Theoretical round. Sketches for solutions. **For jury job ONLY**

These solutions are written for jury members, so they contain a lot of explanations in the form of text. Students are instructed to use only formulas, calculations, drawings and graphs, writing texts is not allowed in the solutions. It is permitted to write a few keywords in English (for example, “Yes”, “No”, “Assume”, “Answer”, “Average”, “Minimum”, “Maximum”, “Impossible situation”, etc.), as well as terms contained in the accompanying tables and names of objects.

Example of the solutions that expected from students, see at the end of the document.

1(αγ). Parallax measurements. The parallax has become 1.379/0.379 = 3.64 times larger. This means that the parallactic base, previously equal to one astronomical unit, has increased by 3.64 times. If an asteroid orbits the Sun in a circular orbit, then the parallax of the object is the angle at which the radius of the orbit of this asteroid is visible from this object. Thus, for a circular orbit, 3.64 au is the radius of the orbit. The orbital period on in this case is calculated according to Kepler's III Law:

$$(T_2/T_1)^2 = (a_2/a_1)^3,$$

respectively, for the case of a circular orbit

$$T_2 = T_1 \cdot (a_2/a_1)^{3/2} = T_1 \cdot (\pi_2/\pi_1)^{3/2} = T_1 \cdot (1.379/0.379)^{3/2} = 6.94 \text{ years.}$$

For non-circular orbits, the answer may differ from the one calculated above. In general, parallax is the amplitude in varying the position of an object relative to distant objects, in other words, half the angle between the extreme positions.

In general, the parallactic base can be equal to:

- maximum – to the major semi-axis of the orbit – in the case when the object is in a plane perpendicular to this minor semi-axis;
- minimum – to the minor semi-axis of the orbit – in the case when the object is in the direction perpendicular to this minor semi-axis and in the plane of the orbit, or near it (that is, on the line of the major semi-axis or near it);
- as well as values in the interval from minimum to maximum (in other cases).

Thus, the obtained above $T_2 = T_1 \cdot (\pi_2/\pi_1)^{3/2} = 6,94$ years is correct for the case when the parallactic base corresponds to the major semi-axis of the orbit, that is, for the case when Sirius is in a plane perpendicular to this major semi-axis.

Let us find the answer for the case when the parallactic base of 3.64 au corresponds to the small semi-axis of the orbit. The ratio of the small and large semi-axes is determined by the ratio

$$b/a = (1-e^2)^{1/2}.$$

(this relation is easily found by the Pythagorean theorem $b^2 + (ea)^2 = a^2$, applied to the right triangle "focus – center – intersection point of the minor axis with the ellipse"). Thus, for an orbit with a minor semi-axis 3.64 au and an eccentricity of 0.59, the major semiaxis is equal to

$$a_3 = b/(1-e^2)^{1/2} = 3.64 \text{ au} / (0.6519)^{1/2} = 4,51 \text{ au.}$$

The orbital period corresponding to such major semi-axis is equal to

$$T_3 = T_1 \cdot (a_3/a_1)^{3/2} = T_1 \cdot 4.51^{3/2} = 9.57 \text{ years.}$$

Thus, in general, we can say that the orbital period of the asteroid lies in the range from 6.94 to 9.57 years.

Answer: $6.94^Y \le T \le 9.57^Y$.

2(αγ). Spacecraft. There are 8 planets in our Solar System. A spacecraft can orbit around each of them in an elliptical orbit. And around each of them, it can orbit with a period $2\tau = 19.28$ days. And orbiting any planet when passing the pericenter of the orbit, the angular size of the planet can be $\alpha_P = 4{,}6°$, and when passing the apocenter – $\alpha_A = 1{,}9°$. So how we can identify the planet?

The thing is, according to the values given in the condition, you can find the average density of the planet. And the densities of the planets are different.

Let the radius of the planet is r. Then the semi-major axis of the spacecraft 's orbit is equal to

$$a = (r/\sin(\alpha_P/2) + r/\sin(\alpha_A/2))/2 = ((1/\sin(\alpha_P/2) + 1/\sin(\alpha_A/2))/2)\ r = 42{,}6\ r.$$

Kepler's III law, the orbital period of the spacecraft ($T = 2\tau = 19{,}28$ days) is equal to the orbital period of a body moving in a circular orbit with the same major semi-axis, i.e. with the radius R of the orbit equals $a = 42{,}6$ r.

Consider the movement of the body in such an orbit.

Centripetal acceleration, on the one hand, is equal to $v^2/R = 4\pi^2R/T^2$, on the other – GM/R^2. Therefore

$$GM/R^2 = 4\pi^2R/T^2,$$

from where we get that the mass of the planet M is equal to

$$M = 4\pi^2R^3/GT^2.$$

The volume of the planet $V = 4\pi R^3/3$.

Density of the planet

$$\rho = M/V = (4\pi^2R^3/GT^2)/(4\pi r^3/3) = 3\pi(R/r)^3/GT^2 = 3\pi((1/\sin(\alpha_P/2) + 1/\sin(\alpha_A/2))/2)^3/G(2\tau)^2.$$

Note. If we use the approximations $\sin(\alpha_P/2) \approx \alpha_P/2$ and $\sin(\alpha_A/2) \approx \alpha_A/2$, then

$$\rho = 3\pi(1/\alpha_P + 1/\alpha_A)^3/G(2\tau)^2.$$

Calculations give the value $\rho = 3{,}94\cdot10^3$ кг/м^3.

This density is an attribute of Mars.

Note that in this problem you need to calculate everything accurately and precisely. Inaccurate calculations can lead to a fundamental error in identifying the planet.

Answer: **Mars.**

3(αγ). NGC of the Year. A human, whose pupil size is 6 mm, sees stars up to the 6th magnitude. From the object $m = 12.1^m$ the light comes in $\exp_{10}(\Delta m/2.5) \approx 280$ times less. This means that a "pupil" 280 times larger in area or $(280)^{1/2} \approx 17$ times in diameter is required for observation.

$$N_1 = (\exp_{10}((12{,}1^m - 6^m)/2.5^m))^{1/2} \approx 17.$$

The lens diameter is 45 mm, it is only in

$$N_2 = 45 \text{ mm} / 6 \text{ mm} = 7{,}5$$

times more.

$$N_2 < N_1.$$

Therefore, regardless of all other circumstances (above or below the horizon, dark or light sky, etc.), no one will be able to see NGC 2021 through a 45 mm diameter lens.

Answer: a, b, c, d, e, f, g, h – “**No**”.

4-5(αβγ). Eclipse at the North Pole. Let's start with calculations.

4.1. The photo will show an image of the disk of the Sun, most of which is eclipsed by the disk of the Moon. This is exactly the picture that needs to be pictured. Since it is required to draw a full-size image, it is necessary to calculate the dimensions. The diameters of the images of the Sun and Moon are easy to calculate if you correctly draw a graph on which the rays of light propagate from the extreme points of each disk.

Note that we are observing not from the center of the Earth, but from the North Pole, so the geocentric angular dimensions should be corrected due to reduce the distances to the Moon and the Sun, which will be approximately

$$\Delta L = 6357 \text{ km} \times \sin 23° \approx 2500 \text{ km}.$$

Compared to the distance to the Sun, this will not give changes within the error limits. But in relation to the distance to the Moon, the correction will be about 0.65% and the apparent angular diameter will not be 29'09".

Thus, the graph can be a schematic drawing of the objective as a lens, from the center of which there are two straight lines, the angle between which is 30.92' (angular diameter of the Sun) and 29.15' (angular diameter of the Moon, visible from the north pole). Rays passing through the center of the lens are not refracted. These straight lines intersect the focal plane located from the lens at a distance of 250 mm, at points, the distance between which is the diameter of the image of the Moon in the focus of the lens. The diameter obtained on the matrix is easy to find from the drawn triangle:

$$d = \beta \cdot F,$$

β must be expressed in radians (that is, dimensionless quantities):

$$d(\text{Sun}) = 30.92' \times \pi / (180 \cdot 60)' \times 250 \text{ mm} \approx 2.25 \text{ mm}.$$

$$d(\text{Moon}) = 29.15' \times \pi / (180 \cdot 60)' \times 250 \text{ mm} \approx 2.12 \text{ mm}.$$

In pixels, this will be:

$$N(\text{Sun}) = d \times 4000 \text{ px} / 14{,}9 \text{ mm} \approx 604 \text{ px}.$$

$$N(\text{Moon}) = d \times 4000 \text{ px} / 14{,}9 \text{ mm} \approx 569 \text{ px}.$$

When printed with a resolution of 300 pixels per inch, the size in the photo will be:

$$D(\text{Sun}) = 25{,}4 \text{ mm} \times N / 300 = 51{,}1 \text{ mm}.$$

$$D(\text{Moon}) = 25{,}4 \text{ mm} \times N / 300 = 48{,}2 \text{ mm}.$$

Thus, you need to draw a ring with an outer radius of 51.1 mm and an inner radius of 48.2 mm. However, the image of the Moon should not be centered, since we are not observing from the center of the strip, but closer to its edge. Having constructed an appropriate drawing or having figured it out in your mind, you can understand that:

-- at the moment of the maximum phase, the centers of the disks are shifted relative to each other strictly vertically,

-- since the observation point is closer to the Moon than the middle of the belt, the disk of the Moon will be shifted upwards,

-- the ratio of the ring thicknesses from above and below will be equal to the ratio of the distances from the observation point to the belt boundaries (in the forward and reverse directions). Having carried out the corresponding measurements (with a ruler), it can be found that these distances correlate approximately as 1:8 (or 11% : 89%).

4.2. Thus, the drawing should be something like this:

The outer size of the ring is 51.1 mm, the thickness of the ring at the top is only 0.3 mm, and at the bottom is 2.6 mm.

4.3. According to the above measurements, the North Pole was located from the central line of the belt at a distance of 0.78 of the belt half-width. This means that here the duration in comparison with the duration on the central line was:

$$t_1 / t_0 = (1–0{,}78^2)^{1/2},$$

For estimation, we can assume that the duration of the eclipse on the central line near the Pole is equal to the maximum duration of the eclipse 3^M51^S. Thus,

$$t_1 = t_0\cdot(1–0{,}78^2)^{1/2} = 3^M51^S\cdot 0.626 = 2^M25^S.$$

5.1. Artistic drawing. The axis of the camera should be directed at an angle of approximately 23° to the horizon (declination of the Sun on June 10).

5.2. Firstly, let us find the height of the Sun above the horizon. At the North Pole, it is strictly equal to the declination of the Sun at the time of observation. Declination can be approximated by the formula

$$\delta = \varepsilon\cdot\cos 12°,$$

where 12° is the approximate angular distance of the Sun from the summer solstice point,

$$\delta = 23.44°\cdot\cos 12° = 22.93°.$$

The required solar power capacity will be equal to

$$W = A\cdot\sin\delta\cdot(\mathbf{d_\odot}^2 – \mathbf{d_1}^2)/\mathbf{d_0}^2 = 1367\ W/m^2\cdot\sin 22.93°\cdot(30.92'^2 – 29.15'^2)/31'^2 \approx 59\ W/m^2.$$

6(αβγ). Closer to the stars. A stratum of one atmosphere increases the stellar magnitude by 0.23 m. First, let us find the height of a small mountain, the climbing on which reduces the magnitude of the star located at the zenith by a very small amount, say, by exactly 0.01^m.

$$H_{0,01} = 0.01^m/0.23^m \times H_0,$$

where H_0 is the height of homogeneous atmosphere, which is (according to the table) 7991 m. So we get

$$H_{0,01} = 0.01^m/0.23^m \times H_0 = 0{,}01^m/0.23^m \times 7991\ m = 347\ m.$$

Thus, for a model where the decrease in magnitude is associated with the approach to the star, we can write the ratio:

$$-2.5^m\cdot\lg[(L-H_{0,01})^2/L^2] = 0.01^m,$$

where L is the distance to the star.

$$-5^m\cdot\lg[(L-H_{0,01})/L] = 0.01^m,$$

$$\lg(1-H_{0,01}/L) = -0.002,$$

Taking into account that $\ln(1+x) \approx x$ is valid for small values of x, we get:

$$\lg(1-H_{0,01}/L) = \ln(1-H_{0,01}/L)/\ln 10 \approx -H_{0,01}/L/\ln 10,$$

$$-H_{0,01}/L = -0.002 \times \ln 10,$$

$$L = H_{0,01} / 0.004605 \approx 75\ km.$$

7(βγ). Variable star. According to the Doppler effect, the relative change (increase/decrease) of the period is proportional to the ratio of the speed of removal/approach of the star to the speed of light. Since the star is moving away from the Sun, we should observe a longer period. Thus, $\Delta P/P = v/c$.

$$\Delta P = P \times v/c = 246\ 357\ s \times 9/300000 = 7{,}39\ c \approx 7\ s.$$

$$P_0 = P_{OBS} – \Delta P = 246\ 357\ s – 7\ s = 246\ 350\ s.$$

However, this is the wrong answer. This would be the answer if observations were made by observers on the Sun, from which this star is moving away at a speed of 9 km/s. But the observations are conducted from the Earth, which itself is moving relative to the Sun with an average speed V_{orb} = 29.8 km/s.

After analyzing the map, it can be understood that at the beginning of November (the last week), the Earth in its annual motion around the Sun is moving just in the direction of the ecliptic longitude at which the tip of the nose of the Ursa Major (the star ο UMa) is located. (Making a drawing, it is easy to understand that the Earth is moving in the direction in the sky where the Sun was a quarter of a year ago in its annual).

Thus, at the beginning of November, relative to the Earth, the variable star is not moving away from us at a speed of v = 9 km/s, but is approaching with a speed

$$v_E = V_{orb}/c \times \cos\beta - v,$$

where β is the ecliptic latitude of ο Uma, equal, as we can estimate from the star map, to about 45°. Thus,

$$v_E = 29.8 \text{ km/s} \times \cos 45° - 9 \text{ km/s} = 12.1 \text{ km/s} \approx 12 \text{ km/s}.$$

That is, correct ΔP is:

$$\Delta P = P \times v_E/c = 246\,357 \text{ s} \times 12/300000 = 9.86 \text{ s} \approx 10 \text{ s}.$$

$$P_0 = P_{OBS} + \Delta P_S = 246\,357 \text{ s} + 10 \text{ s} = 246\,367 \text{ s}.$$

(It is inappropriate to give an answer with an accuracy of more than 1 s).

8(βγ). Alpha Centauri. All necessary data may be taken from the tables.

Distance to Alpha Centauri is

$$D_C = 1 \text{ pc} / \boldsymbol{p} = 206265 \text{ au} / 0.747 \approx 276\,000 \text{ au}.$$

When approaching from such a distance to the place of the Sun, Alpha Centauri A would have a magnitude

$$\mathbf{m_C} = -0{,}01^m - 5^m \lg 276\,000 \approx -0.01^m - 27.21^m \approx -27.22^m.$$

Thus, the difference in the magnitudes of the Sun and α Centauri A when observed from the same distances will be

$$\mathbf{\Delta m} = -26.74^m - (-27.22^m) = 0.48^m.$$

So the ratio of luminosities of Alpha Centauri A L_C and the Sun L_0 is

$$L_C / L_0 = 10^{\Delta m/2.5} = 10^{0.48/2.5} \approx 1.556 \text{ times}.$$

The luminosity of a star L is proportional to its surface area and the fourth power of surface temperature, i.e. $L \sim R^2T^4$. The density of a star is equal to its mass divided by its volume, i.e. it is proportional to M/R^3. Thus, $\rho \sim MT^6/L^{3/2}$. Temperatures of α Centauri A and the Sun, according to the table, are 5810 K and 5777 K respectively. Thus, the ratio of the densities of Alpha Centauri A and the Sun is:

$$\rho_C/\rho_0 = (M_C/M_0)\cdot(T_C/T_0)^6\cdot(L_0/L_C)^{3/2}.$$

From the table of Solar System, $\rho_0 \approx 1409$ kg/m³, so the density of α Centauri A is

$$\rho_C = \rho_0\cdot(M_C/M_0)\cdot(T_C/T_0)^6\cdot(L_0/L_C)^{3/2} \approx 830 \text{ kg/m}^3.$$

9(βγ). Dark matter. According to the data from the table, the temperature of the photosphere of the Sun is approximately T_{sun} = 5777 K, and the stars of 61 Cygni are on average T_{61} = 4480 K. According to Wien's law

$$\lambda_{sun}T_{sun} = \lambda_{61}T_{61},$$

and the redshift of the galaxies

$$z = (\lambda_1 - \lambda_0) / \lambda_0 = (T_{sun} - T_{61}) / T_{61} = 1297 \text{ K} / 4480 \text{ K} \approx 0.29.$$

The speed at which the galaxy is moving away is equal to
According to Hubble's law, $v = HR$,
so $R = v/H = 87\,000$ km/s / 68 km/s/Mpc ≈ 1280 Mpc.
Thus, the distance to these galaxies is approximately 1280 megaparsecs.
From the difference in the apparent temperatures of the components, the relative velocity of the mutual motion of the galaxies can be calculated:

$$\Delta v = c \times (4520\text{ K} - 4440\text{ K}) / 4480\text{ K} \approx 5\,400\text{ km/s}.$$

The apparent distance between the galaxies perpendicular to the line of sight is

$$D = R\cdot\alpha = 1280\text{ Mpc} \times 9'' / 206265'' \approx 56\text{ kpc}.$$

Consider a situation (**) in which the minimum mass of a system of two galaxies is realized: the angular distance of 9" corresponds to the maximum distance between the galaxies (that is, the segment connecting the galaxies is perpendicular to the line of sight), and the line of sight lies in the orbital plane of motion of the galaxies (that is, the velocities of the galaxies are directed along the line of sight, the cosine of the corresponding angle is equal to 1).
From the generalized Kepler's Third Law

$$T^2/D^3 = 4\pi^2 / (G(M_1+M_2))$$

Thus, что

$$M_1+M_2 = 4\pi^2 D^3 / GT^2 = D(2\pi D/T)^2 / G = D(\Delta v)^2/G.$$

Substituting the previously obtained values into this formula (let's not forget to convert kilometers per second into meters per second, and kiloparsecs into meters, the coefficient is $3.09\cdot10^{19}$, we get:

$$M_1+M_2 = 7.6\cdot10^{44}\text{ kg} \approx 3.8\cdot10^{14}\ M_\odot.$$

Very massive galaxies!
Next, we calculate what magnitude a single Sun would have if it were observed from a distance of 1280 Mpc. For a stationary Sun:

$$m = m_{0\odot} + 5^m\cdot\lg(R/10\text{ Пк}) = 4{,}83^m + 5^m\cdot\lg 128\,000\,000 \approx 45{,}4^m.$$

The correction (increase) of the magnitude due to the receding will be

$$\Delta m = -2.5^m\cdot\lg(T_1/T_0) = -2.5^m\cdot\lg(4480\text{ K} / 5777\text{ K}) \approx 0.27^m.$$

Thus, the apparent magnitude of the receding Sun, located at a distance of 1280 Mpc, will be approximately equal to:

$$m_{R\odot} = m + \Delta m = 45.37^m + 0.27^m = 45.64^m \approx 45.6^m.$$

The apparent magnitude of $m_{AB} = 11{,}8^m$ indicates that we see the number of stars similar to the Sun equal to:

$$N = \exp_{10}((m_{R\odot} - m_{AB})/2.5^m) = \exp_{10}((45.6^m - 11.8^m)/2.5^m) = 3.3\cdot10^{13}.$$

Thus, the mass of visible matter in galaxies is $3.3\cdot10^{13}\ M_\odot$, and their total mass is $3.8\cdot10^{14}\ M_\odot$.
Percentage of visible matter:

$$\eta = 3.3\cdot10^{13}\ M_\odot / 3.8\cdot10^{14}\ M_\odot = 8.7\%,$$

and the remaining $1-\eta = 91.7\%$ is dark matter. Note that this percentage of visible matter is the maximum, since above (see **) we assumed conditions that give the minimum total mass of galaxies. Any configuration change will mean the appearance of the sines and cosines of the corresponding angles in the denominator of the final formula.

Examples of the solution that expected from students (without any text):

1(αγ). Parallax measurements.

$\pi_1 = 0{,}379''$, $\pi_2 = 1{,}379''$.

$\pi_2/\pi_1 = 1.379/0.379 = 3.64$.

$$(T_2/T_1)^2 = (a_2/a_1)^3,$$

[picture of circular orbit]

$$T_2 = T_1 \cdot (a_2/a_1)^{3/2} = T_1 \cdot (\pi_2/\pi_1)^{3/2} = T_1 \cdot (1.379/0.379)^{3/2} = 6.94 \text{ years.}$$

[picture with the parallactic bases L]:

L(max) – *[respective picture with major semi-axis of the orbit and the object is in a plane perpendicular to this major semi-axis]*;

L(min) – *[respective picture with minor semi-axis and the object is in the direction perpendicular to this minor semi-axis and in the plane of the orbit]*;

➔ $T_2 = T_1 \cdot (\pi_2/\pi_1)^{3/2} = 6{,}94$ years – minimum.

[picture of ellipse with major and minor semi-axis and triangle "focus – center – intersection point of the minor axis with the ellipse"]

$$b^2 + (ea)^2 = a^2b \quad \rightarrow \quad b/a = (1-e^2)^{1/2}.$$

$$a_3 = b/(1-e^2)^{1/2} = 3.64 \text{ au} / (0.6519)^{1/2} = 4{,}51 \text{ au.}$$

$T_3 = T_1 \cdot (a_3/a_1)^{3/2} = T_1 \cdot 4.51^{3/2} = 9.57$ years – maximum.

$T_2 \le T \le T_3$.

Answer: $6.94^Y \le T \le 9.57^Y$.

2(αγ). Spacecraft. $T = 2\tau = 19.28$ days.

$\alpha_P = 4{,}6°$, $\alpha_A = 1{,}9°$.

[picture of elliptical orbit with angles $\alpha_P = 4{,}6°$ and $\alpha_A = 1{,}9°$ in pericenter and apocenter].

$$a = (r/\sin(\alpha_P/2) + r/\sin(\alpha_A/2))/2 = ((1/\sin(\alpha_P/2) + 1/\sin(\alpha_A/2))/2)\, r = 42{,}6\, r.$$

$$v^2/R = 4\pi^2R/T^2,$$

$$GM/R^2 = 4\pi^2R/T^2 \quad \rightarrow \quad M = 4\pi^2R^3/GT^2.$$

$$V = 4\pi R^3/3.$$

$$\rho(\text{planet}) = M/V = (4\pi^2R^3/GT^2)/(4\pi r^3/3) = 3\pi(R/r)^3/GT^2 = 3\pi((1/\sin(\alpha_P/2) + 1/\sin(\alpha_A/2))/2)^3/G(2\tau)^2.$$

Note: $\sin(\alpha_P/2) \approx \alpha_P/2$, $\sin(\alpha_A/2) \approx \alpha_A/2 \quad \rightarrow \quad \rho = 3\pi(1/\alpha_P + 1/\alpha_A)^3/G(2\tau)^2$.

$$\rho = 3{,}94\cdot 10^3 \text{ кг/м}^3 \quad \rightarrow \quad \text{Mars.}$$

Answer: **Mars.**

3(αγ). NGC of the Year.

[picture of human pupil with mentioned size 6 mm]. 6 mm – 6^m.

$$12.1^m - 6^m = 6.1^m.$$

$$\exp_{10}(\Delta m/2.5) \approx 280,\ (280)^{1/2} \approx 17.$$

$$N_1 = (\exp_{10}((12{,}1^m - 6^m)/2.5^m))^{1/2} \approx 17.$$

[maybe other pictures].

$$N_2 = 45 \text{ mm} / 6 \text{ mm} = 7{,}5$$

$$N_2 < N_1.$$

Answer: a, b, c, d, e, f, g, h – “**No**”.

Below is the marking scheme (Basic Criteria) for the THEORETICAL ROUND.

Each questions carries 8 points. Total number of points for this round for the Beta Group students (Problems 6 to 9) is 48 points. Total number of points for the Gamma Group student (Problems 1 to 9) is 72 points.

Theoretical round. Basic criteria. For work of Jury

Note. Jury members should evaluate the student's solutions in essence, and not by looking on formal existence the mentioned sentences or formulae. The formal presence of the mentioned positions in the text is not necessary to give the respective points. Points should be done if the following steps de facto use these positions. (For example, except the final conclusion, all the points in solution of problem 3 may be in only one common formula.)

Note. Jury members should elaborate more detailed criteria, and also create criteria for other correct ways of the student's solutions.

Note. Everywhere "formula" means "correct formula", "calculation" means "correct calculation", "picture" means "correct picture", etc., and in all cases: based on correct facts and intermediate formulae or calculations.

1. Parallax measurements.

Understanding that the parallax base (a) 1.379/0.379 = 3.64 times larger – 1 pt.
Using Kepler's III Law $(T_2/T_1)^2 = (a_2/a_1)^3$ – 1 pt.
Calculation minimal period $T_2 = T_1 \cdot (a_2/a_1)^{3/2} = T_1 \cdot (\pi_2/\pi_1)^{3/2} = 6.94$ years – 1 pt.
Minor semi-axis $b = a(1-e^2)^{1/2}$ – 1 pt.
Calculation maximum major semi-axis $a_3 = b/(1-e^2)^{1/2} = 3.64$ au / $(0.6519)^{1/2} = 4{,}51$ au – 1 pt.
Formula and calculations $T_3 = T_1 \cdot (a_3/a_1)^{3/2} = T_1 \cdot 4.51^{3/2} = 9.57$ years – 1,5 pt.
Correct full final answer (not two values but range): $6.94^Y \le T \le 9.57^Y$ – 1,5 pt.

2. Spacecraft.

Understanding that calculation of the density necessary – 2 pt.
Finding orbit period $T = 2\tau = 19.28$ days – 0,5 pt.
Finding major semi-axis $a = ((1/\sin(\alpha_P/2) + 1/\sin(\alpha_A/2))/2)\ r = 42{,}6\ r$ – 1 pt.
Relation between density and period – 1 pt.
Final formula (or analogous) $\rho = 3\pi((1/\sin(\alpha_P/2) + 1/\sin(\alpha_A/2))/2)^3/G(2\tau)^2$ – 1,5 pt.
Final calculation $\rho = 3{,}94 \cdot 10^3$ кг/м^3 – 1 pt.
Final answer "Mars" (based on value of the density) – 1 pt.

3. NGC of the Year. For the solution, written in author's solutions:

Understanding the main limiting parameter for all situations is the diameter of the lens – 1,5 pt.
Difference in mag between NGC 2021 and the faintest star visible by humans – 1 pt.
Difference in fluxes of light $\exp_{10}(\Delta m/2.5) \approx 280$ – 1,5 pt.
Lens should be $N_1 = (\exp_{10}((12{,}1^m - 6^m)/2.5^m))^{1/2} \approx 17$ lager than human pupil – 1,5 pt.
Finding N_2 = 45 mm / 6 mm = 7,5 – 1 pt.
$N_2 < N_1$ – 0,5 pt.
Formal answer "No" for all the cases – 1 pt.

Note: there may be many analogous versions, other order of steps possible. For example, may be calculation the limiting lens diameter 17×6 mm ≈ 100 mm, etc.

Other types of solutions (alternative, not addition to author's solution, but instead):

If any other <u>correct</u> criteria were used for proving impossibility to observe NGC 2021 by any animal:
<u>Correct</u> final conclusion for each animal based on <u>correct base</u> – 0,5 pt.

4-5. Eclipse at the North Pole.

4.1. up to 4 points in total, number of points should be proportional to the part of progress in writing correct formulae and correct necessary calculations.

4.2. up to 2 points in total, including

exact correct size of the Sun – 0,5 pt,
exact correct size of the Moon – 0,5 pt,
exact correct displacement of lunar disk relative to solar disk – 0,5 pt,
quality of the picture – 0,5 pt.

4.3. up to 2 points in total, number of points should be proportional to the part of progress in writing correct formulae, correct necessary calculations, and the final result.

5.1. Artistic drawing – up to 2,5 points in total. including:
the Bear – up to 1 pt (depending on the quality of the picture),
photographic equipment – up to 0,5 pt (depending on the quality of the picture),
correct direction of the camera, approx. 23° to the horizon – 1 pt.

5.2. Up to 5,5 pt in total, including:
using solar constant, A = 1367 Вт/м^2 – 1 pt,
conclusion that the height of the Sun is equal to the declination of the Sun – 0,5 pt,
formula $\delta = \varepsilon \cdot \cos 12°$ – 0,5 pt,
calculation $\delta = 23.44° \cdot \cos 12° \approx 22.9°$ – 0,5 pt,
calculation of the area of visible ring relative to full solar disc $\cdot(\mathbf{d_\odot^2 - d_1^2})/\mathbf{d_0^2}$ – 1,5 pt,
final formula, calculations and result – 1,5 pt.

6. Closer to the stars.

Understanding the idea to compare closing to the stars and extinction by atmosphere – 2 pt.
Using value of one atmosphere (maybe approx.) – 1 pt.
Using value of extinction by the terrestrial atmosphere in zenith (maybe approx.) – 1 pt.
Intermediate formulae – up to 1 pt.
Intermediate calculations – up to 1 pt.
Final calculations and answer – 2 pt.

7. Variable star.

Calculation of speed of the star relative to the Earth $v_E = V_{orb}/c \times \cos\beta - v$ – up to 3,5 pt in total (including analyzing the sky map, correct using V_{orb}, correct using cosβ, correct direction of the speed of the Earth around the Sun, etc.)
Correct using Doppler effect – 1,5 pt.
Final correct calculations and answer – 2 pt.
Correct accuracy 1 second (not less ant more) – 1 pt.

Other solutions:
For any solution that does not take into account moving the Earth around the Sun – not more than 1,5 pt.

8. Alpha Centauri.

The distance to Alpha Centauri – 1 pt.
Finding the ratio of luminosities of Alpha Centauri and the Sun – 2 pt.
Using $L \sim R^2T^4$ – 1 pt.
Using $\rho \sim M/R^3$ – 1 pt.
Conclusion $\rho \sim MT^6/L^{3/2}$ – 1 pt.
Final calculations and result – 2 pt.

9. Dark matter.

Wien’s law – 0,5 pt.
Formula and calculations for redshift – 1 pt.
Hubble's law – 0,5 pt.
Formula and calculations for the distance to the galaxies – 1 pt.
Relative velocity of the mutual motion of the galaxies – 1 pt.
Apparent distance between the galaxies – 1 pt.
Generalized Kepler's Third Law – 0,5 pt.
Finding the total mass of the galaxies – 0,5 pt.
Finding the mass of the Sun-like stars in the galaxies (through magnitudes) – 1 pt.
Finding the ratio between mass of the Sun-like stars in total mass – 0,5 pt.
Final result and answer – 0,5 pt.