Theoretical round. Sketches for solutions. For jury job ONLY

Note. The given sketches are not full; the team leaders have to give more detailed explanations to students. But the correct solutions in the students' papers (enough for 8 pts) may be shorter.

Note. Jury members should evaluate the student's solutions in essence, and not by looking on formal existence the mentioned sentences or formulae. The formal presence of the mentioned positions in the text is not necessary to give the respective points. Points should be done if the following steps de facto using these positions.

αβ-1. Culmination of the Moon. The difference in the height of the culmination of the Moon appears from the fact that the Moon in its monthly movement moves approximately along the ecliptic (neglecting inclination angle of the orbit 5°09′), and different points of the ecliptic culminate at different heights (the same as the Sun culminates at different heights during the year). The highest culmination occurs when the Moon is at the highest point of the ecliptic (at the point where the Sun is on June 22).

The December's annular eclipse will occur 4 days after the day of the winter solstice. On the day of the winter solstice, the Sun is at the lowest point of the ecliptic, and 4 days later – about 4° east. This means that the last time at the highest point of the ecliptic, the Moon was 184° back, and in previous times it was more N·360° earlier. The moon moves along the ecliptic with a speed 360°/27.32 days. It is easy to understand without even calculating that in October at the highest point the Moon was 184° + 2·360° = 904° before the solar eclipse. In days:

$$904° / (360° / 27{,}32 \text{ day}) = 68{,}6 \text{ days.}$$

68.6 days before the morning of December 26 is October 18-19. Thus, the approximate answer to question 1.1 is "on October 18-19", but what date exactly, we can find out by answering the question 1.2.

The phases of the Moon change every 29.53 days. Thus, the second new moon before the eclipse occurs on December 26 – 59 days = October 28. So on October 18-19 we have 9-10 days before the new Moon. Culmination of the new Moon occurs at 12 o'clock local solar time, and culmination 9-10 days before the new Moon occurs about 7.3 – 8.1 hours earlier. Thus, the culmination will occur at 3.9 – 4.7 o'clock in the morning of local solar time or at the sixth hour in the morning according to the current UT+03 timezone in Romania. With this in mind, we get that the exact date of the highest culmination of the Moon is on October 19, although on October 18, it culminated at almost the same height.

1.1. 1.2. Answers: October 19, in the 6th hour in the morning.

1.3. $h = 90° - \varphi + \varepsilon = 43°04' + 23°26' = 66.5°$.

1.4. Picture.

α-2. NGC of the Year. Absolute magnitude of NGC 2019 is

$$M = m - 5^m \times \lg (50 \text{ кПк} / 10 \text{ Пк}) = 10.9^m - 18.5^m = -7.6^m.$$

Given that the absolute stellar magnitude of the Sun is 4.8^m, we find that the number of stars N in the cluster is related to the difference in absolute stellar magnitudes as

$$M_\odot - M = 2.5^m \times \lg N, \quad N = 10^{12.4/2.5} \approx 90\,000.$$

By comparing the cluster sizes in the image with the total image size of 4', it can be found that the angular diameter of the central region of the "sphere" is approximately 0,4'–0,5'. We take the value δ = 0,5' for the solution.

The physical diameter of the cluster is:

$$D = L \times \delta = 50\ \text{кПк} \times 0{,}5' / 3438' \approx 7{,}3\ \text{Пк}$$

$$M_{A2} - M = 2.5^m \times \lg N, \quad N = 10^{8.6/2.5} \approx 2\,800.$$

By comparing the cluster size in the image with the total image size of 4′, we can find that the angular diameter of the central region of the "sphere" is approximately 0.4′–0.5′. We may take the value δ = 0.5′ for the solution.

The physical diameter of the central part of the cluster is:

$$D = L \times \delta = 50\ \text{kpc} \times 0{,}5' / 3438' \approx 7.3\ \text{pc}$$

(where 3438 is the number of arc minutes in radians).

Thus, the density of stars in the cluster is equal to:

$$\rho = 90\,000 / (1/6\ \pi D^3) \approx 450 / \text{pc}^3.$$

Along the line of sight passing through the center of the cluster, the number of stars in the picture plane per 1 square parsec is:

$$n = 7.3\ \text{pc} \times 450 / \text{pc}^3 \approx 3300 / \text{pc}^2.$$

Therefore, the average distance between the images of stars in the picture plane is:

$$d = (1/n)^{1/2} \approx 0.017\ \text{pc}.$$

In order to distinguish the stars with the naked eye, it is necessary that:

1. Their apparent magnitude would be no more than 6^m.
2. The angular distance between the stars would be at least 50″.

The first condition gives a distance of no more than $S_1 = 10\ \text{pc} \times 10^{(6-4{,}8)/5} \approx 17$ pc.

The second condition. The angular distance 50″ for the length of 0.017 pc turns out at the distance $S_2 = 206265''/50'' \times 0{,}017\ \text{pc} \approx 70$ pc.

Thus, to fulfill both conditions, it is necessary to approach the cluster stars by a distance S = 17 pc. Given that this distance is comparable to the size of the cluster, and we need to distinguish any stars, for example, the nearest stars of the cluster, this distance is the distance to the near edge of the cluster. Accordingly, the distance to the center is X = S + D/2 ≈ 21 pc.

β-2. NGC of the Year. Absolute magnitude of NGC 2019 is

$$M = m - 5^m \times \lg (50\ \text{кПк} / 10\ \text{Пк}) = 10.9^m - 18.5^m = -7.6^m.$$

Compared to main sequence stars, white dwarfs make a very insignificant contribution to luminosity. We know color indexes of Vega and Altair (see the Table) and the position of these stars in the Hertzsprung-Russell diagram, by interpolation it can be obtained that the stars of the NGC 2019 cluster have a spectral class A2 and absolute magnitude of about $M_{A2} = +1^m$. Thus, the number of stars of the main sequence N in the cluster is related to the difference in absolute magnitudes as

$$M_{A2} - M = 2.5^m \times \lg N, \quad N = 10^{8.6/2.5} \approx 2\,800.$$ (1.5 points)

Since the percentage of white dwarfs is insignificant, we can assume that the total number of stars is also equal to 2800.

By comparing the cluster size in the image with the total image size of 4′, we can find that the angular diameter of the central region of the "sphere" is approximately 0.4′–0.5′. We may take the value δ = 0.5′ for the solution.

The physical diameter of the central part of the cluster is:

$$D = L \times \delta = 50\ \text{kpc} \times 0{,}5' / 3438' \approx 7.3\ \text{pc}$$ (0.5 point)

(where 3438 is the number of arc minutes in radians).

Thus, the density of stars in the cluster is equal to:

$$\rho = 2\,800 / (1/6\ \pi D^3) \approx 14 / \text{pc}^3.$$ (1 point)

Along the line of sight passing through the center of the cluster, the number of stars in the picture plane per 1 square parsec is:

$$n = 7.3\ \text{pc} \times 14 / \text{pc}^3 \approx 100 / \text{pc}^2.$$ (1 point)

Therefore, the average distance between the images of stars in the picture plane is:

$$d = (1/n)^{1/2} \approx 0.1\ \text{pc}.$$ (0.5 point)

In order to distinguish the white dwarfs with the naked eye, it is necessary that:

1. Their apparent magnitude of the white dwarfs would be no more than 6^m.
2. The angular distance between the stars (any stars) would be at least 50″.

The first condition gives a distance of no more than $S_1 = 10\ \text{pc} \times 10^{(6-11)/5} \approx 1$ pc. (1 point)

The second condition. The angular distance 50″ for the length of 0.1 pc turns out at the distance $S_2 = 206265''/50'' \times 0{,}1\ \text{pc} \approx 400$ pc. (1 point)

Thus, to fulfill both conditions, it is necessary to approach the cluster stars by a distance $S \approx 1$ pc. Given that this distance is comparable to the size of the cluster (and even less than it), and we need to distinguish any stars, for example, the nearest stars of the cluster, this distance is the distance to the near edge of the cluster. Accordingly, the distance to the center is $X = S + D/2 \approx 5$ pc. ~~(0.5 point)~~ (1 point) (0.5 point)

α-3. Sunset in Chukotka.

3.1. In the supplement table we find that the longitudes of Cape Dezhnev and the opposite point on the coast of Alaska (respectively 169°39' W and 166°40' W) differ by 2°59′ ≈ 3°. Since the Bear-Chukchi is located 3° west of the Bear-Eskimo, for him, all astronomical events associated with the rotation of the Earth occur 12 minutes later (1 degree in an hourly measure is 4 minutes). So, 12 minutes after the described moment, the Bear-Chukchi will also see the sunrise. That is, the night (twilight) for him will last only $\tau \approx 12$ minutes. Such a short duration of the Sun's stay under the horizon means that at the time of its lower culmination, the solar disk barely hides behind the horizon. Let us calculate the necessary value of the declination of the sun δ. If we take the radius of the Sun $\rho \approx 16'$, then we get that the "visible" height of the center of the Sun must satisfy the condition

$$h_{min} = -\rho = \varphi - 90° + \delta,$$

whence
$$\delta = 90° - \varphi - \rho = 23°39'.$$

However, we know that the declination of the Sun during a year does not exceed 23°26′. Does it mean that the situation is impossible?

However, possible. The fact is that in previous arguments refraction was not taken into account, which "raises" the visible position of the stars, at the horizon it is approximately $r = 35'$. If we take it into account, it turns out:

$$h_{min} = -\rho = \varphi - 90° + \delta + r,$$

whence
$$\delta = 90° - \varphi - \rho - r = 23°04'.$$

Thus, the case takes place on dates close to the day of the summer solstice. The approximate difference in days N between the desired dates and June 22 can be found from the ratio:

$$\delta = \varepsilon \cdot \cos(0.986° \cdot N),$$

where $\varepsilon = 23°26'$ is the angle of inclination of the ecliptic plane to the celestial equator, 0.986° is the coefficient for converting days into degrees (360°/365). We get

$$N = \arccos(23°04' / 23°26') / 0{,}986 \approx 10.3.$$

Thus, the observations occur 10 days before the summer solstice, or 10 days after, that is, about June 12 or July 2.

Note. The value of refraction depends on atmospheric pressure and temperature. The value of $r = 35'$ at sea level (observers are sitting on the shore) at a normal pressure of 760 mmHg is realized at a temperature of +6° C. This is a very likely temperature for the month of June in the Bering Strait.

3.2. On the same or different dates (local time for each the Bear) do the Bears-observers see sunrise and sunset? Based on so large difference in the time zones (UT+12 and UT–09, 21 zone difference), we can conclude that between them (that is, between Chukotka and Alaska) there is a line for changing dates. Indeed, the longer part of day Chukotka and Alaska live on different dates.

But consider our case. As we found out, the event takes place around local true midnight (or rather, for the Bear-Chukchi, $\tau/2 = 6$ minutes earlier, for the Bear-Eskimo, $\tau / 2 = 6$ minutes later).

For the Chukchi Bear, the mean solar midnight occurs

$$24^H \times (360° - 169°39')/360° = 12{,}69^H$$

earlier than on the Greenwich meridian, that is, at 11^H19^M Greenwich time, adding the difference $+12^H$, we get 23^H19^M of the same date as in Greenwich. The 6^M correction for sunrise and a possible correction by the equation of time never exceeding 16^M cannot change the date.

For the Eskimo Bear, the mean solar midnight occurs

$$24^H \times 166°40'/360° = 11.11^H$$

later than on the Greenwich meridian, that is, at 11^H07^M Greenwich time, subtracting the difference 09^H, we get 02^H07^M of the same date as in Greenwich. Here, the 6^M correction for sunset and a possible correction of the time equation all the more cannot change the date.

Conclusion: for both the Bear-Chukchi and the Bear-Eskimo, the event occurs on the same date as at that moment in Greenwich, that is, on the same date.

3.3. Picture.

β-3. UY Scuti.

3.1. In order to plot the position of the star on the Hertzsprung-Russell diagram, we need to know its absolute magnitude and temperature.

The absolute stellar magnitude of UY Scuti, if we consider it only by the light reaching us, is found from the data of its apparent magnitude and the distance that can be obtained by knowing parallax.

$$M = m + 5^m - 5^m \log D = m + 5^m + 5^m \log \boldsymbol{p}.$$

$$M_2 = 9.1^m + 5^m + 5^m \log(0.00034) = -3.25^m,$$

Given the correction for the fact that only 0.5% of the light passes through the envelope surrounding it, we get the true value of the absolute magnitude in the visible range:

$$M_1 = M_2 + 2.5^m \log(0.005) = -9^m.$$

This value should be used to find to plot the position of the star at the Hertzsprung-Russell diagram on a vertical scale. It also follows from this that the luminosity of UY Scuti in the visible range is greater than the solar one L_0 in

$$L_S / L_0 = 100^{(4.8+9)/5} \approx 330\ 000 \text{ times}.$$

On the other hand,

$$L_S / L_0 = (R_S/R_0)^2 (T_S/T_0)^4 = (V_S/V_0)^{2/3} (T_S/T_0)^4,$$

where from

$$T_S = T_0 \cdot (V_0/V_S)^{1/6} (L_S/L_0)^{1/4} = 5778 \text{ K} \times (5000000000)^{-1/6} (330000)^{1/4} \approx 3350 \text{ K},$$

Thus, the position of the star in the diagram should be plotted at the point with approximate coordinates [3350 K, –9^m].

3.2. The Sun radiates

$$W_0 = A \cdot 4\pi a^2 = 1367\ W/m^2 \times 4\pi (1.496 \cdot 10^{11}\ m)^2 = 3{,}84 \cdot 10^{26}\ W.$$

UY Scuti star emits ≈330,000 times more, that is,

$$W_S = 330000 \cdot 3.84 \cdot 10^{26}\ W \approx 1.26 \cdot 10^{32}\ W.$$

For this reason, per unit time, the UY Scuti loses mass

$$\Delta m_R/\Delta t = (\Delta E/c^2)/\Delta t = W_S/c^2 = 1.26 \cdot 10^{32} / (3 \cdot 10^8\ м/c)^2\ W \approx 1.4 \cdot 10^{15}\ kg/s.$$

The total mass loss per unit time is

$$\Delta m_0/\Delta t = (\Delta m_R/\Delta t)/0.0004 \approx 3.5 \cdot 10^{18}\ kg/s.$$

Over the year, UY Scuti is losing

$$\Delta m_Y = 3.5 \cdot 10^{18}\ kg/s \times 3600 \times 24 \times 365.25\ s \approx 1.1 \cdot 10^{26}\ kg.$$

For evaluation, we should assume that the rate of mass loss of the star will continue and it will exist until the complete loss of mass. In this case, the remaining life time of the UY Scuti will be

$$\Delta T = 8\ M_\odot / \Delta m_Y = 8 \cdot 2 \cdot 10^{30}\ kg / 1.1 \cdot 10^{26}\ kg \approx 1.5 \cdot 10^5\ years.$$

Only 150 thousand years. Very transient on a cosmic scale.

αβ-4. Comet particles. Let us consider, that at some point, when the comet is at a distance R from the Sun, as shown in Figure 4.1, comet particles with characteristic size ***d*** the mass ***m*** have detached from the comet, and their velocity with respect to the Sun being equal to the velocity of the comet, $\vec{v}$. Let us consider the shape of the particles as spherical with the radius $r = d/2$, and that they reflect all the light falling on them.

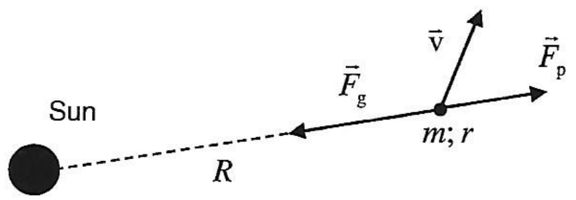

Fig. 4.1

The forces acting on the comet particle are: $\vec{F}_g$ – the gravitational force of the Sun; $\vec{F}_p$ – the rejection force due to the solar radiation pressure;

$$F_g = G\frac{mM_S}{R^2}.$$

Let us find formula for the force of solar radiation.

Let's admit, as in Figure 4.2, that the Sun is a sphere with Σ_0 surface, having R_S radius. It is called integral luminosity of Sun, L, the total radiation energy emitted per unit of time by Sun, on its entire surface, across all wavelengths in all directions ($L = A \cdot 4\pi \cdot (au)^2 = 3.86 \cdot 10^{26}$ W). Dimension of luminosity is a power.

If Σ is the circumferential sphere, whose radius R represents the instantaneous distance between the ice particle and the center of the Sun, then, obviously, the solar radiation energy per unit of time through Σ surface is equal to L.

If we use mirror reflection model of a disk, the solar radiation energy per unit of time which arrives on the surface πr^2 of the particle, is:

$$x = L \cdot \frac{\pi r^2}{4\pi R^2}.$$

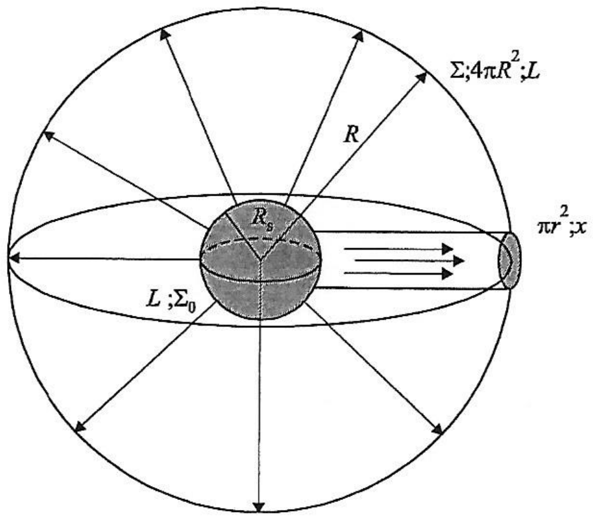

Fig. 4.2

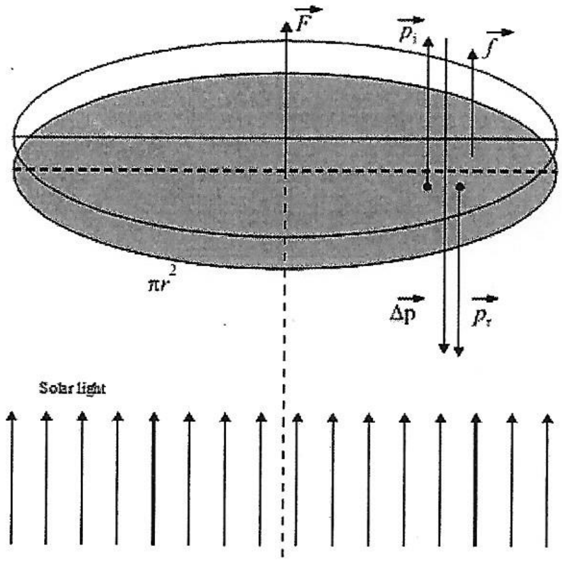

Fig. 4.3

As a result, the illumination of the surface of the circular disk (the solar radiation energy per unit of time, per unit area on the disk) is:

$$E = \frac{x}{\pi r^2} = \frac{L}{4\pi R^2};$$

$$\langle E \rangle_{SI} = \frac{W}{m^2}.$$

Using Figure 4.3, let's calculate the variation of photon momentum due to solar light reflection on the disk surface into a point:

$$\Delta\vec{p} = \vec{p}_r - \vec{p}_i;$$

$$\Delta p = p_r + p_i;$$

$$p_r = p_i = p_0 = \frac{h\nu}{c};$$

$$\Delta p = 2\frac{h\nu}{c},$$

where h – the Planck constant, ν – light frequency, and c – speed of light in vacuum.

As a result of principle of reciprocal action, on the disk will act a force:

If in Δt time on the disk surface is reflecting ΔN_k photons with ν_k frequency, then the force that will act on ice particle will be:

$$F_k = f_k \cdot \Delta N_k = \Delta N_k \cdot \frac{\Delta p_k}{\Delta t} = N_k \pi r^2 \cdot \Delta t \cdot \frac{2\frac{h\nu_k}{c}}{\Delta t},$$

where N_k is the photon number with ν_k frequency which arrive on the unit surface area of the disk in unit time;

$$F_k = 2 \cdot \frac{N_k h\nu_k}{c} \cdot \pi r^2;$$

$$\langle N_k h\nu_k \rangle_{SI} = \frac{W}{m^2};$$

$$N_k h\nu_k = E_k,$$

representing the illumination of the surface of the disk due to its component with ν_k frequency of the solar radiation;

$$F_k = 2\frac{E_k}{c}\pi r^2;$$

$$p_k = \frac{F_k}{\pi r^2} = 2\frac{E_k}{c},$$

representing the pressure exerted on disk by the light solar with ν_k frequency. According to all solar radiation components (ν_1, ν_2, ... ν_n), we calculate the resulting force acting on the disk:

$$F_\text{p} = \sum_{k=1}^{n} F_k = 2\frac{\sum_{k=1}^{n} E_k}{c}\pi r^2;$$

$$\sum_{k=1}^{n} E_k = E,$$

representing the total illumination of the disk due to all solar radiation components:

$$F = 2F_\text{p} = 2\cdot\frac{E}{c}\pi r^2;$$

$$p = \frac{F_\text{P}}{\pi r^2} = 2\frac{E}{c},$$

representing the solar light pressure on the ice particle. So for mirror plate:

$$F_\text{p} = pS = 2\frac{L}{4\pi R^2 c}\times\pi r^2 = \frac{Lr^2}{2cR^2}.$$

where: p – the solar radiation pressure, L – the Sun luminosity, and c – speed of light in vacuum, so that the result of the two collinear and opposite forces, acting on the ice particle is:

This issue is correct for the photons reflect just to back direction (back to the Sun). But this coefficient $k = 2$ is different for spherical shape of the ice particles. To calculate the average coefficient for income of all the directions of the photons reflected from the sphere of radius r, you should use integral:

$$\int_0^{\pi/2} 2\pi r\sin\alpha\cdot r\cos\alpha(1+\cos 2\alpha)d\alpha;$$

(where 1 is the part of momentum due to coming photons, and cos2α is the is the part due to reflecting photons);

$$\text{so}\quad k = \frac{\int_0^{\pi/2} 2\pi r\sin\alpha\cdot r\cos\alpha(1+\cos 2\alpha)d\alpha}{\pi r^2};$$

$$k = \int_0^{\pi/2} 2\sin\alpha\cdot\cos\alpha(1+\cos 2\alpha)d\alpha = \int_0^{\pi/2}\sin 2\alpha(1+\cos 2\alpha)d\alpha = \int_0^{\pi/2}(\sin 2\alpha+\sin 2\alpha\cdot\cos 2\alpha)d\alpha =$$

$$= \int_0^{\pi/2} (\sin 2\alpha) d\alpha + \frac{1}{2}\int_0^{\pi/2} (\sin 4\alpha) d\alpha = 1 + 0 = 1;$$

so for spherical shape the average coefficient is k = 1, and .

$$F_\text{p} = pS = \frac{L}{4\pi R^2 c} \times \pi r^2 = \frac{Lr^2}{4cR^2}.$$

The total force,

$$F = F_\text{g} - F_\text{p} = G\frac{mM_\text{S}}{R^2} - \frac{Lr^2}{4cR^2};$$

$$F = G\frac{m}{R^2}\left(M_\text{S} - \frac{Lr^2}{4cGm}\right),$$

which is a central force resulting from the gravitational interaction of the comet particle and an equivalent Sun, having an effective mass:

$$M_\text{S,effective} = M_\text{S} - \frac{Lr^2}{4cGm}.$$

Those two interactions (gravitational and electromagnetic) between the ice particle and the Sun, with M_S mass, can be replaced by a single interaction (gravitational) between the ice particle and an effective equivalent Sun, with mass, $M_\text{S,effective}$, as shown in Figure 4.4.

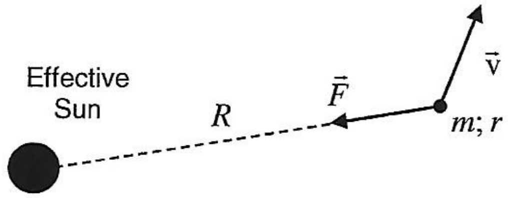

Fig. 4.4

Because $F < F_\text{g}$, the trajectory of the ice particle will no longer be the same as the comet trajectory. From that moment on, the ice particle is in the gravitational field of an effective Sun with a smaller mass, so that the particle trajectory may be an another ellipse, a parabola, or a hiperbolic, depending on the total energy of the particle – effective Sun system.

The total mechanical energy of the equivalent system, immediately after its formation (after detachment of the ice particle on the comet), is:

$$E = \frac{m\text{v}^2}{2} - G\frac{mM_\text{ef}}{R}.$$

Depending on the algebraic sign of *E*, the ice particle leaves the Solar System or not. In order for the ice particle not to leave the Solar System, the condition must be fulfilled:

$$E < 0,$$

so that, it follows:

$$\frac{m\text{v}^2}{2} - G\frac{m}{R}\left(M_\text{S} - \frac{Lr^2}{4cGm}\right) < 0.$$

Let's admit that the detachment of the ice particle from the comet occurred when the comet was at aphelion, where velocity is very small, so it follows:

$$v \approx 0;\ M_S - \frac{Lr^2}{4cGm} > 0;\ M_S > \frac{Lr^2}{4cGm};\ m = \rho \cdot \frac{4\pi r^3}{3};$$

$$r > \frac{3L}{16\pi c\rho GM_S}.$$

The value of luminosity L we may get from the solar constant A = 1367 W/m² and 1 au distance:

$$L = A \cdot 4\pi \cdot (au)^2;$$

$$\text{so} \quad r > \frac{3A \cdot (au)^2}{4c\rho GM_S}.$$

We may estimate the density of comet particles as the characteristic density of the ice matter in comets, about ~ 200-400 kg/m³. Let us take for calculation ρ = 300 kg/m³. Also we should take the values of speed of light in vacuum: c = 299 792 458 m/s, constant of gravitation: $G = 6.674 \cdot 10^{-11}$ N·m²/kg², and solar constant: A = 1367 W/m², from the table of constants; 1 astronomical unit: 1 au = 149.6 mln.km, and mass of the Sun: $M_S = 1.989 \cdot 10^{30}$ kg, from the table of Solar System. The calculations give us the critical radius,

$$r \approx 1.9 \cdot 10^{-6}\ \text{m}, \quad \text{so} \quad d = 2r \approx 3.8 \cdot 10^{-6}\ \text{m}.$$

As the minimum accuracy of the used data is 1 significant digit (we consider ρ = 300 kg/m³) the final answer should be done with 1 significant digit as well:

$$d > 4 \cdot 10^{-6}\ \text{m} = 4\ \mu\text{m}.$$

α-5. Interstellar comet.

5.1. Sure, we can. The entire constellation of Cassiopeia is circumpolar in Piatra Neamt. To confirm, you can find the height of Ruchbah at the lower culmination:

$$h = \varphi - 90° + \delta = 17°10'.$$

Thus, the star is always above the horizon, it can be observed at any clearly night.

5.2. According to the law of conservation of energy, the perihelion speed of the comet and its speed at infinity are related by the equation:

$$mV_P^2/2 - GMm/R_P = mV_\infty^2/2,$$

$$V_\infty^2 = V_P^2 - 2GM/R_P,$$

$$V_\infty = (V_P^2 - 2GM/R_P)^{1/2} = (43000^2 - 2 \cdot 6.67 \cdot 10^{-11} \cdot 2 \cdot 10^{30}/(2.01 \cdot 1.5 \cdot 10^{11})^{1/2} \approx 31\ \text{km/s}.$$

The comet makes almost its entire journey away from the stars. Therefore, most of the way (namely, relatively speaking, outside the Solar System) it traveled from the vicinity of the Ruchbah star to us with a speed $V_\infty \approx 31$ km/s relative to the Earth.

Let us find the distance to Ruchbah.

$$D = 1/p = 1/0{,}0328 \approx 30{,}5\ \text{Пк}.$$

It would seem that it is enough to calculate the speed V in units of pc/year ($3.2 \cdot 10^{-5}$ pc/year is obtained), divide 30.5 pc by this value and get an answer: approximately 960,000 years. Alas, we forgot that Ruchbah also moves relative to the Sun, as indicated by the radial velocity given in the table $V_r = -6{,}7$ km/s (the star is approaching us). Thus, this star is approaching us, earlier it was at a farther distance and it took more time to overcome the path to the Solar system. Relative Ruchbah, the comet moves at a speed

$$V_1 = V_\infty + V_r \approx 24.3\ \text{km/s},$$

Or in another units, about $3,2\cdot10^{-5}$ pc/year. Dividing 30.5 pc by this value, we get the answer: about 1.2 million years.

5.3. As we previously found out, the distance to Ruchbah is:

$$D = 1/p = 1/0.0328 \approx 30.5 \text{ pc.}$$

The sun at such a distance will have a magnitude

$$m_1 = m_0 + 5^m\log(D/R_0) = -26{,}8^m - 5^m\log(pR_0) = -26.8^m + 5^m\log(206265/0.0328) =$$
$$= -26.8^m + 34.0^m \approx 7.2^m.$$

Невооружённым глазом можно видеть только звёзды $7{,}2^m$ и ярче. Ответ: «**No**».
With the naked eye, only stars 7.2^m and brighter can be seen. The answer is «**No**».

β-5. Two satellites. See solution in the separate file.

Problem β5. Two satellites. Solution

5.1.

1) Satellite visibility S_1

According to the notations in Figure 5.2, results:

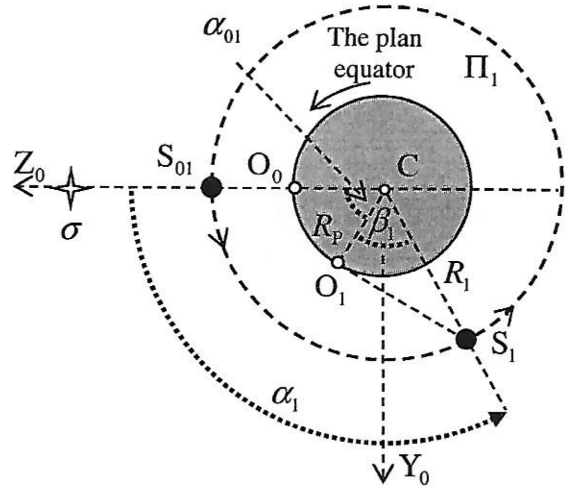

Fig. 5.2

$$\alpha_1 = \omega_1 t_1;\ \omega_1 = \sqrt{\frac{GM_P}{R_1^3}};\ R_1 = \sqrt[3]{\frac{GM_P}{\omega_1^2}};$$

$$\alpha_{01} = \omega_0 t_1;\ \cos\beta_1 = \frac{R_P}{R_1};\ \beta_1 = \arccos\left(\frac{R_P}{R_1}\right) = \arccos\left(R_P \cdot \sqrt[3]{\frac{\omega_1^2}{GM_P}}\right);$$

$$\alpha_1 = \alpha_{01} + \beta_1;$$

$$t_1(\omega_1 - \omega_0) = \beta_1;$$

$$t_1 = \frac{\beta_1}{\omega_1 - \omega_0};$$

$$t_1 = \frac{\arccos\left(R_P \cdot \sqrt[3]{\frac{\omega_1^2}{GM_P}}\right)}{\omega_1 - \omega_0},$$

being the duration of the satellite's visibility S_1.

2) Satellite visibility S_2

According to the notations in Figure 3, at the begining (initial moment), the observer, located in O_0, has direct visibility to the satellite in S_0. From this moment, due to Planet's rotation, the observer on Planet's equator has visual acces to the left space of the plane Π_P, tangent plane on Planet's surface at the point on the equator where the observer is. If the satellite is in this space, of course it will be visible. While the Π_P plane rotates, being solidary with the observer on the Planet's equator, the satellite „get down" into its orbit in Π_2 plane.

The two planes, Π_P and Π_2, intersects, the direction of this intersection (AD direction from Figure 3) are moving, being constantly parallel to the Planet's axis rotation. The dihedral angle between the two planes first decreases from the 90^0, corresponding to the initial moment, when the two planes are perpendicular, to the final value 0^0, when the two planes become parallel, so that this angle rises again until the two planes are again perpendicular, etc.

At some point in its evolution, the satellite dissapears from the observer's field of view. This occurs when the observer is in O_2 position, and the satellite is in S_2 position, located on the intersection axis of the two planes.

The t_2 represents the time in which S_2 satellite are visible, when satellite comes out from the observer's field of view, which is in O_2 position, and passes behind the Π_P plane, tangent to the Planet's surface in the point on the equator where the observer is located, but remaining in the plane of its orbit, Π_2.

Under these circumstances, from rectangular triangles ACS_2 and ACO_2, results:

$$AC = R_2 \cos\alpha_2;\ AC = \frac{R_P}{\cos\alpha_{02}};$$

$$R_2 \cos\alpha_2 = \frac{R_P}{\cos\alpha_{02}};$$

$$\cos\alpha_2 \cdot \cos\alpha_{02} = \frac{R_P}{R_2};\ \alpha_2 = \omega_2 t_2;\ \alpha_{02} = \omega_0 t_2;\ \omega_2 = \sqrt{\frac{GM_P}{R_2^3}};\ R_2 = \sqrt[3]{\frac{GM_P}{\omega_2^2}};$$

$$\cos\omega_2 t_2 \cdot \cos\omega_0 t_2 = \frac{R_P}{R_2},$$

$$\cos\omega_2 t_2 \cdot \cos\omega_0 t_2 = R_P \cdot \sqrt[3]{\frac{\omega_2^2}{GM_P}},$$

where t_2 is the duration of the satellite's visibility S_2;

$$R_P = \sqrt[3]{\frac{GM_P}{\omega_2^2}} \cdot \cos\omega_2 t_2 \cdot \cos\omega_0 t_2.$$

Results:

$$t_1 = \frac{\arccos\left(R_P \cdot \sqrt[3]{\frac{\omega_1^2}{GM_P}}\right)}{\omega_1 - \omega_0};$$

$$t_1 = \frac{\arccos\left(\sqrt[3]{\frac{\omega_1^2}{GM_P}} \cdot \sqrt[3]{\frac{GM_P}{\omega_2^2}} \cdot \cos\omega_2 t_2 \cdot \cos\omega_0 t_2\right)}{\omega_1 - \omega_0};$$

$$t_1 = \frac{\arccos\left(\sqrt[3]{\frac{\omega_1^2}{\omega_2^2}} \cdot \cos\omega_2 t_2 \cdot \cos\omega_0 t_2\right)}{\omega_1 - \omega_0},$$

being the duration of the satellite's visibility S_1.

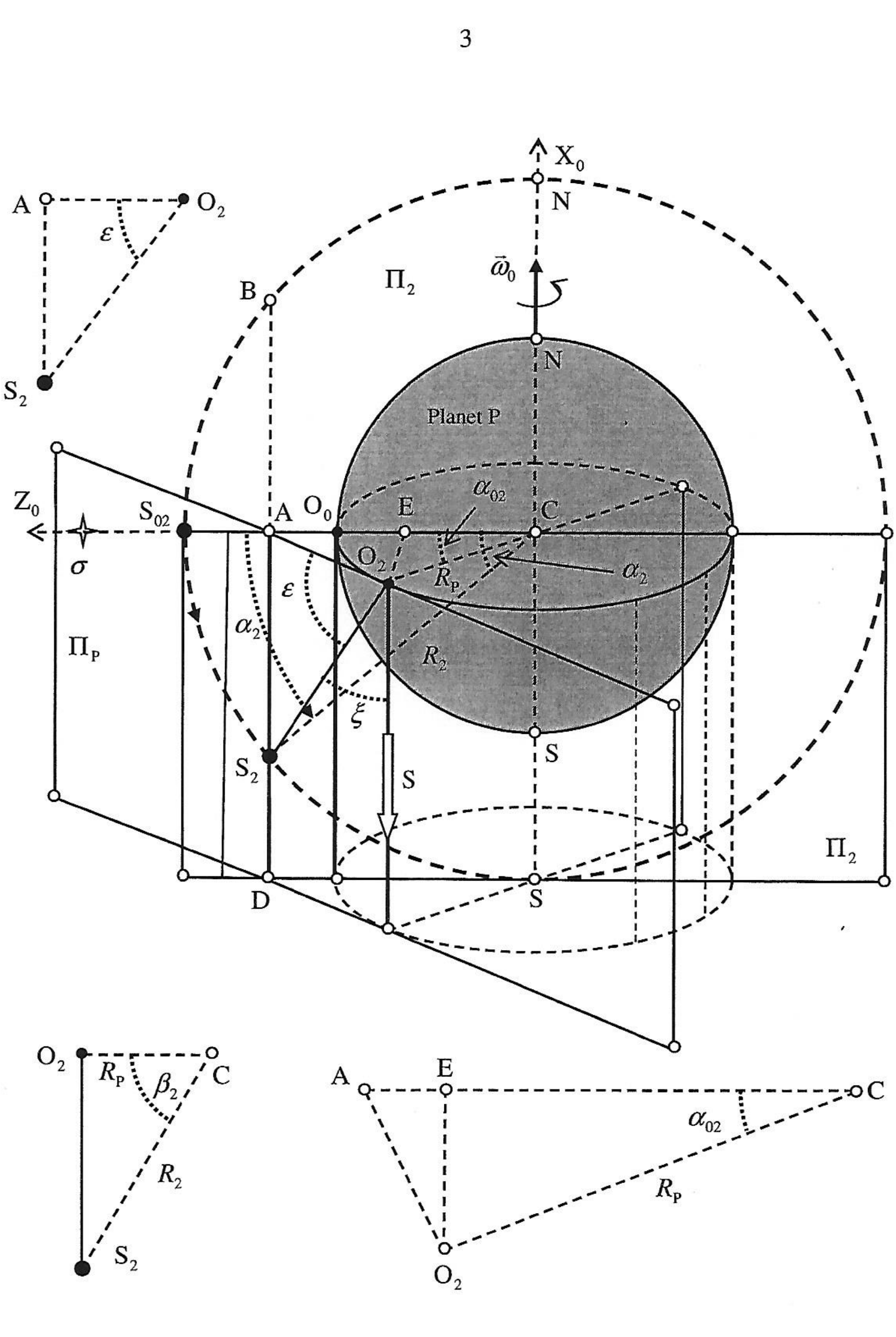

Fig. 3

5.2.

From rectangular triangles AO_2S_2, CO_2S_2 and ACO_2, in Figure 5.3, results:

$$\cos\varepsilon = \frac{AO_2}{O_2S_2};\ \sin\beta_2 = \frac{O_2S_2}{R_2};\ \tan\alpha_{02} = \frac{AO_2}{R_P};$$

$$O_2S_2 = R_2\sin\beta_2;\ AO_2 = R_P\tan\alpha_{02};$$

$$\cos\varepsilon = \frac{AO_2}{O_2S_2};$$

$$\cos\varepsilon = \frac{R_P \tan\alpha_{02}}{R_2 \sin\beta_2};$$

$$O_2S_2 = \sqrt{(CS_2)^2 - (CO_2)^2} = \sqrt{R_2^2 - R_P^2} = R_2\sqrt{1-\left(\frac{R_P}{R_2}\right)^2};$$

$$\sin\beta_2 = \frac{O_2S_2}{R_2};\ \sin\beta_2 = \sqrt{1-\left(\frac{R_P}{R_2}\right)^2};\ \alpha_{02} = \omega_0 t_2;$$

$$\cos\varepsilon = \frac{R_P \tan\alpha_{02}}{R_2 \sin\beta_2};\ \alpha_{02} = \omega_0 t_2;$$

$$\cos\varepsilon = \frac{R_P}{R_2}\cdot\frac{\tan\omega_0 t_2}{\sqrt{1-\left(\frac{R_P}{R_2}\right)^2}};$$

$$R_P = \sqrt[3]{\frac{GM_P}{\omega_2^2}}\cdot\cos\omega_2 t_2\cdot\cos\omega_0 t_2;$$

$$\frac{R_P}{\sqrt[3]{GM_P}} = \frac{1}{\sqrt[3]{\omega_2^2}}\cdot\cos\omega_2 t_2\cdot\cos\omega_0 t_2;$$

$$R_2 = \sqrt[3]{\frac{GM_P}{\omega_2^2}};\ \sqrt[3]{GM_P} = R_2\cdot\sqrt[3]{\omega_2^2};$$

$$\frac{R_P}{R_2\cdot\sqrt[3]{\omega_2^2}} = \frac{1}{\sqrt[3]{\omega_2^2}}\cdot\cos\omega_2 t_2\cdot\cos\omega_0 t_2;$$

$$\frac{R_P}{R_2} = \cos\omega_2 t_2\cdot\cos\omega_0 t_2;$$

$$\cos\varepsilon = \frac{R_P}{R_2}\cdot\frac{\tan\omega_0 t_2}{\sqrt{1-\left(\frac{R_P}{R_2}\right)^2}};$$

$$\cos\varepsilon = \frac{\cos\omega_2 t_2\cdot\cos\omega_0 t_2\cdot\tan\omega_0 t_2}{\sqrt{1-(\cos\omega_2 t_2\cdot\cos\omega_0 t_2)^2}};$$

$$\varepsilon = 90^0 - \xi,$$

where ξ is the azimuth of the satellite when it disappears from the field of view of the observer;

$$\cos\left(90^0 - \xi\right) = \frac{\cos\omega_2 t_2\cdot\cos\omega_0 t_2\cdot\tan\omega_0 t_2}{\sqrt{1-(\cos\omega_2 t_2\cdot\cos\omega_0 t_2)^2}};$$

$$\sin\xi = \frac{\cos\omega_2 t_2\cdot\cos\omega_0 t_2\cdot\tan\omega_0 t_2}{\sqrt{1-(\cos\omega_2 t_2\cdot\cos\omega_0 t_2)^2}}.$$

Theoretical round. Basic criteria. For work of Jury

Note. The given sketches of solutions are not full; the team leaders have to give more detailed explanations to students. The correct solutions in the students' papers (enough for 8 pts) may be shorter.

Note. Jury members should evaluate the student's solutions in essence, and not by looking on formal existence the mentioned sentences or formulae. The formal presence of the mentioned positions in the text is not necessary to give the respective points. Points should be done if the following steps de facto using these positions.

Note. Jury members should elaborate more detailed criteria, and also create criteria for other correct ways of the student's solutions.

αβ-1. Culmination of the Moon.

1.1. Using idea about declination of the Moon during month – 1 pt.
Correct using reference point (like solstice for lowest point of the ecliptic) – 1 pt.
Using the necessary period and phases of the Moon, calculations – 1 pt.
Corect final answer – 1 pt.

1.2. Conclusions and result – 2 pt.

1.3. Formula – 0,5 pt.
Result – 0,5 pt.

1.4. Artistic picture of the Bear-astronomer – 1 pt.

αβ-2. NGC of the Year. (The same criteria for α and β)

Number of stars in cluster – 1,5 pt.
Size of the cluster – 0.5 pt.
Distance to the cluster – 0.5 pt.
The density of stars (3D) – 1 pt.
The number of stars in the picture plane per square parsec (or similar) – 1 pt.
Distance, enough to see the (sunlike for **α** and white dwarfs for **β**) stars as 6^m (criterion 1) – 1 pt.
Distance to resolve (separate) two stars (criterion 2) – 1 pt.
The answer using the both criteria – 1 pt.
Using (adding) half of size of the cluster – 0.5 pt.

Additionally:

- Wrong number of significant digits in the answer: α – minus 0.5 pt, β – minus 1 pt.

α-3. Sunset in Chukotka.

3.1. Explanations and conclusion: sunset and sunrise near lower culmination – 2,5 pt.
Calculation lower culmination height at summer solstice – 0.5 pt.
Using refraction – 1 pt.
Calculation the possible dates – 2 pt.

3.2. Conclusions and result – 1 pt.

3.3. Artistic picture of the Bears-astronomers – 1 pt.

β-3. UY Scuti.

3.1. Understanding, which parameters are necessary – 1 pt.
Formulae and calculation the absolute magnitude – 1,5 pt.
Formulae and calculation the temperature – 3.5 pt.

3.2. 2 pt.
Additionally:
- Wrong number of significant digits in the answer – minus 0.5 pt.

α-4. Comet particles.

Comparing the gravitational force and force due to solar radiation – 2.5 pt.
Correct taking the value of density of comet substance – 1 pt.
Final result – 2.5 pt.
Additionally for the method:
- If the student make calculation after each point – 0 pt.
- If the student first derive the algebraic formula of the answer and only then get the numerical answer by inserting the numerical data into this formula +2 pt.

Additionally:
- Wrong number of significant digits in the answer: minus 0.5 pt.

β-4. Comet particles.

Formula for the force (or pressure) due to solar radiation – 2 pt.
Comparing the gravitational force and force due to solar radiation – 1.5 pt.
Correct taking the value of density of comet substance – 0.5 pt.
Final result – 2 pt.
Additionally for the method:
- If the student make calculation after each point – 0 pt.
- If the student first derive the algebraic formula of the answer and only then get the numerical answer by inserting the numerical data into this formula: +2 pt.

Additionally:
- Wrong number of significant digits in the answer: minus 1 pt.

α-5. Interstellar comet.

5.1. 1 pt.

5.2. Formula and calculation the velocity outside Solar System – 2 pt.
Formula and calculation the distance to Ruchbah – 1 pt.
Using the radial velocity of Ruchbah – 1 pt.
Calculation and the final answer – 1.5 pt.
Additionally:
- Wrong number of significant digits in the answer – minus 0.5 pt.

5.3. 1,5 pt.

β-5. Two satellites.

5.1. Formula for t_1 dependence on ω_1, ω_0, R_P, G, M_P – 2 pt.
Formula R_P dependence on t_2, ω_2, ω_0, G, M_P – 2 pt.
Formula t_1 dependence on t_2, ω_2, ω_1, ω_0 – 1 pt.

5.2. up to 3 pt (depending on progress in solution).