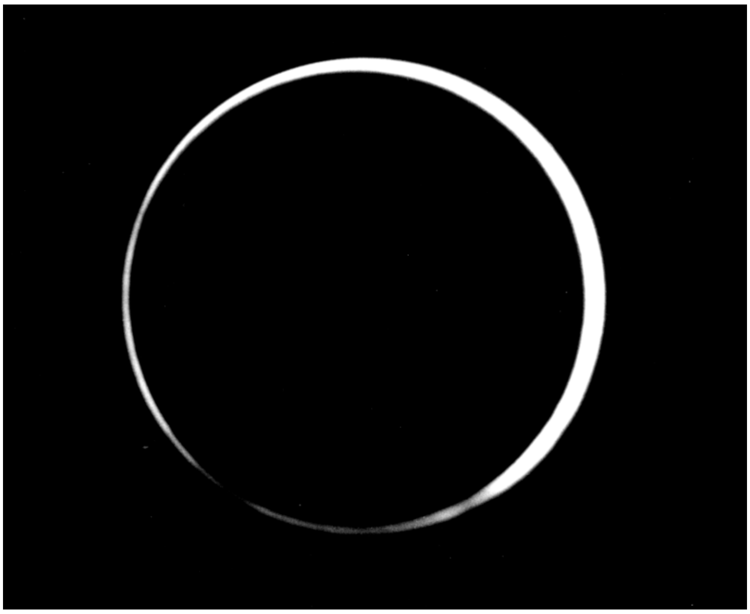
and $\frac{\mathcal{M}_1}{\mathcal{M}_2} = \frac{a_2}{a_1}$

Since $i = 90^o$, $\sin i = \sin 90^o = 1$, and then

$$\mathcal{M}_1 = \frac{a^3}{P^2(1 + a_1/a_2)} = \frac{(0.1930)^3}{(0.0821)^2(1 + 0.0827/0.1103)} = 0.61\ \mathcal{M}_\odot \quad (15\%)$$

$$\mathcal{M}_2 = \frac{a^3}{P^2(1 + a_2/a_1)} = \frac{(0.1930)^3}{(0.0821)^2(1 + 0.1103/0.0827)} = 0.46\ \mathcal{M}_\odot \quad (10\%)$$

11. Below is a picture on a 35 mm film of annular eclipse in Dumai, North Sumatra on August 22, 1998, taken with a telescope having effective diameter 10 cm and f-ratio 15. The diameter of the Sun's disk in original picture on the film is 13.817 mm and the diameter of the Moon's disk is 13.235 mm. Estimate the distances of the Sun and the Moon (expressed in km) from the Earth and the percentage of the solar disk covered by the Moon during the annular eclipse.



Solution:

D = 10 cm, f-ratio 15 or $f = F/D = 15$, then F = 150 cm

(D = diameter of the telescope and F is the focal length of the telescope).

S = scale of the image on the focal plane of the telescope = (206265/F) (″/mm)

= (206265/1500) = 137.51 ″/mm (25%)

Angular diameter of the Sun = 137.51 ″/mm × 13.817 mm

= 1899.97567 ″ = 0º.527771019 = 0º 31' 39″.98

Distance of the Sun = 1 392 000/ ((0º.527771019/180º) x π)

= 151 118 045.9 km (25%)

Angular diameter of the Moon = 137.51 ″/mm × 13.235 mm

=1819.94485 ″ = 0º.505540236 = 0º 30' 19″.94

Distance of the Moon = 3476/(0º.505540236/180º × π) = 393955.0515 km. (25%)

The percentage of the solar disk covered by the Moon is (13.235 mm/13.817 mm)^2 × 100% = 91.75 %. (25%)

12. Consider a type Ia supernova in a distant galaxy which has a luminosity of $5.8 \times 10^9 L_\odot$ at maximum light. Suppose you observe this supernova using your telescope and find that its brightness is 1.6×10^{-7} times the brightness of Vega. The redshift of its host

galaxy is known to be z = 0.03. Calculate the distance of this galaxy (in pc) and also the Hubble time.

Solution:

The relation between flux F and luminosity L: $F = \dfrac{L}{4\pi d^2}$

For Vega : $F_V = \dfrac{L_V}{4\pi\, d_V^2}$

For Supernova : $F_{SN} = \dfrac{L_{SN}}{4\pi\, d_{SN}^2}$

$F_{SN} = 1.6 \times 10^{-7} F_V \implies \dfrac{L_{SN}}{4\pi\, d_{SN}^2} = \left(1.6\times10^{-7}\right)\dfrac{L_V}{4\pi\, d_V^2}$

$$L_{SN}\, d_V^2 = \left(1.6\times10^{-7}\right) L_V\, d_{SN}^2 \qquad (25\%)$$

$$d_{SN} = d_V\left[\frac{1}{1.6\times10^{-7}}\frac{L_{SN}}{L_V}\right]^{1/2} = 7.76\left[\frac{1}{1.6\times10^{-7}}\frac{5.8\times10^{9}}{130}\right]^{1/2} = 129581812.59\ \text{pc}$$

$$= 1.3 \text{ x } 10^8 \text{ pc} \qquad (35\%)$$

If t_H is the Hubble time, then the relation between redshift, distance, and Hubble time is:

$$t_H = \frac{d_{SN}}{cz} = \left[\frac{1.3\times10^{8}\,\text{pc}}{\left(1\,\text{lyr}/\text{yr}\right)\left(0.03\right)}\right]\left(\frac{3.26\,\text{lyr}}{1\,\text{pc}}\right) = 1.41\times10^{10}\,\text{years} \qquad (40\%)$$

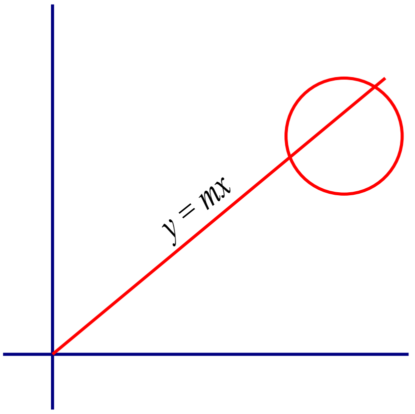
13. In the journey of a space craft, scientists make a close encounter with an object and they would like to investigate the object more carefully using their on-board telescope. For simplicity, we assume that the position of the space craft is stationary in (0,0) and the shape of the object is a disk and the boundary has the equation

$$x^2 + y^2 - 10x - 8y + 40 = 0.$$

Find the exact values of maximum and minimum of tan φ where φ is the elevation angle of the telescope with respect to the "horizontal" direction (x-axis) during investigation from one edge to the other edge.

Solution:

I. Algebraic Solution



Let $y = mx$ be the line of sight of the telescope. (20%)

Then, the intersection of the line of sight and the circle is

$$x^2 + (mx)^2 - 10x - 8(mx) + 40 = 0$$

$$(1 + m^2)x^2 - (10 + 8m)x + 40 = 0$$ (20%)

The equation will have solution if its discriminant

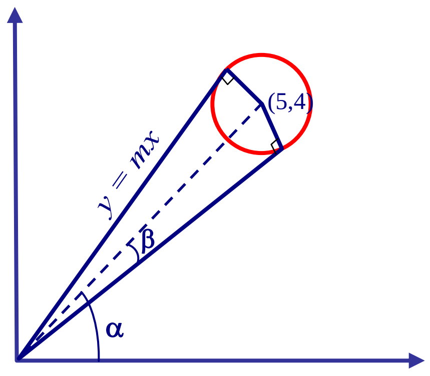
$$D = b^2 - 4ac \geq 0,$$
$$= (-(10 + 8m))^2 - 4(1 + m^2)40 \geq 0.$$

The solution of the inequality is $\frac{5}{6} - \frac{\sqrt{10}}{12} \leq m \leq \frac{5}{6} + \frac{\sqrt{10}}{12}$ (20%)

Therefore the exact value of the maximum value of $\tan\varphi$ is $\frac{5}{6} + \frac{\sqrt{10}}{12}$ (20%)

and the minimum value is $\frac{5}{6} - \frac{\sqrt{10}}{12}$ (20%)

II. Geometric Solution



Let us write the equation of the circle in the following form (20%)

$$(x-5)^2 + (y-4)^2 = -40 + 25 + 16$$
$$= 1$$

The centre of the circle is (5,4) and the radius is 1

First we obtain $\tan\alpha = \frac{4}{5}$ and $\tan\beta = \frac{1}{\sqrt{40}}$ (20%)

The minimum value of tan of elevation angle is (30%)

$$\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta} = \frac{\frac{4}{5}-\frac{1}{\sqrt{40}}}{1+\frac{4}{5\sqrt{40}}} = \frac{5}{6} - \frac{\sqrt{10}}{12}$$

The maximum value of tan of elevation angle is

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta} = \frac{\frac{4}{5}+\frac{1}{\sqrt{40}}}{1-\frac{4}{5\sqrt{40}}} = \frac{5}{6} + \frac{\sqrt{10}}{12}$$ (30%)

14. Consider a Potential Hazardous Object (PHO) moving in a closed orbit under the influence of the Earth's gravitational force. Let *u* be the inverse of the distance of the object from the Earth and *p* be the magnitude of its linear momentum. As the object travels, the graph of *u* as a function of *p* passes through points A and B as shown in the following table. Find the mass and the total energy of the object, and sketch the shape of *u* curve as a function of *p* from A to B.

	p ($\times 10^9$ kg m s^{-1})	u ($\times 10^{-8}$ m^{-1})
A	0.052	5.15
B	1.94	194.17

Solution:

The mechanical energy is conserved and has the form (10%)

$$E = \frac{p^2}{2\boldsymbol{m}} - \frac{G\boldsymbol{\mathcal{M}m}}{r} = \frac{p^2}{2\boldsymbol{m}} - G\boldsymbol{\mathcal{M}m}u$$

Insert the values for *p* and *u* for points A and B and solve for *m* and *E*.

(20%)

$$E_A = \frac{p_A^{\ 2}}{2m} - G\mathcal{M}mu_A$$

$$E_B = \frac{p_B^{\ 2}}{2m} - G\mathcal{M}mu_B$$

$$\frac{p_A^{\ 2}}{2m} - G\mathcal{M}mu_A = \frac{p_B^{\ 2}}{2m} - G\mathcal{M}mu_B$$

$$m = \sqrt{\frac{p_A^{\ 2} - p_B^{\ 2}}{2G\mathcal{M}(u_A - u_B)}}$$

$$m = \sqrt{\frac{(5.2\times10^{7})^2 - (1.94\times10^{9})^2}{2(6.6726\times10^{-11})(5.9736\times10^{24})(5.15\times10^{-8} - 1.94\times10^{-6})}} = 50 \text{ tons.}$$ (20%)

Thus,

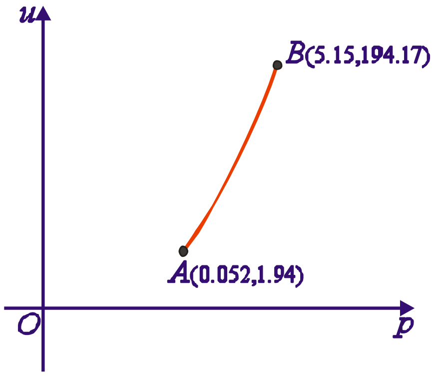
$$E = E_A = \frac{p_A^{\ 2}}{2m} - G\mathcal{M}mu_A$$

$$E = \frac{(5.20\times10^{7})^2}{2(50000)} - (6.6726\times10^{-11})(5.9736\times10^{24})(50000)(5.15\times10^{-8})$$ (20%)

$$= -1.0\times10^{12}\ \text{J}$$

To sketch the curve, we use the equation

$$E = \frac{p^2}{2m} - G\mathcal{M}mu \longrightarrow u = \frac{1}{2G\mathcal{M}m^2}p^2 - \frac{E}{G\mathcal{M}m} \longrightarrow \text{Parabolic curve}$$ (10%)



(20%)

15. Galaxy NGC 2639 is morphologically identified as an Sa galaxy with measured maximum rotational velocity v_{max} of 324 km/s. After corrections for any extinction, its apparent magnitude in B is m_B = 12.22. It is customary to measure a radius R_{25} (in units of kpc) at which the galaxy's surface brightness falls to 25 $mag_B/arcsec^2$. Spiral galaxies tend to follow a typical relation:

$$\log R_{25} = -0.249 M_B - 4.00,$$

where M_B is the absolute magnitude in B. Apply the B-band Tully-Fisher relation for Sa spirals

$$M_B = -9.95 \log v_{max} + 3.15 \quad (v_{max} \text{ in km/s})$$

to calculate the mass of NGC 2639 out to R_{25}. If colour index of the sun is $(m_{B\odot} - m_{V\odot}) = 0.64$, write the mass in units of solar mass $\mathcal{M}_\odot$ and its luminosity B-band in unit of $L_\odot$.

Solution:

Absolute B magnitude of NGC 2639:

$M_B = -9.95 \log v_{max} + 3.15 = -9.95 \log(324) + 3.15 - 21.83$ (20%)

$\log R_{25} = -0.249 M_B - 4.00 = 1.4357 \longrightarrow R_{25} = 27.2678 \text{ pc} = 8.41 \times 10^{19} \text{cm}$ (20%)

Mass estimation of NGC 2639:

$$\mathcal{M} = \frac{v^2 R_{25}}{G} = \frac{(324\times10^5)^2(8.41\times10^{19})}{6.6726\times10^{-8}} = 1.32 \times 10^{42} \text{ kg} = 6.62 \times 10^{11}\ \mathcal{M}_\odot$$ (20%)

The B magnitude of the Sun:

$M_{B\odot} = M_{V\odot} + (m_{B\odot} - m_{V\odot}) = 4.82 + 0.64 = 5.46$ (20%)

meaning the Sun is fainter in the blue band relative to the A0 standard star.

Luminosity of NGC 2639 in solar luminosity ($L_\odot$):

$$L = 10^{0.4(M_{B\odot} - M_B)} = 10^{0.4(5.46+21.83)} L_\odot = 8.24\times10^{10} L_\odot$$ (20%)