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Practical round

6. Comet observer. Table 1 shows 20 observations (ephemerides) of the comet P/2007 R2 (Gibbs), taken in equal time intervals of 121 days and 18 hours, so that there are 3 measurements per year. The orbit's inclination angle is $i = 1.4339^\circ$ and can be neglected. The following notations are used:

- N – number of observation.
- Date (UT) – date of observation, HR and MN are hour and minutes
- R. A. (hh:mm:ss) and DEC (deg:mm:ss) – right ascension and declination of comet's centre, respectively.
- T-mag – the comet's approximate apparent total magnitude.
- Delta – the distance of comet's centre with respect to the observer at the observation moment, measured in au (astronomical units)
- S-O-C (Sun-Observer-Comet) – Sun-Observer-Comet angle in degrees (the comet's apparent *solar elongation,* in the range 0º – 180º).
- Column /r – the comet's apparent position relative to the Sun in the observer's sky.

  /T indicates that the comet *trails* the Sun (the comet rises and sets *after* the Sun),

  /L indicates that the comet *leads* the Sun (the comet rises and sets *before* the Sun), as shown on Figure 1.

Fig. 1: *The Earth's orbit around the Sun, as seen from the north ecliptic pole. The direction of Earth's orbital motion is shown with an arrow.*

Use the data from the table to:

6.1. Draw the comet's orbit on the provided graph paper as seen from the north ecliptic pole. Denote the points (comet's positions) with their relevant numbers ***N***. Assume that the Earth moves with constant orbital speed.

6.2. Calculate the values of the orbit's semi-major axis a and eccentricity e.

6.3. Without using any additional data (such as the mass of the Sun or Kepler's Third Law), estimate the comet's orbital period T.

6.4. Without using any additional data (such as the mass of the Sun), calculate the orbital speed of the comet in perihelion v_p and aphelion v_a.

6.5. Calculate the solar mass M. The gravitational constant is $G = 6.67 \cdot 10^{-11}\ N \cdot m^2/kg^2$.

6.6. Calculate the orbital velocity v and the escape velocity v_e at the position with $N = 7$.

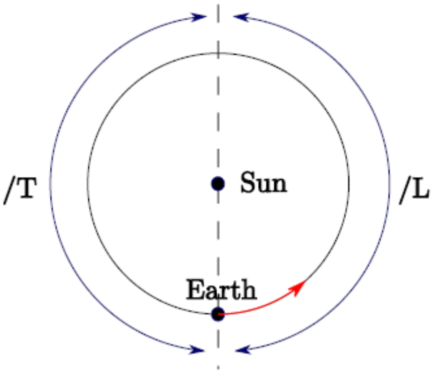

Fig.1. **Рис.1.**

N	Date (UT)	HR:MN	R. A.			DEC			T-mag	Delta	S-O-C	/r
1	2004-09-30	00:00	12	26	37.40	-02	35	00.4	33.12	6.485	0.271	/T
2	2005-01-29	18:00	13	28	32.58	-09	15	06.5	32.32	4.967	105.966	/L
3	2005-05-31	12:00	12	38	34.29	-04	26	57.7	31.77	4.493	120.555	/T
4	2005-09-30	06:00	13	29	55.02	-09	49	15.1	31.82	5.715	17.327	/T
5	2006-01-30	00:00	15	07	52.88	-18	16	04.6	30.60	4.419	80.257	/L
6	2006-05-31	18:00	14	28	41.00	-15	54	56.7	28.80	2.940	149.891	/T
7	2006-09-30	12:00	15	08	57.98	-18	38	54.3	28.09	3.884	42.866	/T
8	2007-01-30	06:00	18	31	38.72	-24	18	09.1	25.77	3.252	32.669	/L
9	2007-06-01	00:00	23	02	49.26	-07	42	38.0	21.35	1.464	86.189	/L
10	2007-09-30	18:00	03	31	15.59	+20	03	36.3	18.62	0.649	131.671	/L
11	2008-01-30	12:00	03	43	46.19	+21	38	57.9	23.18	1.616	108.756	/T
12	2008-05-31	06:00	06	55	11.41	+23	55	34.8	27.27	3.724	32.564	/T
13	2008-09-30	00:00	09	38	45.54	+15	09	48.2	29.09	4.234	45.116	/L
14	2009-01-29	18:00	09	57	24.96	+13	53	18.2	29.55	3.204	163.127	/L
15	2009-05-31	12:00	09	31	40.74	+15	30	19.4	31.20	4.845	70.213	/T
16	2009-09-30	06:00	11	04	18.66	+06	35	02.1	32.13	5.854	22.454	/L
17	2010-01-30	00:00	11	42	42.32	+02	32	28.1	31.90	4.447	134.801	/L
18	2010-05-31	18:00	10	57	35.25	+07	03	49.9	32.45	5.196	92.885	/T
19	2010-09-30	12:00	12	04	42.28	-00	18	36.2	33.00	6.407	5.875	/L
20	2011-01-30	06:00	12	58	53.76	-06	12	50.7	32.40	4.920	113.868	/L

Table 1. **Таблица 1.**

Practical round

6. The first gravitational wave detection. In 1916, the year after the final formulation of the equations of general relativity, Albert Einstein predicted the existence of gravitational waves. They are emitted by accelerating masses in a way similar to the emission of electromagnetic waves by accelerating charges. In the case of gravity, like electromagnetism,.when a stationary mass moves suddenly then the gravitational force exerted by it on the test masses changes. However, that change will not happen immediately. Instead, the information that the mass moved will propagate at the speed of light and will take the form of gravitational radiation. When the gravitational wave reaches the test masses, those test masses accelerate, because the gravitational force suddenly changes. For a mass undergoing periodic motion, the gravitational waves it produces will be periodic, and those can accelerate surrounding test masses periodically.

On September 14, 2015, the two detectors of the Laser Interferometer Gravitational-Wave Observatory (LIGO) detected the coincident signal GW150914 as shown in Fig. 1. It was the first direct detection of gravitational waves and the first direct observation of a binary black hole system merging to form a single black hole. The distance to the merging black hole binary system was determined to lie somewhere between 250 and 600 Mpc, and the two initial black hole masses as being 36±4 M$\odot$ and 28±4 M$\odot$, respectively. When those two black holes merged, they formed a single black hole of mass 62±4M$\odot$. The missing mass was emitted as energy in gravitational radiation, which was calculated to be 3.0±0.5 M$\odot c^2$. As usual, 1 M$\odot$ is one solar mass equal to 2.0×10^{30}kg, and c = 3.0×10^{8} m/s is the speed of light in vacuum. The peak emitted power of gravitational radiation was calculated at several times of 10^{49} W – more than 10 times greater than the combined power of all light radiated by all the stars in the observable universe.

Due to the tidal forces gravitational waves cause relative displacements (ΔL) in the test masses that are proportional to the distance (L) between those test masses. Physicists define the *amplitude* of gravitational waves by using the *dimensionless* quantity h, defined as

$$h = \Delta L/L$$

which is usually called the dimensionless strain of the gravitational wave. This is the quantity plotted on the vertical axis of Fig.1.

Practical round

6. The first gravitational wave detection. Continue.

Fig. 1. *The gravitational-wave event GW150914 observed by the two LIGO detectors. The vertical axis shows the amplitude of the gravitational wave (the strain, h). The black holes go through the following stages (images placed along the horizontal axis at their roughly corresponding times):*
(1) the inspiral, as the two black holes approach each other;
(2) the merger, as the black holes join together; and
(3) the ringdown, as the single black hole that has newly formed briefly oscillates before settling down.

Using a Newtonian (classical) mechanics approximation do the following tasks:

6.1. Using the masses quoted by the LIGO team, calculate the radii of the horizons of the two black holes before they merged. Include an estimate of the error for the radii..

6.2. From the measurements of the gravitational wave signal presented in the graph above, find the orbital period of the black hole binary at the moment of the merger.

6.3. Estimate the total mass of the initial black hole binary. Use ***only*** information of the observed gravitational radiation from the graph. Do ***not*** use any of the other LIGO results quoted above.

6.4. Find an expression for the energy released in gravitational waves until the time of the black hole merger in terms of the initial black hole masses (M_1 and M_2) only. Using the result from 6.3., calculate the total released gravitational energy if you assume $M_1 = M_2$.

6.5. Find the average power P of gravitational wave emission during the last 0.10s before the merger.

6.6. As with most waves, the gravitational wave flux is proportional to the square of the amplitude of the wave. The amplitude (strain) right next to the two merging black holes is approximately $h = (v/c)^2$, where $\boldsymbol{v}$ is the orbital velocity of the black holes. Estimate the distance to the merging black holes detected by LIGO.

6.7. The LISA experiment aims to measure gravitational waves from colliding supermassive black holes at high cosmological redshift. Estimate the required sensitivity of LISA in terms of the dimensionless strain.

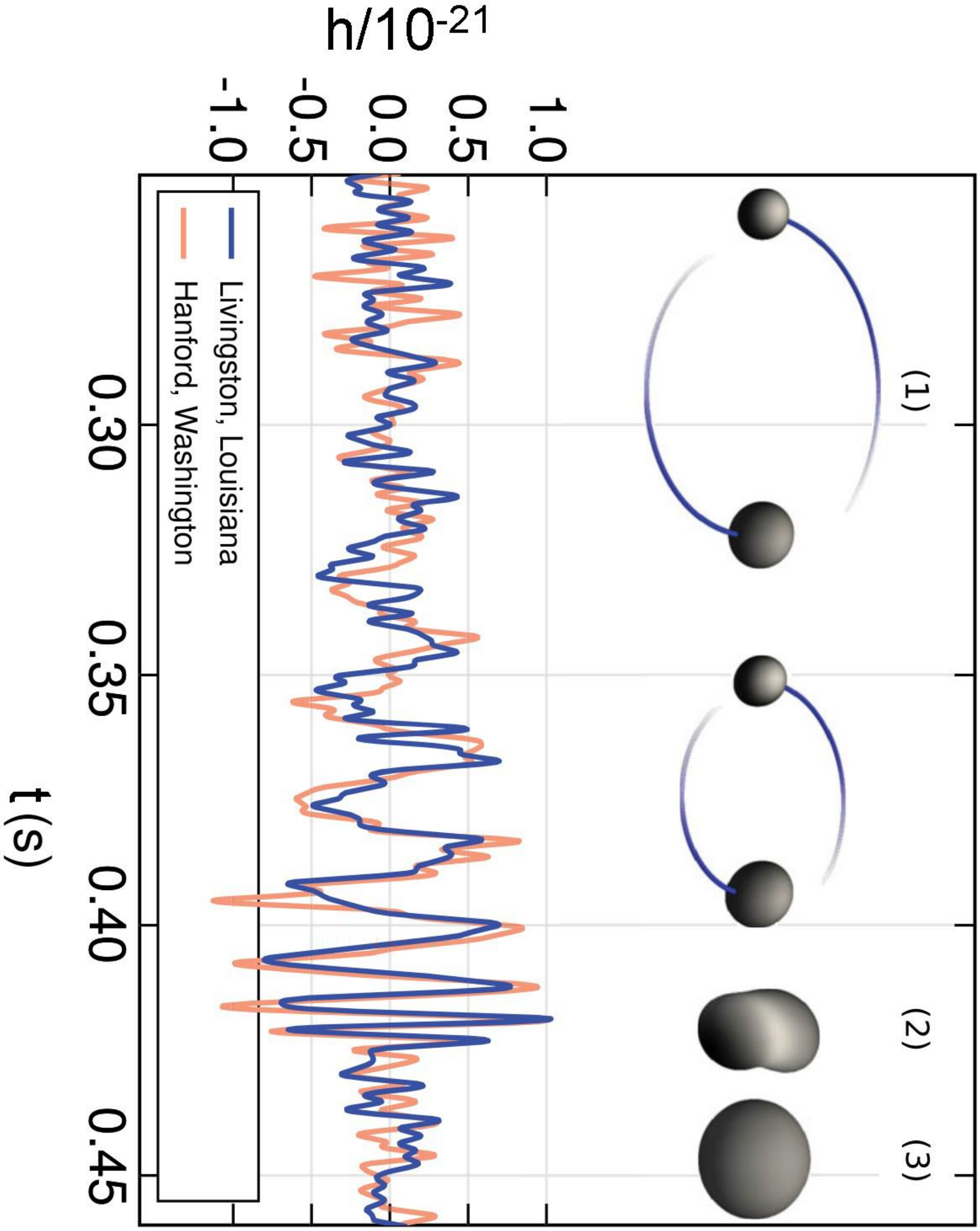
h/10^{-21}
-1.0
-0.5
0.0
0.5
1.0
Livingston, Louisiana
Hanford, Washington
0.30
0.35
0.40
0.45
t(s)
(1)
(2)
(3)

Fig.1

Practical round

7. **The Initial Mass Function and supernovae.** The evolution of a single star depends solely on its mass, therefore mass is the most important parameter of stars. It is believed that the distribution of stars by mass at the time of their birth (also known as Initial Mass Function, IMF) is universal.

Figure 2. *The Initial Mass Function (IMF) in logarithmic scale, according to two different models. The observational data is shown in equally sized bins, with error bars. The y-axis shows the relative number of stars (Δn/ΔlgM) with a given mass.*

7.1. The star formation rate in our galaxy is $\Delta M/\Delta t = 8\ M_\odot$ / year. Stars born with mass greater than $8M_\odot$ explode as core-collapse supernovae. Estimate the core-collapse supernova rate in the galaxy. (Or: how often do core-collapse supernovae explode in our galaxy?)

Hint: *What is the average initial mass* $\mathbf{M_{SN}}$ *of a core-collapse supernova? What fraction* $\mathbf{q}$ *of the total star formation mass goes into supernovae (0<q<1)? Find the answers to these questions by doing measurements on the IMF figure (Fig.2).*

7.2. What is the frequency f [yr^{-1}] of directly observed supernovae in the Milky Way? If there is a significant difference to your result from 7.1., provide an explanation by drawing schemes and, if necessary, by including a very short text (not more than 20 words).

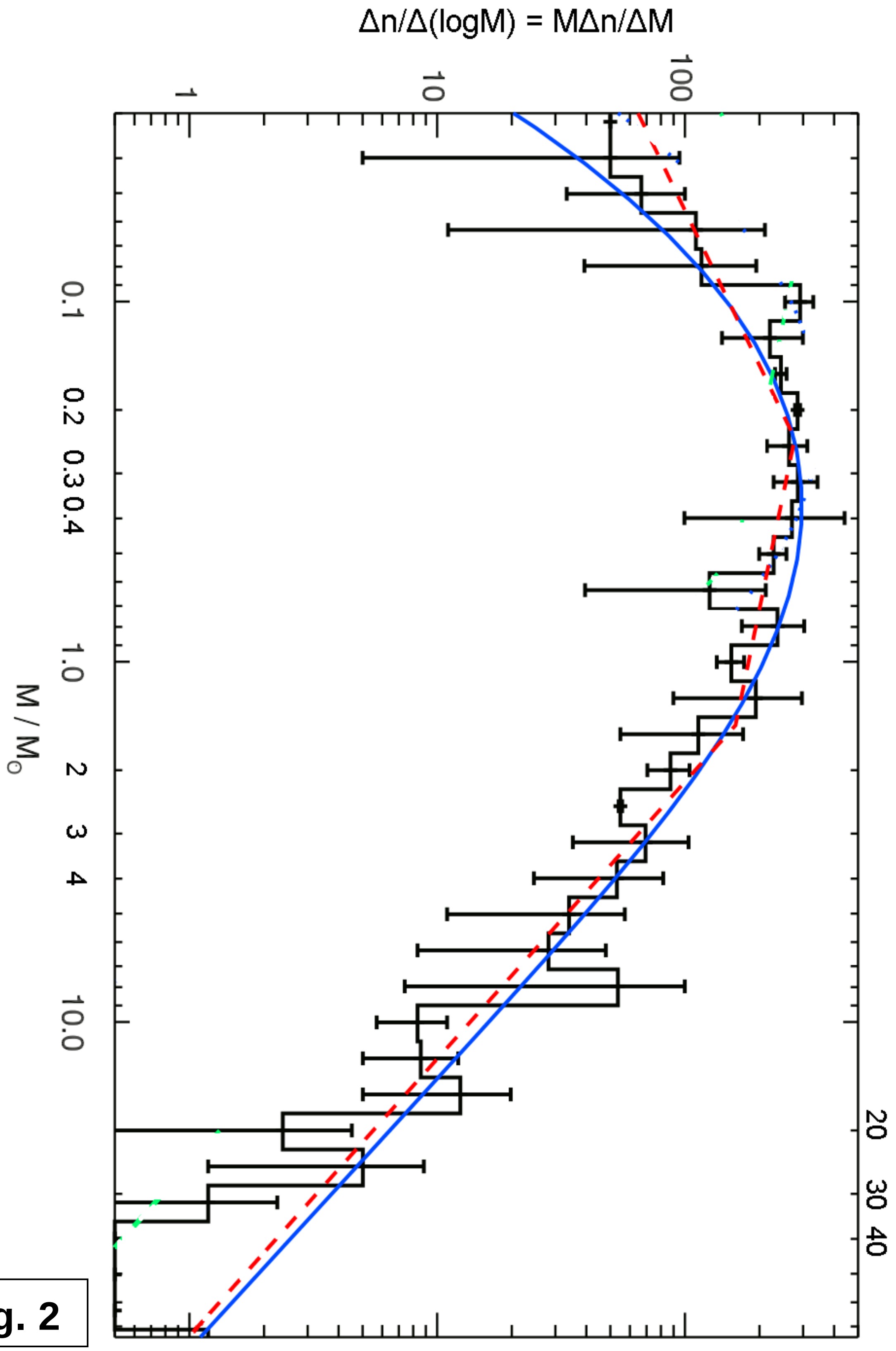

Fig. 2