АСТРОНОМИЧЕСКОЕ ОБЩЕСТВО | EURO–ASIAN ASTRONOMICAL SOCIETY

Round **Theo**

Group **α** **β**

XIV Международная астрономическая олимпиада
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Theoretical round. Sketches for solutions

Note for jury and team leaders. The proposed sketches are not full; the team leaders have to give more detailed explanations for students. But the correct solutions in the students' papers (enough for 8 pts) may be shorter.

αβ-1. Sirius. Sirius is the brightest star (in historical-classical meaning of a star) on the Earth's sky. Therefore in a first approximation it will be the brightest star in those districts on the Earth where it is visible, that is, at least sometimes appears over horizon. As the declination of Sirius is $\delta = -16°43'$, it will be visible in all southern hemisphere, and also in northern hemisphere at latitudes not nearer 16°43' to the pole, that is, at latitudes not higher 73°17' North. Theoretically, in the second approximation refraction may be taken into account (35' at horizon). With the refraction Sirius can be above horizon at districts till latitudes of 73°52' North. But refraction doesn't matter for our goal. There is a talk about real (not theoretical) sky in the text. In the third approximation light absorption should be taken into account. It is obvious that Vega or Arcturus in high sky essentially are more bright than Sirius at horizon. Heights on which Sirius becomes weaker than Vega or Arcturus may be roughly estimated as ~5÷8°. Thus, Sirius is the brightest star in the sky for districts to the south from the latitudes of 65°÷ 69° North.

Note for jury: the solution without a correct mention of refraction (that is, without mentions that it not effected here) should be estimated with a deduction of 0.5 points. Values ~5÷8° are approximate, it is important the understanding of the effect and a correct order of this height.

αβ-2. Number of molecules. The pressure of an atmosphere is the total weight of all number (N) of its molecules, distributed over the whole surface of a planet, that is,

$$P = N \cdot m \cdot g / S,$$

where m is the mean value of mass of molecules in atmosphere. Taking into account that the surface of the Earth is

$$S = 4\pi R_E^2$$

and mass of molecule is

$$m = \mu / N_A,$$

where N_A is Avogadro number, we can write

$$N = N_A \cdot P \cdot 4\pi R_E^2 / \mu \cdot g$$

Since the Earth atmosphere consists mostly on N_2 ($\mu_{N2} = 28$ g/mol) and O_2 ($\mu_{O2} = 32$ g/mol) in approximate proportion 3:1, the mean value of μ is 29 g/mol or in SI: $2.9 \cdot 10^{-2}$ kg/mol. Calculations:

$$N = 6.022 \cdot 10^{23}\ \text{mol}^{-1} \cdot 10^5\ \text{N} \cdot \text{m}^{-2} \cdot 12.6 \cdot (6.37\ \text{m} \cdot 10^6)^2 / 2.9 \cdot 10^{-2}\ \text{kg} \cdot \text{mol}^{-1} \cdot 9.81\ \text{N} \cdot \text{kg}^{-1} = 1.08 \cdot 10^{44} \approx 1.1 \cdot 10^{44}.$$

α-3. Efficiency of eye. For detecting a star the human retina should uninterruptedly remember the image of the star. That is the eye should receive from this star at least 7 photons per second. Human eyepiece is about d = 6 mm in the very dark, so the limit of illumination is:

$$7\ \text{photons} / \pi d^2/4 \approx 250\,000\ \text{photons/m}^2\ \text{(very roughly, of course)}$$

It is about 10 000 000 000 / 250 000 = 40 000 less than the illumination from 0^m star. In stellar magnitudes the difference is:

$$\Delta m = 2^m.5\ \lg 40\,000 \approx 11^m.5.$$

So the theoretical limit for human eye in the very dark is to see stars of $11^m.5$. Of course, it is quite far from reality. Nobody reported about possibility to see stars fainter than 8^m even in the absolute dark.

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β-3. Eris. Let us at first answer for the question about the "Great opposition". And what does it mean the word "Great"? The Great oppositions appear for the situation when the distance to opposition planet is smaller than during usual (mean) oppositions. In our case it is an opposition when the outer planet is near its perihelion. Now Eris is near its aphelion point. From the value of the semiaxis of the Eris' orbit it is evident that its period is of order of centuries and we may talk about the period of the Great oppositions as "years" (in comparison with "weeks" for the Great opposition of Mars) near after of half revolution of this dwarf planet around the Sun.
The period of Eris can be taken from the table (days should be recalculated into years)

$$T_{Ed} = 204852 \text{ days} / 365.25 \text{ days/year} = 560.85 \text{ years} \approx 560 \text{ years}$$

or found from the the III Kepler law

$$(T_{Ed}/T_E)^2 = (A_{Ed}/A_E)^3,$$

(here and below A means the major semiaxis, T is period, e is eccentricity, α is albedo, D are diameters of the bodies, R are distances, and indexes E, Ed, Sa and S correspond to Earth, Eris, Saturn and Sun).

$$T_{Ed} = T_E \cdot (A_{Ed}/A_E)^{3/2} = 1 \cdot (68.01)^{3/2} \text{ years} \approx 560 \text{ years}.$$

So the years of the "Great oppositions of Eris" will be after $T_{Ed} / 2 \approx 280$ years.

For the questions of the stellar magnitude there may be a few ways to solve this problem. The main two of them is to compare Eris with some other planet (data can be given from the table) or to calculate it directly using formulae of the total solar irradiation.

Let us use the first one and compare Eris and Saturn.
The following aspects should be taken into account:

1. $\mathbf{R_{S\text{-}>}}$. Distance from the Sun. The intensity of light coming to the bodies is reverse proportional to the square of these distances.
2. **D**. Size (diameter) of the body. The intensity of light reflecting by the body is proportional to the to the square of these sizes.
3. **α**. Albedo. The intensity of light reflecting by the body is proportional to the albedos.
4. $\mathbf{R_{E\text{-}>}}$. Distance from the observer (i.e. from the Earth): 1.8 a.u. for the asteroid of the main belt (in oppositions) and about 40 a.u. for asteroid of the Kuiper belt. The intensity of light coming to the telescope is reverse proportional to the square of these distances.

The flux to Earth from Eris $\quad F_{Ed} \sim \alpha_{Ed} \cdot D_{Ed}^2 \cdot (1/R_{E\text{-}Ed})^2 \cdot (1/R_{S\text{-}Ed})^2.$

The flux to Earth from Saturn $\quad F_{Sa} \sim \alpha_{Sa} \cdot D_{Sa}^2 \cdot (1/R_{E\text{-}Sa})^2 \cdot (1/R_{S\text{-}Sa})^2.$

The ratio of fluxes $\quad F_{Ed}/F_{Sa} = (\alpha_{Ed}/\alpha_{Sa}) \cdot (D_{Ed}^2/D_{Sa}^2) \cdot (R_{E\text{-}Sa} \cdot R_{S\text{-}Sa})^2/(R_{E\text{-}Ed} \cdot R_{S\text{-}Ed})^2.$

For the current situation the following distances should be used:
Since Eris is near its aphelion, $R_{S\text{-}Ed} = (1+e)A_{Ed}$, and $R_{E\text{-}Ed} = (1+e_{Ed})A_{Ed} \pm A_E$, but, since we do not know the real configuration of Earth, Sun and Eris (including we do not know the inclination of the Eris' orbit), we should use the average value of $R_{E\text{-}Ed} = (1+e_{Ed})A_{Ed}$ as well.
The stellar magnitude for Saturn is given for the mean opposition, so we should not use eccentricity here, $R_{S\text{-}Sa} = A_{Sa}$ and $R_{E\text{-}Sa} = A_{Sa} - A_E$.

$$F_{Ed}/F_{Sa} = (0.86/0.68) \cdot (2600/120536)^2 \cdot (8.584 \cdot 9.584)^2/(1.434 \cdot 68.01)^4.$$

$$F_{Ed}/F_{Sa} \approx 4.4 \cdot 10^{-8}.$$

So the difference in magnitudes between the current visible Eris and Saturn in mean opposition is

$$\Delta m = -2^m.5 \cdot \lg(F_{Ed}/F_{Sa}) \approx 18^m.4,$$

and the magnitude of Eris

$$m_{Ed} = m_{Sa} + \Delta m = 0^m.7 + 18^m.4 \approx 19^m.1.$$

The real values that can be found in reports of Eris observations are $18^m.7 \div 18^m.8$.

$$F_{Ed}/F_{Sa} \approx 1.91\cdot10^{-6}.$$

$$\Delta m = -2^m.5\cdot\lg(F_{Ed}/F_{Sa}) \approx 14^m.3,$$

and the magnitude of Eris in the "Great opposition" is

$$m_{Ed} = m_{Sa} + \Delta m = 0^m.7 + 14^m.3 \approx 15^m.0.$$

And we should repeat that this way with comparing the Eris and Saturn it is not the only possible correct way for solution.

αβ-4. Catastrophe. At first we should realise that "suddenly decreasing of solar mass to half its original value" is a hypothetic process and many physical laws of conservation cannot be used to compare parameters before and after since the system is not closed. Since no any other changing done, we should assume that at the moment of the mass decreasing other parameters of the Sun and the Earth have not changed: Positions of the Sun and the Earth, velocity of the Earth. Taking into account this postulate, a first approximation, if mass of a central body is reduced to half its original value, the circular speed becomes parabolic one. That is the Earth will move on a parabolic orbit and will never return. The answer to the problem is meaningless (the period is equal to infinity).

Nevertheless, it is known, that on July 5 the Earth is near the aphelion of its orbit. Its speed in this case is less than circular. Thus, in this case if mass of a central body to reduce twice, the speed of the Earth already is less than parabolic, the Earth will be rotate around the Sun on a prolate elliptic orbit.

For the beginning let us find a relation between the speed of the Earth in aphelions V_{aph} and circular speed V_0 for a motion on a circular orbit with the same semi-axis.

Using the II Kepler law one may write

$$V_{per}\cdot R_{per} = V_{aph}\cdot R_{aph},$$

using the law of conservation of energy

$$V^2_{per}/2 - GM/R_{per} = V^2_{aph}/2 - GM/R_{aph},$$

and also taking into account, that

$$R_{per} = a_0\,(1-e),\ R_{aph} = a_0\,(1+e),\ GM = V_0^2\cdot R_0\,,$$

it is possible to find

$$V_{per} = V_0\cdot\{(1+e)/(1-e)\}^{1/2}\,,$$

$$V_{aph} = V_0\cdot\{(1-e)/(1+e)\}^{1/2}\,.$$

The orbit parameters of the Earth after that the mass of Sun has decreased to half its present value are designate as:

a_N – semi-axis of an orbit,

T_N – period of revolution,

e_N – eccentricity,

V_N – speed for a motion on a circular orbit with a semi-axis a_N.

Let us write three additional equations:

distance R_{aph}, former distance in aphlion, became now the distance in perihelion

$$a_0\,(1+e) = R_{aph} = R_{per\text{-}N} = a_N\cdot(1-e_N),$$

speed V_{aph}, former speed in aphlion, will now become the speed in perihelion

$$V_{aph} = V_{per\text{-}N} = V_N\cdot\{(1+e_N)/(1-e_N)\}^{1/2},$$

and the relation

$$V_N = 2\pi a_N / T_N\,.$$

By solving all equations together it is possible to receive a beautiful expression

$$a_N = a_0(1+e)/2e.$$

According to the general III Kepler law

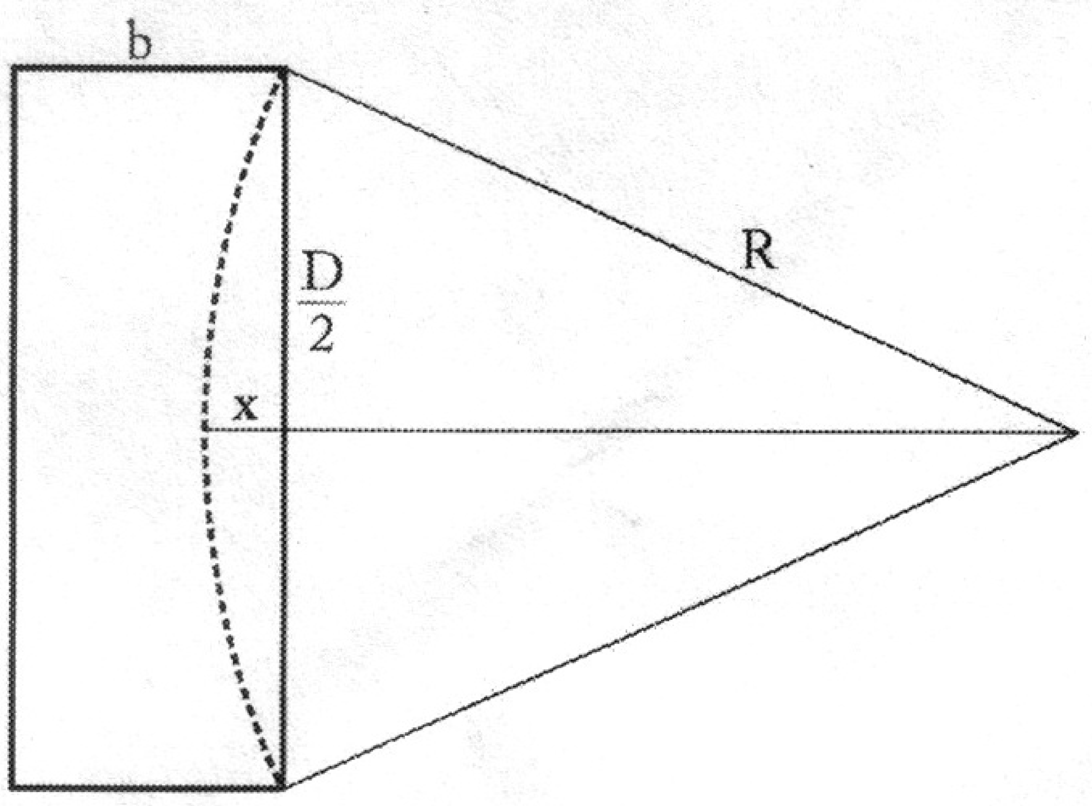
$$T_N = T_0\cdot 2^{1/2}\cdot(a_N/a_0)^{3/2},$$

one can find

$$T_N = T_0\cdot 2^{1/2}\cdot((1+e)/2e)^{3/2},$$

$$T_N = 2^{1/2} \cdot (1.017/0.034)^{3/2} \approx 230 \text{ years.}$$

α-5. Mirror for a telescope. It is evident that the depth '**x**' in the centre of the disc must be less than b = 40 mm. Let R be the curvature radius of the mirror. It means (from the Pythagorean theorem) that the minimum R is defined by the equation:



$$R^2 = D^2/4 + (R-b)^2$$

$$2Rb = b^2 + D^2/4$$

$$R = D^2/8b + b/2$$

The relation between the focal length F and the curvature radius of the mirrow is R = 2F, i.e. for zero thickness in the centre of the disc

$$F = R/2 = D^2/16b + b/4$$

$$F = 500^2/(16 \cdot 40) + 40/4 \approx 401 \text{ mm}$$

So, theoretically the focal length may be in the interval from 401 mm to infinity. Nevertheless, the thickness in the centre of the disc cannot be zero, let us take it at least 2 mm. In this case we should use b* = 38 mm instead b = 40 mm

$$F = R/2 = D^2/16b^* + b^*/4$$

$$F = 500^2/(16 \cdot 38) + 38/4 \approx 421 \text{ mm}$$

The focal length may be in the interval from about 420 mm to infinity.

β-5. Galaxy pair.

5.1. In April and September, the right ascension of the sun is about 0.5-2.5 h and 10.5-12.5 h respectively. And the right ascension of the galaxy is RA ≈ 147° ~ 10h. Thus, April is the better choice because in September the visual position of the galaxies is too close to the sun. Answer: **Apr.**

~ 1 p

According to the DECs of the two galaxies, it's easy to find that the upper one is IC564.

~ 0.5 p

5.2. The wavelength of Hα emission line measured in figure 2 is approximately 6700 Å.

~ 1 p

Taking into account (see table of constants) that the laboratory wavelength of Hα emission is 6563 Å, the redshift of the galaxy is:

$$z = (\lambda - \lambda_0)/\lambda_0 = (6700-6563)/6563 = 0.02$$

~ 1 p

5.3. Since z << 1, the recession velocity of the galaxy is:

$$v = cz = 6000 \text{ km/s}$$

~ 1 p

We should choose the filter whose peak value of redshift is close to 6000 km/s. The suitable one is № C4.

~ 1 p

5.4. According to Hubble's law, the distance to the galaxies is:

$$r = v/H_0 = 6000 \text{ km/s} / 70 \text{ (km/s)/Mpc} \approx 85 \text{ Mpc, i.e. } 85\,000\,000 \text{ parsec.}$$

~ 1 p

The two galaxies are close to each other and the curvature of the celestial sphere can be neglected. The angular distance of the centers of the galaxies is:

$$\alpha = (\Delta RA^2 + \Delta DEC^2)^{0.5}$$

Where, ΔRA = (RA1-RA2) × cos(DEC) = 0.00303°, ΔDEC = (DEC1-DEC2) = 0.02579°. As a result,

$$\alpha = 0.02597° = 93.5''.$$

~ 1 p

The distance between the two galaxies is:

$$d = r \cdot \alpha = 85\,000\,000 \cdot 93.5 / 3600 / 57.3 = 38.5 \text{ kpc}$$

~ 0.5 p