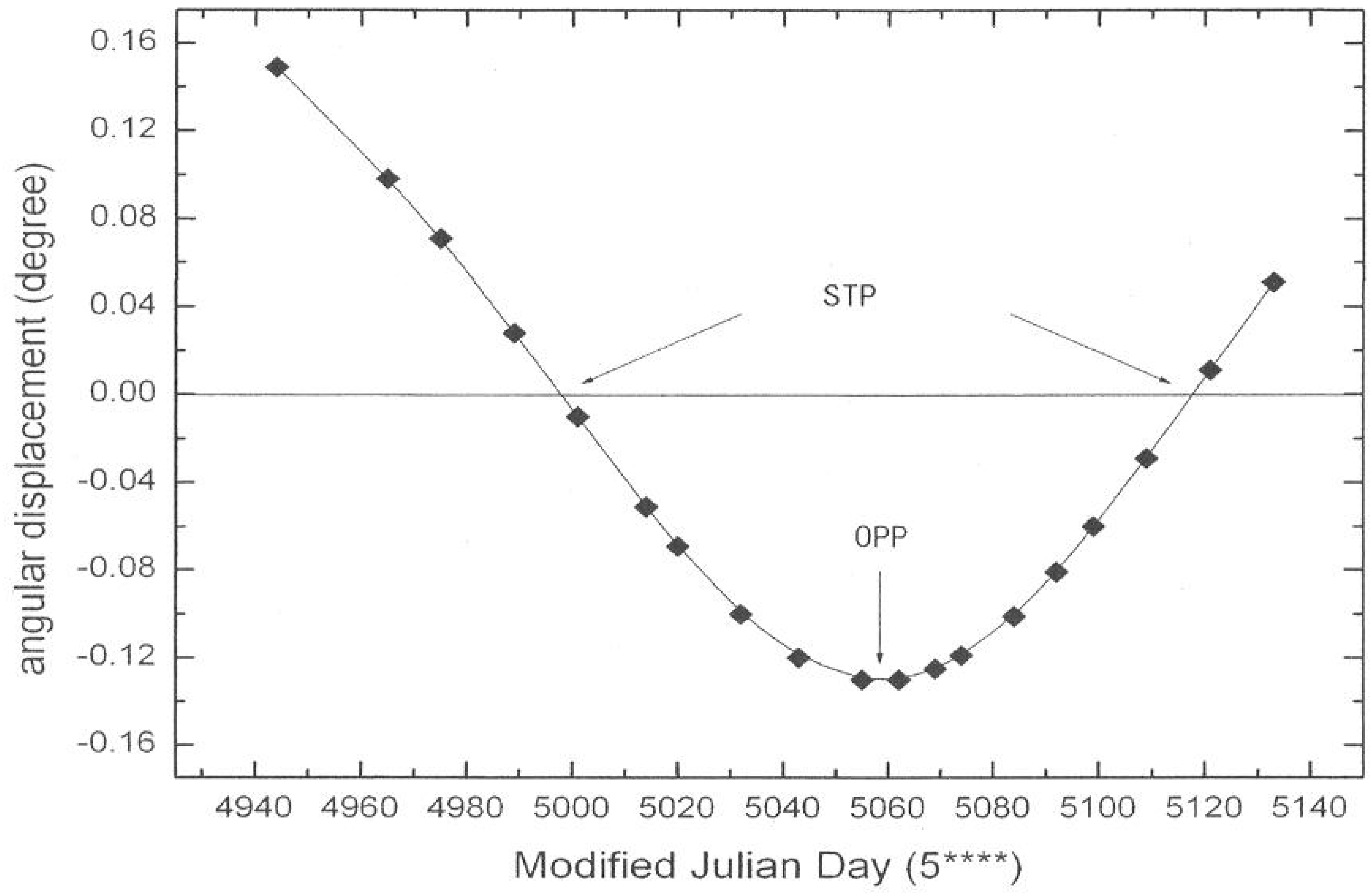
2009 IAO, Practical Round, Sketches for Solution

6. (Group α) Motion of a Planet.

1) The plotted figure is shown as below.



~2 p

At stationary points, the planet's angular displacement is zero. It's easy to find the two days of stationary points: MJD 54998 and MJD 55118.

~2 p

From one stationary point to another, the opposition of the planet happens when the angular displacement of the planet relative to the background stars peaks. The date can be read out through figure 1 : MJD 55058, i.e. the lowest point of figure 1.

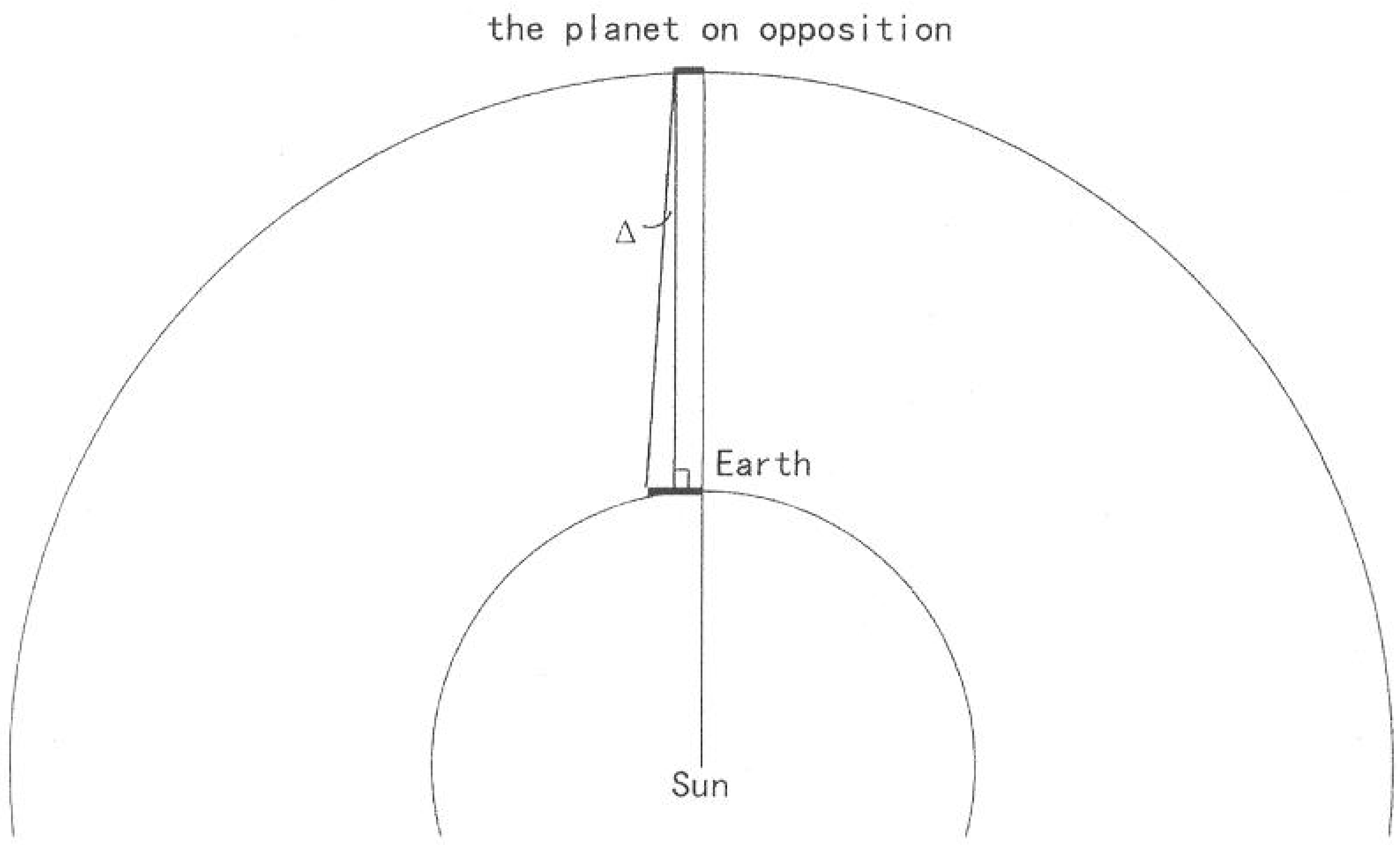
~2 p

2) The angular displacement on MJD 55058 is about Δ=-0.132°(±0.002°).

~1 p

MJD of STP	MJD of OPP	ADRS on the day of OPP
54998±2 & 55118±2	55058±2	-0.132°±0.002°

3)



As shown in the figure above, when the planet is on opposition :

$$\frac{v_E - v_p}{a_p - a_E} \times t = \tan\Delta \qquad (1)$$

where v and a are the orbital speed and radius, t is a short time interval from the moment of opposition. If we set the value of t to one day, since Δ=0.132°=0.002304<<1 (the value of Δ is found from figure 1), (1) can be rewritten as:

$$v_E - v_p = (a_p - a_E)\cdot \Delta / t \qquad (2)$$

According to Kepler's Law :

$$a^3{}_E / T^2{}_E = a^3{}_p / T^2{}_p$$

$$\text{and } v = 2\pi \cdot a / T \;\; => \;\; a_p / a_E = v^2{}_E / v^2{}_p \qquad (3)$$

Substitute the value of a_E=1 AU into (2) and (3), we can find that:

$$(v_p / v_E + 1)\cdot \Delta = (v_p / v_E)^2 \cdot v_E$$

where Δ=0.002304, v_E=0.0172 AU/day. Thus, v_p=0.439v_E, a_p=5.2 AU (±0.2AU).

~ 3 p

6. (group β) Light Echo.

1) The intrinsic color of the stars is B_0-V_0 = $(B$-$A_B)$ - $(V$-$A_V)$, and the M_V can be obtained through the H-R diagram.

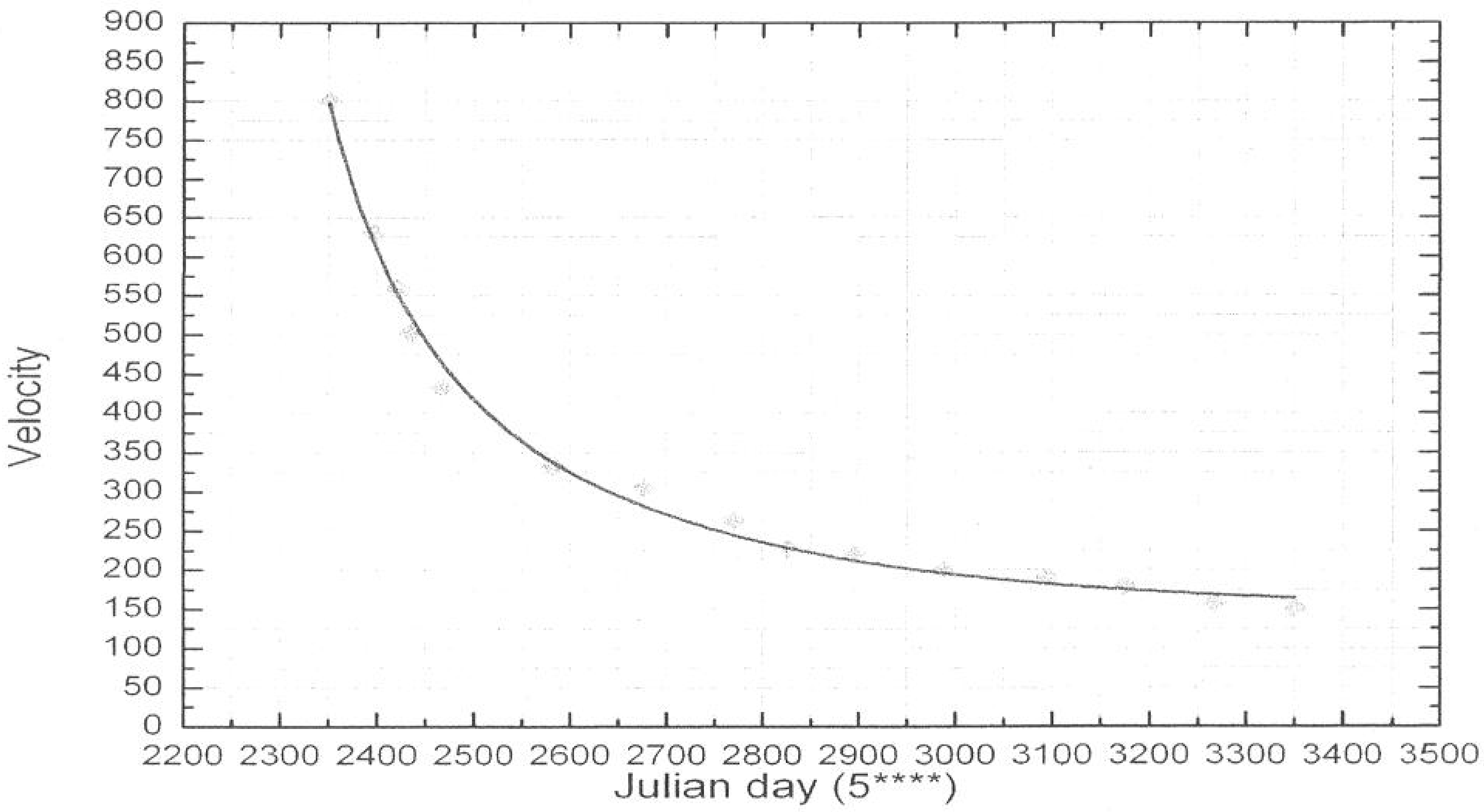
Number	Spectral type	V	B	$(B\text{-}V)_0$	V_0	M_v
N1	B6 V	16.02	16.73	-0.15	13.42	-0.5±0.2
N2	B4 V	15.00	15.63	-0.21	12.40	-1.1±0.2
N3	B3 V	14.79	15.41	-0.22	12.19	-1.3±0.2

~ 1 p

According to m-M=5lg$r$-5, we can get the distance modules of the three stars. And the distance of the cluster is the mean value of these stars, i.e. d≈18 kly (±2 kly).

~ 2 p

2) This question is a hint to remind the students that the shells in figure 1 are not envelopes of V838 Mon but interstellar dusts that brightened by the outburst of V838 Mon.



~ 2 p

The graph is plotted here. The black solid line is a fitted curve through the data in table 2.

The area of the curve in figure 1 indicates the distance that the envelope expanded. The area of a unit rectangle is:

$$A = 25 \text{ km/s} * 50 \text{ day} = 1.08*10^8 \text{ km} = 0.72 \text{ AU}$$

The julian day of May 20, 2002 and Sep. 2, 2002 is 52414 and 52519. It can be counted in the figure that on May 20, 2002 and Sep. 2, 2002, the envelope expands about 34A(25 AU) and 78A(56 AU). Thus, the angular diameters of the envelopes are respectively:

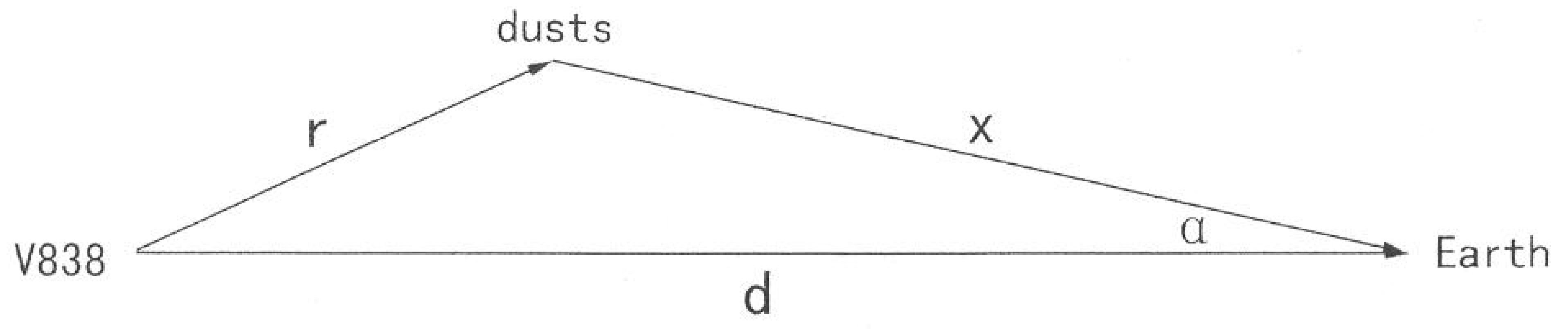
$$\beta_1 = 2*25 \text{ AU}/18000 \text{ ly} \approx 0.01''$$

$$\beta_2 = 2*56 \text{ AU}/18000 \text{ ly} \approx 0.02''$$

~ 2 p

3) Let "0" be the time of the observation of the original outburst, i.e. March 09, 2002, and t be the time

when the outermost light echos were observed. The light echos delayed because of the speed of light is limited, as displayed by the following figure.



From the figure, we get:

$$ct+d=r+x \qquad (1)$$
$$r^2=x^2+d^2-2xd\cdot\cos\alpha \qquad (2)$$

Then we can find:

$$r=\frac{d(d+ct)(1-\cos\alpha)+c^2t^2/2}{d(1-\cos\alpha)+ct}$$

where α is the visual radius of the outermost dusts shell and can be measured from the HST figure, c is the speed of light, d is the distance of the star. $2\alpha_1=(21.9-12.0)/22.4\times97''=42.9''$, $2\alpha_2=(59.1-45.1)/22.4\times97''=60.6''$. The final result is listed in the following table.

time (days from 3/9/2002)	Diameter of stellar envelope ($''$)	Visual diameter of dust shell($''$)	Radius of dust shell (ly)
72 (0.1971 year)	0.01	40 (±5)	9.0
177 (0.4846 year)	0.02	60 (±5)	7.5

~3 p

7. (Group α & β) The Vernal Equinox Day of Saturn.

1) On the vernal equinox day, the sun-observer will see a 0° inclination angle of the ring of Saturn. It can be found that the day is Julian day 2455055, and the corresponding date is August 11, 2009.

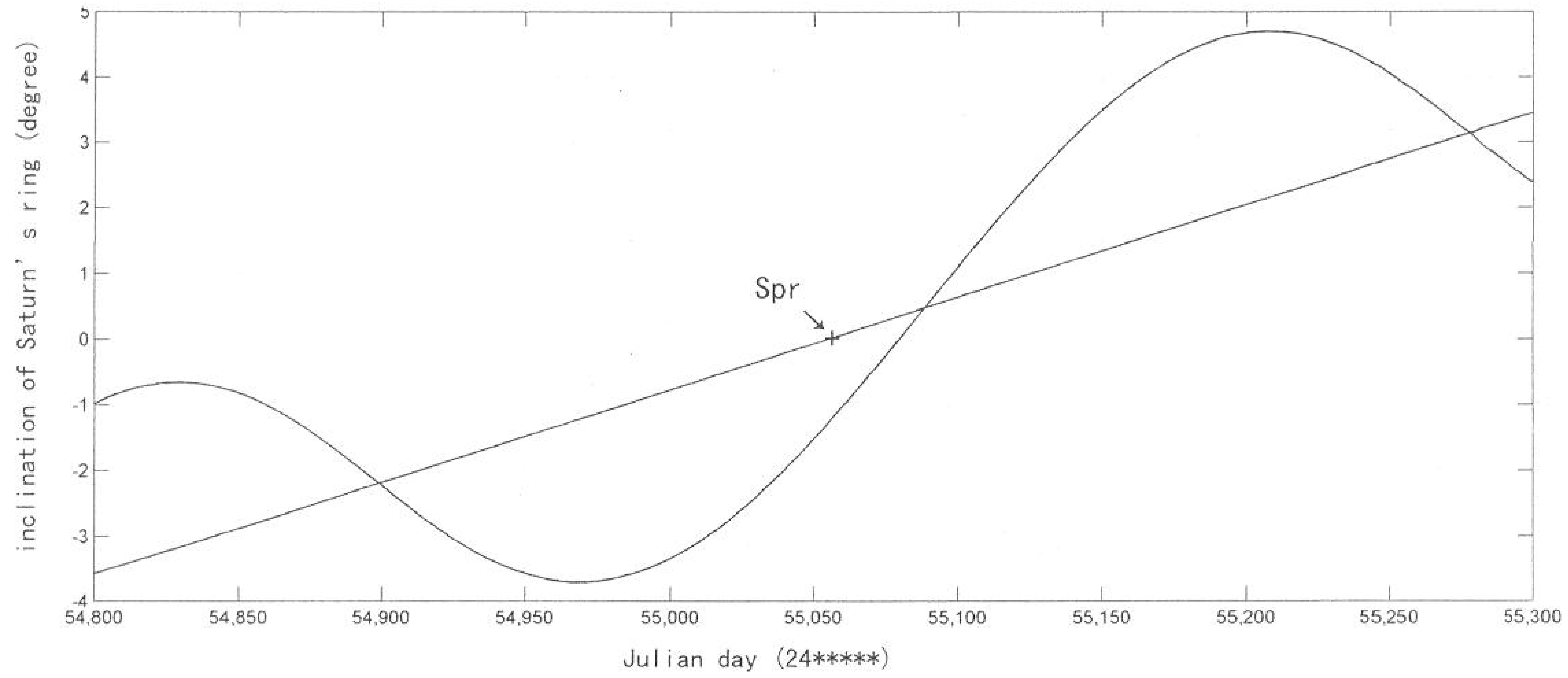


figure 1

group α ~2 p
group β ~1 p

2) From figure1, the maximum difference of the observed inclination angle of Saturn's ring between Beijing Planetarium and the sun-observer can be measured and its value is about 2.7 °. The maximum inclination angle of Saturn's ring observed by the sun-observer is equal to the inclination angle of its equator, ie, 26.7° (when Saturn is on solstices).

Thus the maximum inclination angle of Saturn's ring observed by Beijing Planetarium is about:

$$\gamma=26.7°+2.7°=29.4°.$$

~2 p

Note: In fact, when Saturn is on solstices, the Earth is not exactly at the right position that makes the difference maximum in the past decades. Actually, the maximum angle observed is about 28 °.

3) The angle that Saturn revolute from Jan 13, 2005 to Aug 11, 2009 is:

$$\theta = (2455055-2453383)*360°/T = 56°$$

The angle that Earth revolute from Jan 13 to Aug 11 is about:

$$\lambda = 210*360°/365 = 207°$$

Thus, on Aug 11, the angle of Earth-Sun-Saturn is about 207°- 56°≈150°.

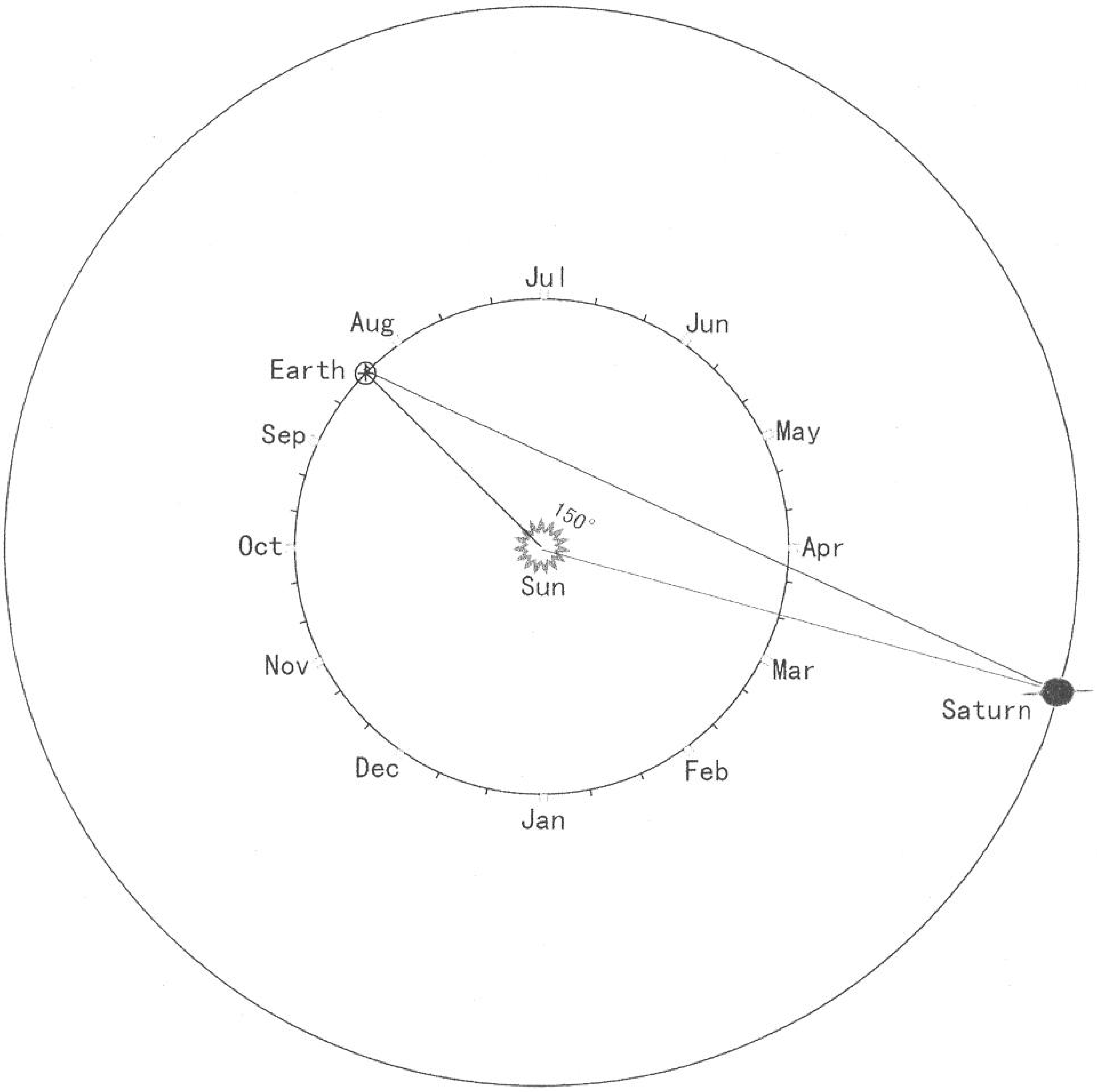


figure 2

group α ~4 p
group β ~2 p

4) On the vernal equinox day of Saturn, the distance between Saturn and Earth can be derived from the triangle Earth-Sun-Saturn in figure 2:

$$E\_Sa^2 = E\_Su^2 + Su\_Sa^2 - 2E\_Su*Su\_Sa*\cos 150°,\ E\_Sa = 10.4\ AU.$$

~2 p

5) (group β) For the sun-observer, the inclination angle of the ring of Saturn (noted as η) varies with time and follows the law below:

$$\sin\eta = \sin\theta' \times \sin\alpha$$

Where θ' is the angle that Saturn revolute since its vernal equinox and α is 26.7°.

Ignoring the inclination of Saturn's orbit, for Beijing Planetarium, the inclination angle of the ring of Saturn (noted as η') changes following the law below (see figure 3):

$$\sin\eta' = \sin(\theta' + \Delta) \times \sin\alpha$$

where Δ is the angular distance between Earth and Sun as seen on Saturn. At the same moment, when Δ is maximum, the difference of the inclination angle of Saturn's ring between the earth-observer and the sun-observer reaches maximum. It's easy to find that $\Delta_{max}=6°$.

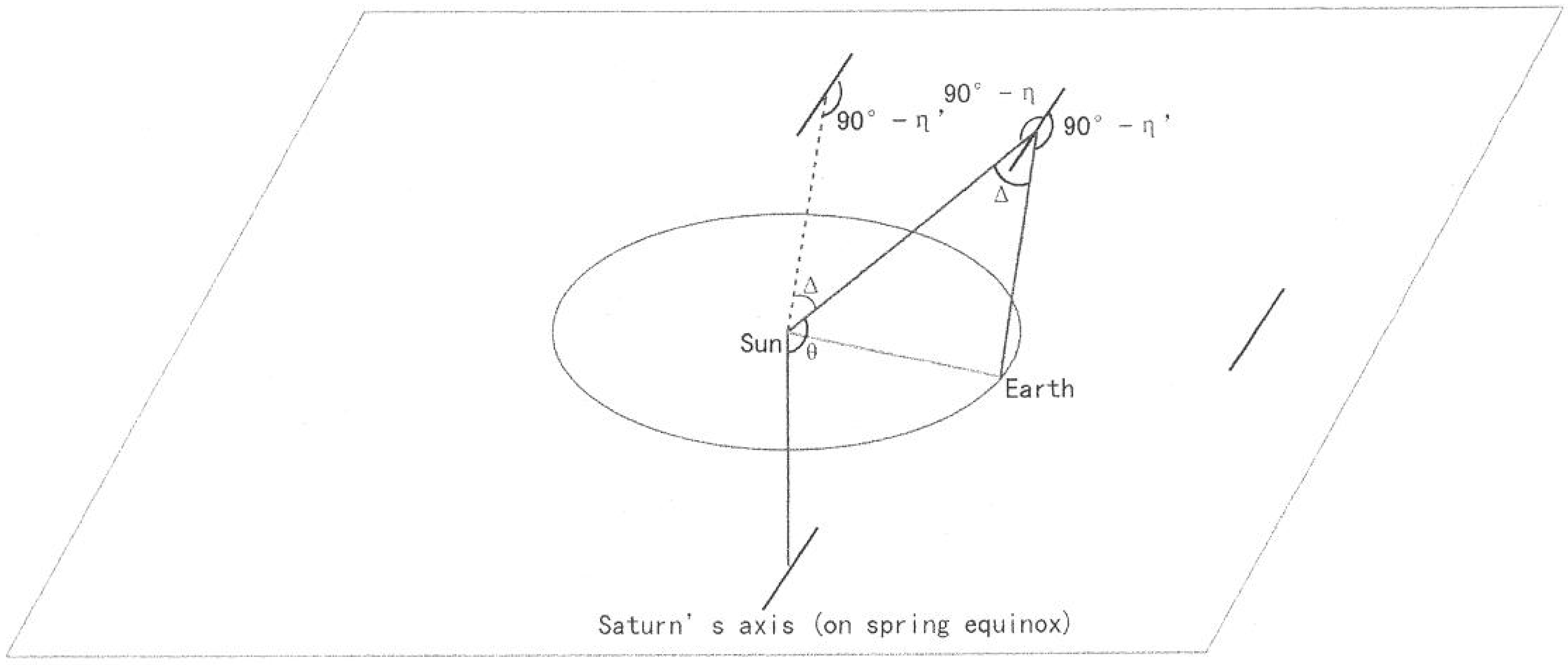


figure 3

Now we need to find the value $|\eta'-\eta|_{max}$. According to symmetry, it's enough to consider the situation that θ ~ [0, 1/2pi].

On the vernal equinox day of Saturn: $\theta = 0°$, $\eta = 0°$, $\eta' = 2.7°$, $|\eta'-\eta| = 2.7°$;

On the summer solstice day of Saturn: $\theta = 90°$, $\eta = \alpha = 26.7°$, $\eta' = 26.56°$, $|\eta'-\eta| = 0.14°$. It can be deduced that $(\eta'-\eta)$ decreases monotonously from vernal equinox to around summer solstice. Thus $|\eta'-\eta|_{max}$ =2.7 °, and it happens when Saturn is on vernal equinox and the angular distance between Earth and the Sun seen on Saturn reaches maximum.

group β ~3 p

2009 IAO, Observational Round, Sketches for Solution

Group α & β

8.

8.1 ~4 p

8.2 ~3 p