IAO for people in a hurry

IAO Malaysia Team 2021

October 2021

More resources here

1 Positional astronomy

1.1 Parsecs

$$d = 1/\theta \tag{1}$$

θ is the parallax in arcseconds. $[d]$ =pc.

1.2 Altitude, declination and latitude

Must learn to derive these equations by drawing diagrams!

The maximum altitude if object culminates north of zenith

$$a_{max} = 90^\circ + \phi - \delta \tag{2}$$

The maximum altitude if object culminates south of zenith

$$a_{max} = 90^\circ - \phi + \delta \tag{3}$$

Altitude of lower culmination

$$a_{min} = \delta + \phi - 90^\circ \tag{4}$$

Careful when doing this in the southern hemisphere: derive accordingly

From above you can derive,

$$\delta = \frac{1}{2}(a_{max} + a_{min}) \tag{5}$$

$$\phi = 90^\circ - \frac{1}{2}(a_{max} - a_{min}) \tag{6}$$

1.3 Local sidereal time, hour angle and right ascension

You must remember the definitions of these quantities.

LST is the hour angle of vernal equinox. HA is the angle measured westwards from the meridian. RA is the eastwards angle from the vernal equinox.

$$\mathrm{LST} = \mathrm{HA} + \mathrm{RA} \tag{7}$$

1.4 Sign of Equation of Time

NYSS (pronounce "NICE"): New Year Sundial Slow, ie apparent solar time < mean solar time near January.

1.5 Declination of the Sun

$$\sin\delta = \sin\epsilon \sin\frac{2\pi t}{T} \tag{8}$$

IAO officially uses the approximation:

$$\delta = \epsilon \sin\frac{2\pi t}{T} \tag{9}$$

T is one year. t is time since vernal equinox (March 21). Make sure T and t is same units (usually days). You should remember $\epsilon = 23^\circ 26'$ is the axial tilt of Earth obit, also note that the maximum declination is thus $23^\circ 26'$

1.6 Planetary configuration

Phase angle (α) is the Sun-object-observer angle. Elongation (ϵ) is the Sun-observer-object angle. Usually observer is at Earth. See Fig 1.

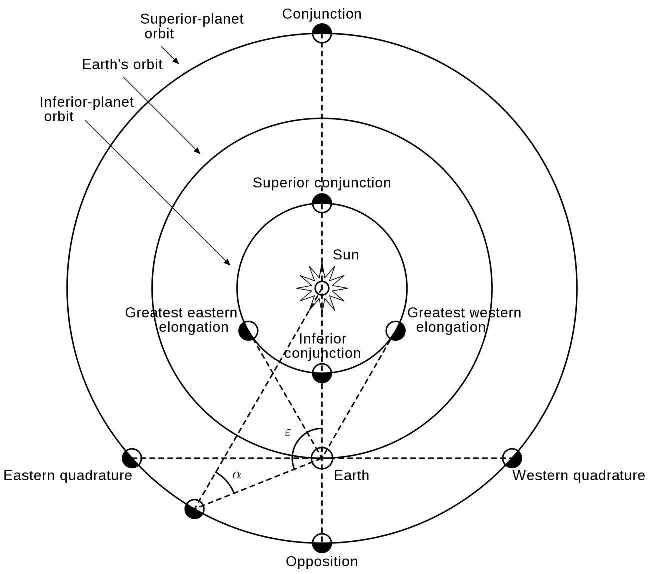

Figure 1: Planetary configuration

1.7 Sidereal v. solar

The following can be derived easily. See Roy Clarke. If you can't remember, just memorise/understand that one solar day is longer than one sidereal day.

$$\text{24h of solar time} = \frac{366.25}{365.25} \times \text{24h of sidereal time} \tag{10}$$

1.8 Synodic period

in: inner, out: outer. If you can't remember, just make sure that $P_{syn} > 0$

$$\frac{1}{P_{syn}} = \frac{1}{P_{in}} - \frac{1}{P_{out}} \tag{11}$$

2 Radiation

For more information, check out Planck's radiation law, but not required.

2.1 Flux intensity

$$L = 4\pi R^2 F \tag{12}$$

2.2 Pogson's magnitudes

$$m_1 - m_2 = -2.5 \log \frac{F_1}{F_2} \tag{13}$$

Use the relationship with luminosity to get the distance modulus. Absolute magnitude: star at 10pc.

$$m_{app} - m_{abs} = 5 \log \frac{d}{10} \tag{14}$$

$[d]$ =pc

2.3 Stefan-Boltzmann law

Derive (hard, unnecessary) from Planck's. Relates flux intensity to temperature

$$F = \sigma T^4 \tag{15}$$

$\sigma = 5.670 \times 10^{-8}$ is given

2.4 Wien's displacement law

Derive (hard, unnecessary) from Planck's. Gives the peak wavelength of a blackbody at a temperature. Note this is not the *only* wavelength, the blackbody emits a continuous spectrum.

$$\lambda = \frac{b}{T} \tag{16}$$

$b = 2.898 \times 10^{-3}$ is given

3 Celestial Mechanics

3.1 Newton's gravitation

$$F = \frac{GMm}{r^2} \tag{17}$$

3.2 Gravitational energy

$$U = -\frac{GMm}{r} \tag{18}$$

Potential energies are always negative (you need to pay energy to send something out of space into infinity, so if you are near Earth, you are already owe energy to Earth. You need to pay your debts before able to escape Earth. The further away from Earth, the lesser your debt, but still negative)

3.3 The axes and eccentricity

Derive from Pythagoras. See Fig 2.

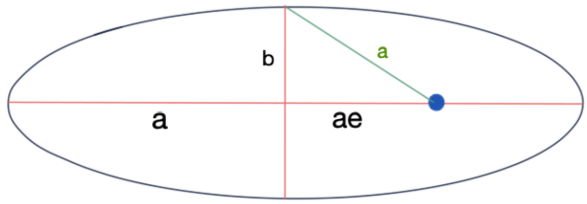

Figure 2: The axes and eccentricity

$$a^2 = (ae)^2 + b^2 \tag{19}$$

Remembering the position of ae allows you to easily find the perihelion and aphelion distances. Also, the sum of distances from each foci to a point on an ellipse is always $d_1 + d_2 = 2a$.

3.4 Eccentricity

Conic sections include ellipses ($e < 1$), parabola ($e = 1$), hyperbola ($e > 1$). Circles are special case ellipse ($e = 0$).

Ellipse is when the object cannot escape (hence in orbit). Parabola is when an object has **exactly** the amount of energy to escape. Hyperbola is when the object has excess energy even after escaping.

3.5 Kepler's Laws

All Kepler's laws can be derived from Newton's gravitational law, but not required.

3.5.1 First: Law of Orbits

Gravitational orbits are conic sections. See Fig 3.

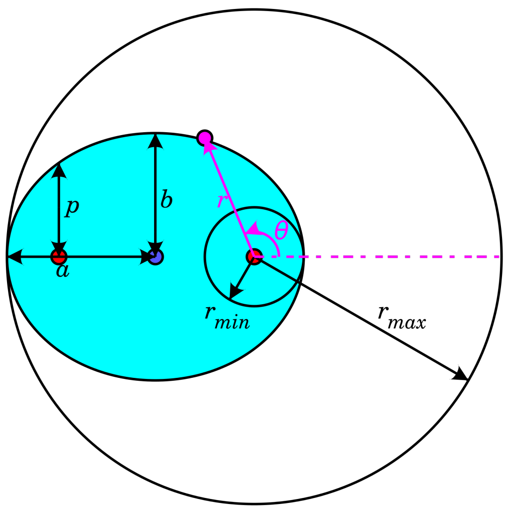

Figure 3: Kepler's First Law

Equation is optional for IAO. No need memorise.

$$r = \frac{a(1-e^2)}{1 + e\cos\theta} \tag{20}$$

Geometric interpret $p = a(1-e^2)$ of Fig 3.

3.5.2 Second: Law of Areas

Equal area is swept at equal duration. See Fig 4.

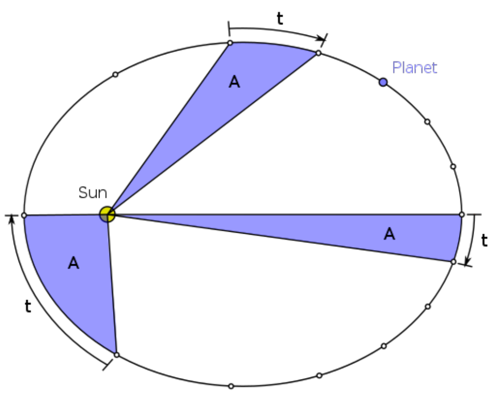

Figure 4: Kepler's Second Law

The total area of ellipse is $A = \pi ab$. There is no elementary formula for the perimeter of the ellipse.

3.5.3 Third: Law of Periods

Relates period to semi-major axis

$$P^2 = \frac{4\pi^2}{G(M+m)} a^3 \tag{21}$$

Usually m is negligible

3.6 Virial Theorem

For a large system like a nebula

$$K + U = \frac{U}{2} \tag{22}$$

where U is potential energy. Note that total energy is half the potential energy, and is negative since $U \leq 0$.

For two objects in orbit,

$$\frac{1}{2}mv^2 + \left(-\frac{GMm}{r}\right) = -\frac{GMm}{2a} \tag{23}$$

3.7 Vis-viva

Derive from Virial Theorem. Optional and not required, but usable when you are out of ideas.

$$v^2 = GM\left(\frac{2}{r} - \frac{1}{a}\right) \tag{24}$$

3.8 Escape velocity

Don't memorise, you can derive from vis-viva by setting $a = \infty$, or derive from energy conservation.

$$v = \sqrt{\frac{2GM}{R}} \tag{25}$$

3.9 Schwarzschild radius

Radius of a (over simplified) black hole. Don't memorise, you can derive from escape velocity above (erroneous, but the rigorous derivation is very complicated).

$$r = \frac{2GM}{c^2} \tag{26}$$

3.10 Hill Sphere

Region of gravitational dominance of secondary. Optional derivation.

$$r_H \approx r\sqrt[3]{\frac{m}{3M}} \tag{27}$$

r is distance between primary and secondary. Note that this means r_H also changes throughout the orbit. M mass of primary, m mass of secondary.

3.11 Roche Limit

Distance where the secondary will start to disintegrate due to tidal forces from primary. Optional derivation.

$$d \approx R_s\sqrt[3]{\frac{2M}{m}} \tag{28}$$

R_s is the radius of the secondary. Note that this means Roche limit is unique to each primary-secondary pair. M mass of primary, m mass of secondary.

4 Optics

4.1 General Snell's law

See Fig 5. Also the definition of $n = \frac{c}{v}$. Just make sure $n \geq 1$ always. (Optional: derive from Huygens)

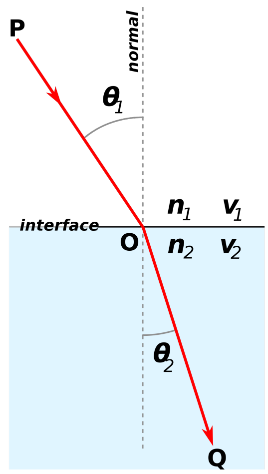

Figure 5: Snell's Law

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \tag{29}$$

4.2 Thin lens

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v} \tag{30}$$

4.3 Rayleigh's Criterion

Minimum angular distance to discern two objects.

$$\theta = 1.22\frac{\lambda}{D} \tag{31}$$

The constant 1.22 is for circular aperture (constant is different for, eg rectangular). Sometimes, and in practice, you can use $\theta \approx \frac{\lambda}{D}$

4.4 Magnification of telescopes

Objective over eyepiece. If you can't remember, just make sure $m > 1$

$$m = \frac{f_o}{f_e} \tag{32}$$

4.5 Field of view

$$FOV_{telescope} \approx \frac{FOV_{eyepiece}}{m} \tag{33}$$

5 Cosmology

5.1 Hubble's Law

Get expansion velocity of **extragalactic** objects.

$$v = H_0 D \tag{34}$$

$H = 70(\text{km/s})/\text{Mpc}$ but the value is controversial, usually given. Can also get the age of the universe through $t = 1/H_0$. **Be careful of units**, use D in Mpc, v is km/s.

5.2 Doppler effect

For stationary observer, define z

$$z = \frac{\lambda_{\text{obsv}} - \lambda_{\text{emit}}}{\lambda_{\text{emit}}} \tag{35}$$

For low speeds ($v \ll 0.1c$):

$$z \approx \frac{v}{c} \tag{36}$$

Remember that redshift (longer wavelength, lower frequency) occurs when the object moves away (positive velocity). Use this fact to check your answers.

The relativistic version:

$$1 + z = \left(1 + \frac{v}{c}\right)\gamma \tag{37}$$

$\gamma = 1/\sqrt{1 - (v/c)^2}$ aka Lorentz factor.

6 Observation

Use a star chart, Stellarium or any stargazing apps to help in this section.

6.1 Constellations

6.1.1 Equatorial System

15 at plane. Notably, Psc ($RA \approx 0$, intersect with ecliptic as vernal equinox), Orion (belt $RA \approx 6$h), Vir (intersect as autumnal equinox, Spica $RA \approx 13$h), Aqr (Altair $RA \approx 20$h).

North CE pole at UMi (Polaris). South CE pole at Oct (near Sigma Oct).

6.1.2 Ecliptic System

12+1 at plane. Famous mnemonic: *The ramble twins crab liverish, scaly scorpions are good water fish.* Ram (Ari), Bull (Tau), Twins (Gem), Crab (Cnc), Li-on (Leo), Ver-gin (Vir), Scaly (Lib), Scorpions (Sco), Are-cher (Sgr), Good/goat (Cap), Water (Aqr), Fish (Psc).

RA=0 at Psc, and increases in this order. Note the ecliptic plane also cuts Ophiuchus (Oph), between Sco and Sgr.

North ecliptic pole at Dra. South ecliptic pole at Dor.

Note that Sun is at Psc with (RA=0) on vernal equinox. This info can help you estimate the date of an unknown star chart. (eg if Sun at Virgo, it's around September (March+6)). Note that it is one month later than the your civil "zodiac signs". (eg Sun intersects Lib at 23rd Oct - 22nd Nov instead of 23rd Sep - 22nd Oct)

6.1.3 Galactic System

27 at plane. Galactic centre at Sgr. Notable constellations on the galactic plane (not exhaustive): Cyg, Cas, Per, Ori, CMa, Cru, Sco.

North galactic pole at Com. South ecliptic pole at Scl.

6.2 Messier objects

Memorise the type, appearance, and positions of all 110 Messier objects if possible. Here are the famous.

MX	Name	Cons
M31	Andromeda Galaxy	And
M1	Crab Nebula	Tau
M42	Orion Nebula	Ori
M27	Dumbbell Nebula	Vul
M16	Eagle Nebula	Ser
M13	Hercules Globular Cluster	Her
M8	Lagoon Nebula	Sgr
M45	Pleiades	Tau
M7	Ptolemy Cluster	Sco
M57	Ring Nebula	Lyr

Then check out Caldwell.

6.3 Asterisms/patterns

Look these up on Google. Recognising them will greatly help with map/observational problems. Also note the date/season when these constellations culminates at midnight.

- Summer triangle
- Winter hexagon
- Winter triangle
- Pegasus' square
- Sagittarius' teapot
- Orion's belt
- Lyra's parallelogram
- (Aquila) Altair's two friends
- The crowns (Corona Borealis, Australis)
- The southern cross (Crux)

6.4 Precession circle

26,000 year cycle. See Fig 6. The red dot is the North Ecliptic Pole. Note Polaris is at +2000 (the year of the Gregorian calendar).

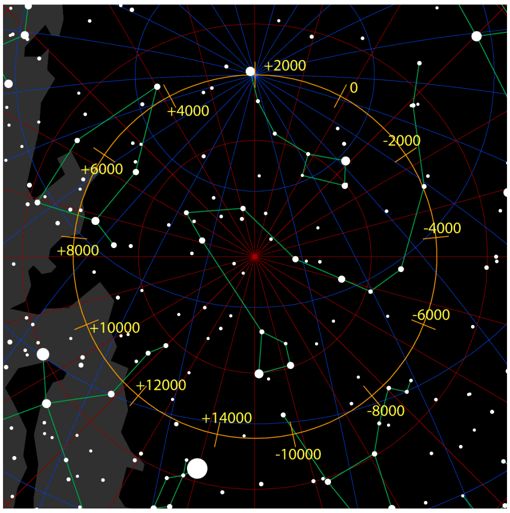

Figure 6: North precession circle

Check out the south precession circle.

6.5 Stars/others

Memorise the top 10 brightest stars: colours, common name, constellation, spectral class, apparent magnitude, eq coordinates.

Memorise double stars (Albireo, ϵ Lyr): same characteristics as above. Check whether its optical double or visual binary.

Misc flashcards
Train relative positioning of constellations
SFA star chart
SFA star chart with notes
Messier flashcards (prioritise starred)
Astro symbols: Planets, Zodiac

7 Misc

7.1 Small angle approximations

θ in radians.

$$\sin\theta \approx \theta \tag{38}$$

$$\cos\theta \approx 1 - \frac{\theta^2}{2} \approx 1 \tag{39}$$

$$\tan\theta \approx \theta \tag{40}$$

7.2 Planar trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \tag{41}$$

$$c^2 = a^2 + b^2 - 2ab\cos C \tag{42}$$

7.3 Conservation of angular momentum

It is equivalent to Kepler's second law. All orbits must follow this.

$$m_1 v_1 r_1 = m_2 v_2 r_2 \tag{43}$$

7.4 Centripetal force

Useful for geostationary orbits. (Note: geosynchronous = same period as Earth. geostationary = geosynchronous and always above a certain point on Earth. Geostationary is circular, geosynchronous can be elliptical).

$$F = \frac{mv^2}{r} \tag{44}$$

7.5 Arc length

θ in radians

$$s = r\theta \tag{45}$$

7.6 Exoplanet transit

$$(\frac{R_p}{R_s})^2 = \frac{\Delta F}{F} \tag{46}$$

Usually F is normalised to 1. See also deriving and calculating inclination via impact parameter

7.7 Assumptions/knowledge

Here are some values from past papers that IAO expects you to know/assume. If you think an important value is not given, just assume and set your significant figures appropriately (usually one sig fig).

- Atmospheric refraction at horizon (standard conditions): $35'$
- Thickness of Milky Way: 400pc
- Time taken for a Moon to return to a phase: 29.53 days
- Axial tilt/max solar declination: $23°26'$
- Limiting magnitude for the naked eye: 6^{m}

- Characteristic density of ice in comets: $200 - 400\text{kg/m}^3$
- Twilight. Civil ($0° - 6°$), nautical ($6° - 12°$), astronomical ($12° - 18°$): angle of the true Sun centre below horizon.
- Ironically, Earth perihelion in winter (early Jan), perihelion in summer (early Jun). (valid this epoch, will change due to apsidal precession).

7.8 Others

See the IAO syllabus. Special relativity (easier than it sounds, just understand the gamma factor and multiply to the mass/length/time). Coordinate transformation between alta-azimuth, equatorial, ecliptic, galactic. Read on Roy Clarke for more positional astronomy (distance to horizon, how longitude affects timing). Spherical trigonometry to get the distance between two points on a sphere. Distance ladder (moving clusters method, HR diagram fitting). Qualitative understanding of turn-off points on HR diagram and thus the age of cluster. Obtain colour excess from comparing main sequence and thus getting interstellar extinction via the empirical ratio 3.0. Estimate FOV by recording the time taken for a star to cross the eyepiece. Contact chooijqweb@gmail.com for suggestions/corrections.

IAO papers
Eclipsing binary star simulator
Exoplanet transit simulator
Other simulations

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