The 2nd International Olympiad on Astronomy and Astrophysics

Bandung, Indonesia

Thursday, 21 August 2008

Practical Competition: Data Analysis

Please read this carefully:

1. *Every student receives problem sheets in English and/or in his/her native language, answer sheets, millimeter block papers, and scratch sheets.*
2. *The time available is five hours for the data analysis and observation competitions. There are three data analysis problems, and two observation problems.*
3. *Use only the materials provided.*
4. *Fill in the boxes at the top of each sheet of paper with your country code and your student code.*
5. *Use only the front side of* ***answer sheets****. Write only inside the boxed area.*
6. *Begin answering each question on a separate sheet.*
7. *Numerical results should be written with as many digits as are appropriate.*
8. *Write on* ***the answer sheets and the millimeter block papers*** *whatever you consider is required for the solution of each question. Please express your answer primarily in term of equations, numbers, figures, and plots. If necessary provide your answers with concise text. Full credit will be given to correct answer with detailed steps for each question. Underline your final result.*
9. *At the end of the exam place the answer sheets and the millimeter block papers inside the envelope and leave everything on your desk.*

Astronomical and Physical Constants

Quantity	Value
Astronomical unit (AU)	149,597,870.691 km
Light year (ly)	9.4605×10^{17} cm = 63,240 AU
Parsec (pc)	3.0860×10^{18} cm = 206,265 AU
Sidereal year	365.2564 days
Tropical year	365.2422 days
Gregorian year	365.2425 days
Sidereal month	27.3217 days
Synodic month	29.5306 days
Mean sidereal day	$23^h56^m4^s.091$ of mean solar time
Mean solar day	$24^h3^m56^s.555$ of sidereal time
Mean distance, Earth to Moon	384,399 km
Earth mass ($\mathfrak{M}_\oplus$)	5.9736×10^{27} g
Earth's mean radius	6,371.0 km
Earth's mean velocity in orbit	29.783 km/s
Moon's mass ($\mathfrak{M}_☽$)	7.3490×10^{25} g
Moon's mean radius	1,738 km
Sun mass ($\mathfrak{M}_\odot$)	1.9891×10^{33} g
Mean Earth radius	6.3710×10^{6} cm
Sun radius ($R_\odot$)	6.96×10^{10} cm
Sun luminosity ($L_\odot$)	3.96×10^{33} erg s^{-1}
Sun effective temperature ($T_{eff_\odot}$)	5 800 ^{o}K
Sun apparent magnitude ($m_\odot$)	-26.8
Sun bolometric magnitude ($m_{bol_\odot}$)	-26.79
Sun absolute magnitude ($M_\odot$)	4.82
Sun absolute bolometric magnitude ($M_{bol_\odot}$)	4.72
Speed of light (c)	2.9979×10^{10} cm/s
Gravitational constant (G)	6.6726×10^{-8} dyne cm^2 g^{-2}
Boltzmann constant (k)	1.3807×10^{-16} erg. K^{-1}
Stefan-Boltzmann constant (σ)	5.6705×10^{-5} erg cm^{-2} K^{-4} s^{-1}
Planck constant (h)	6.6261×10^{-27} erg s
Electron charge (e)	1.602×10^{-19} C = 4.803×10^{-10} esu
Electron mass (m_e)	$5.48579903 \times 10^{-4}$ amu

Proton mass (m_p)	1.007276470 amu
Neutron mass (m_n)	1.008664904 amu
Deuterium nucleus mass (m_d)	2.013553214 amu
Hydrogen mass	1.00794 amu
Helium mass	4.002603 amu
Carbon mass	12.01070 amu

Conversion table	
1 Å	0.1 nm
1 barn	10^{-28} m^2
1 G	10^{-4} T
1 erg	10^{-7} J = 1 dyne cm
1 esu	3.3356×10^{-10} C
1 amu (atomic mass unit)	1.6606×10^{-24} g
1 atm (atmosphere)	101,325 Pa = 1.01325 bar
1 dyne	10^{-5} N

300 points for 3 problems, 100 points for each problem

I. Virgo Cluster

The Virgo cluster of galaxies is the nearest large cluster which extends over nearly 10 degrees across the sky and contains a number of bright galaxies. It is very interesting to find the distance to Virgo and to deduce certain cosmological information from it. The table below provides the distance estimates using various distance indicators (listed in the left column). The right column lists the mean distance $d_i \pm$ the standard deviation s_i.

i	**Distance Indicator**	**Virgo Distance (Mpc**
1	Cepheids	14.9 ± 1.2
2	Novae	21.1 ± 3.9
3	Planetary Nebulae	15.2 ± 1.1
4	Globular Cluster	18.8 ± 3.8
5	Surface Brightness Fluctuation	15.9 ± 0.9
6	Tully-Fisher relation	15.8 ± 1.5
7	Faber-Jackson relation	16.8 ± 2.4
8	Type Ia Supernovae	19.4 ± 5.0

1. By applying a weighted mean, compute the average distance (which can be taken as an estimate to the distance to Virgo)

$$d_{avg} = \frac{\sum_i \frac{d_i}{s_i^2}}{\sum_i \frac{1}{s_i^2}}$$

where the sum runs over the eight distance indicator used.

2. What is the uncertainty (rms) (in unit of Mpc) in that estimate?
3. Spectra of the galaxies in Virgo indicate an average recession velocity of 1136 km/sec for the cluster. Can you estimate the Hubble constant H_0 and its uncertainty (rms)?
4. What is the Hubble Time (age of the universe) using the value of Hubble constant you found and the uncertainty (rms)?

II. Determination of stellar masses in a visual binary system

The star α-Centauri (Rigel Kentaurus) is a triple star which consists of two main-sequence stars α-Centauri A and α-Centauri B representing visual binary system, and the third star, called Proxima Centauri, which is smaller and fainter than the other two stars. The angular distance between α-Centauri A and α-Centauri B is 17.59". The binary system has an orbital period of 79.24 years. The visual magnitudes of α-Centauri A and α-Centauri B are -0.01 and 1.34 respectively. Their color indices are 0.65 and 0.85 respectively. Use the data below to answer the following questions.

Data for main-sequence stars

$(B-V)_0$	$T_{\rm eff}$	BC
-0.25	24500	2.30
-0.23	21000	2.15
-0.20	17700	1.80
-0.15	14000	1.20
-0.10	11800	0.61
-0.05	10500	0.33
0.00	9480	0.15
0.10	8530	0.04
0.20	7910	0
0.30	7450	0
0.40	6800	0
0.50	6310	0.03
0.60	5910	0.07
0.70	5540	0.12
0.80	5330	0.19
0.90	5090	0.28
1.00	4840	0.40
1.20	4350	0.75

BC=Bolometric Correction, $(B\text{-}V)_0$=Intrinsic Color

Questions:

1. Plot the curve BC versus $(B\text{-}V)_0$.

2. Determine the apparent bolometric magnitudes of α-Centauri A and α-Centauri B using the corresponding curve.
3. Calculate the mass of each star.

Notes:

1. **Bolometric correction** (BC) is a correction that must be made to the apparent magnitude of an object in order to convert an object's visible magnitude to its bolometric magnitude:

$$BC = m_v - m_{bol} \text{ or } BC = M_v - M_{bol}$$

2. **Luminosity mass relation** : $M_{bol} = -10.2 \log\left(\frac{M}{M_\odot}\right) + 4.9$

III. The Age of Meteorite

The basic equation of radioactive decay can be expressed as:

$$N(t) = N_0 \exp(-\lambda t)$$

where $N(t)$ and N_0 are the number of remaining atoms of the radioactive isotope (or parent isotope) at time t and its initial number at $t = 0$, respectively, while λ is the decay constant. The decay of the parent produces daughter nuclides $D(t)$, or radiogenics, which is defined as

$$D(t) = N_0 - N(t) .$$

Based on those ideas, a group of astronomers investigates a number of meteorite samples to determine their ages. They have two kinds of samples: allende chondrite (A) and basaltic achondrite (B). From the samples, they measure the abundance of ^{87}Rb and ^{87}Sr, where it is assumed that ^{87}Sr is entirely produced by the decay of ^{87}Rb. The value of λ is 1.42×10^{-11} per year for this isotopic decay. In addition, non-radiogenic element ^{86}Sr is also measured. Results of measurement are given in the table below, expressed in ppm (part per million).

Sample No	Meteorite type	^{86}Sr (ppm)	^{87}Rb (ppm)	^{87}Sr (ppm)
1	A	29.6	0.3	20.7
2	B	58.7	68.5	44.7
3	B	74.2	14.4	52.9
4	A	40.2	7.0	28.6
5	A	19.7	0.4	13.8
6	B	37.9	31.6	28.4
7	A	33.4	4.0	23.6
8	B	29.8	105.0	26.4
9	A	9.8	0.8	6.9
10	B	18.5	44.0	15.4

Questions:

1. Express the time t in term of $\dfrac{D(t)}{N(t)}$
2. Determine the half-life $t_{1/2}$, i.e., the time needed to obtain a half number of parents after decay.
3. Knowledge on the ratio between two isotopes is more valuable than just the absolute abundance of each isotope. It is quite likely that there was some initial strontium present. By taking $\left(\dfrac{^{87}\mathrm{Rb}}{^{86}\mathrm{Sr}}\right)$ as independent variable and $\left(\dfrac{^{87}\mathrm{Sr}}{^{86}\mathrm{Sr}}\right)$ as dependent variable, estimate the simple linear regression model to represent the data.
4. Plot $\left(\dfrac{^{87}\mathrm{Rb}}{^{86}\mathrm{Sr}}\right)$ versus $\left(\dfrac{^{87}\mathrm{Sr}}{^{86}\mathrm{Sr}}\right)$ and also the regression line (isochrone) for each type of the meteorites. (Please use minimum 7 decimal digits for intermediate calculations)
5. Subsequently, help this astronomer to determine the age of each type of the meteorites and its error. Which type is older?
6. Determine the initial value of $\left(\dfrac{^{87}\mathrm{Sr}}{^{86}\mathrm{Sr}}\right)_0$ for each type of the meteorites and its error.

Glossary:

A simple linear regression line $y=a+bx$ can be fitted to a set of data (X_i,Y_i), $i=1,\ldots,n$, in which

$$b = \frac{SS_{xy}}{SS_{xx}}$$

$$a = \bar{y} - b\bar{x}$$

where

SS_{xx} : sum of square for $X = \sum_{i=1}^{n} X_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} X_i\right)^2$

SS_{yy} : sum of square for $Y = \sum_{i=1}^{n} Y_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} Y_i\right)^2$

SS_{xy} : sum of square for both X and $Y = \sum_{i=1}^{n} X_i Y_i - \frac{1}{n}\sum_{i=1}^{n} X_i \sum_{i=1}^{n} Y_i$

Standard deviation of each parameter, a and b can be calculated by

$$S_a = \sqrt{\frac{SS_{YY} - \frac{(SS_{XY})^2}{SS_{XX}}}{(n-2)SS_{XX}} \times \sum_{i=1}^{n} X_i^2}$$

$$S_b = \sqrt{\frac{SS_{YY} - \frac{(SS_{XY})^2}{SS_{XX}}}{(n-2)SS_{XX}}}$$